COSMOGRAPHIA,

OR

A VIEW

OF THE

Terrestrial and Cœlestial

GLOBES,

IN A

Brief Explanation

OF THE

PRINCIPLES

Of plain and solid

GEOMETRY,

Applied to Surveying and Gauging of CASK.

The Doctrine of the Primum Mobile.

With an Account of the Julian & Gregorian Calendars, and the Computation of the Places of of the Sun, Moon, and Fixed Stars, from such Decimal Tables of their Middle Motion, as supposeth the whole Circle to be divided into an hundred Degrees or Parts.

To which is added an Introduction unto GEOGRAPHY.

By John Newton, D. D.

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Fellowes

TO THE

Moſt Honourable

HENRY SOMERSET,

Lord Herbert, *Baron of* Chepſtow, Raglan, *and* Gower, *Earl and Marqueſs of* Worceſter, *Lord Preſident and Lord Lieutenant of* Wales *and the* Marches, *Lord Lieutenant of* Glouceſter, Hereford *and* Monmouth, *and of the City and County of* Briſtol, *Knight of the Moſt Noble Order of the Garter, and one of His Majeſtie's Moſt Honourable Privy Council.*

HE that adventures upon any thing contrary to the General received practice, what ever his own courage and reſolutions are, had need to be ſupported, not only by the moſt Wiſe and Honourable, but alſo

the moſt Powerful Perſons that are in a Nation or Kingdom; For let the Propoſals be never ſo advantagious to the Publick, they ſhall not only be decried and neglected, but it is well, if the Promoter be not both abuſed and ruined: Yet I, notwithſtanding all theſe diſcouragements, have not been ſilent, but in order to Childrens better Education, have long ſince publiſhed my thoughts, *and have and do declare, that the multitude of Schools for the learning of the Latine and Greek Tongues, are deſtructive both to our youth and the Commonwealth*; and if the Opinion of Sir *Francis Bacon* in his Advice to King *James* concerning *Sutton*'s Hoſpital, be not ſufficient to warrant my Aſſertion, I could heartily wiſh that no ſuch Evidence could have been produced, as the late unhappy Wars, in the Bowels of this Kingdom hath afforded us; for what he ſaith there by way of Advice, we by woful Experience have found too true; that by reaſon of the multitude of Grammar Schools, more Scholars are dayly brought up, than all the Preferments in this Nation can provide for, and ſo they become uncapable of other Profeſſions, and unprofitable in their own, and at laſt become, *materia rerum novarum*; whether this be an eſſential or an accidental Effect.

I will

I will not here diſpute; the truth of it, I am ſure, cannot be denied: but that is not all; by this means it comes to paſs, that four of the ſeven Liberal Arts, are almoſt wholly neglected, as well in both Univerſities, as in all Inferiour Schools; and ſetting aſide the City of *London*, there are but few Places in this Nation, where a man can put his Son, to be well inſtructed in *Arithmetick*, *Geometry*, *Muſick* and *Aſtronomy*; and even that Famous City was without a Publick School for Mathematical Learning, till His preſent Majeſty was pleaſed to lay the Foundation; nay ſo averſe are men in the general to theſe Arts (which are the ſupport of all Trade) that without a high hand, it will be almoſt impoſſible, to make this People wiſe for their own good; I come therefore to your Honour, humbly to beg your Countenance and Aſſiſtance, that the Stream of Learning may be a little diverted, in thoſe Schools that are already erected, and to be inſtrumental for the erecting more, when they ſhall be wanting; that we may not be permitted ſtill to begin at the wrong end; but that according to the practice of the Ancient Philoſophers, Children may be inſtructed in *Arithmetick*, *Geometry*, *Muſick* and *Aſtronomy*; before the Latine and Greek

Grammars are thought on, these Arts in themselves, are much more easie to be learned, tend more to a general good, and will in a great measure facilitate the Learning of the Tongues, to as many as shall after this Foundation laid, be continued at School, and provided for in either Universities. Your Honour was instrumental to enlarge the Maintenance for God's Minister in the Place where I live, and perhaps it may please God to make you so, not only in making this Place in particular, but many other Places in this Land happy, by procuring Schools for these Sciences, and not only so, but by your Loyal and Prudent managing the several Trusts committed to you, you may do much for God's Glory, your Countries Good, and the continuance of your own Honour to all Future Generations, which is, and shall be the Prayer of,

Your Honour's Obliged and

Devoted Servant,

JOHN NEWTON.

TO THE READER.

MY Design in publishing these *Introductions to* Geometry *and* Astronomy, *is so well known by all the Epistles, to my other Treatises of* Grammar, Arithmetick, Rhetorick, *and* Logick, *that I think it needless to tell thee here, that it is my Opinion, that all the Arts should be taught our Children in the English Tongue, before they begin to learn the Greek or Latin Grammar, by which means many thousands of Children would be fitted for all Trades, enabled to earn their own Livings, and made useful in the Commonwealth; and that before they attain to twelve years of age; and by consequence the swarming of Bees would be prevented, who being compelled to leave their Hives, for want of room, do spread themselves abroad, and instead of gathering of Honey, do sting all that come in their way. We should not have such innumerable company of Gown-men to the loss and prejudice of themselves and the Commonwealth, and those we had would probably be more learned, and better regarded.*

His

His Majesty being pleased to begin this Work, by His Bounty towards a Mathematical School in Christ's Church *London; I am not now without hopes, to see the same effected in many other Places in this Kingdom; and to this purpose I have to my Introductions to the other Arts, added these also to* Geometry *and* Astronomy; *which I call by the name of* Cosmographia; *and this I have divided into four Parts; in the first I have briefly laid down the first Principles belonging to the three kinds of Magnitude or continued Quantity, Lines, Planes and Solids; which ought in some measure to be known, before we enter upon* Astronomy, *and this part I call an Introduction unto* Geometry.

The second and third Parts treat of Astronomy; *the first of which sheweth the Doctrine of the* Primum Mobile, *that is, the Declination, Right Ascension, and Oblique Ascensions of the Sun and Stars, and such other Problems, as do depend upon the Doctrine of Spherical Triangles.*

The second Part of Astronomy, *treateth of the motion of the Sun, Moon and Fixed Stars; in order whereunto, I have first given thee a brief account of the Civil Year, with the cause of the difference between our* Julian *and* Gregorian Calendar, *and of both from the true; for it must be acknowledged*

that

that both are erroneous, though one be the worse of the two; yet not so bad, but that our Dissenting Brethren have I hope some better Arguments to justifie their Non-conformity, than what I see published in a little Book without any name to it, concerning two Easters in one Year; by the General Table, saith this learned man, who owneth the Feast of Easter was to be observed Anno 1674. *upon the 19 day of* April, *so the Almanacks for that Year, as well as the General Table set before the Book of Common Prayer, but by the Rule in the said Book of Common Prayer given, the Feast of Easter should have been upon the twelfth of* April, for Easter-Day must always be the first Sunday after the first Full Moon, which happeneth next after the one and twentieth day of *March*, and if the Full Moon happen upon a Sunday, Easter-Day is the Sunday after; *Now in the Year* 1674. *the* 19 *of* April *being Friday was Full Moon, therefore by this Rule, Easter-Day should be the twelfth, and by the Table and the Common Almanacks* April *the tenth, but this learned man must know, that the mistake is in himself, and not in the Rule or Table set down in the Book of Common Prayer; for if he please to look into the Calendar, he will find that the Golden Number Three, (which was the Golden Number for that Year) is*

placed

placed against the last day of March, and therefore according to the supposed motion of the Moon, that Day was New Moon; and then the Full Moon will fall upon the fourteenth day of April, and not upon the tenth, and so by consequence the Sunday following the first Full Moon after the 21 day of March was the nineteenth of April and not the twelfth. And thus the Rule and the Table in the Book of Common Prayer for finding the Feast of Easter are reconciled; and when Authority shall think fit, the Calendar may be corrected and all the moveable Feasts be observed upon the days and times at first appointed; but till that be, a greater difference than one Week will be found in the Feast of Easter between the Observation thereof according to the Moons true motion, and that upon which the Tables are grounded; for by the Fathers of the Nicene Council it was appointed, that the Feast of Easter should be observed upon the Sunday following the first Full Moon after the Vernal Equinox, which then indeed was the 21 of March; but now the tenth, and in the Year 1674. Wednesday the 11 of March was Full Moon, and therefore by this Rule, Easter-Day should have been upon March the fifteenth, whereas according to the Rules we go by, it was not till April the nineteenth.

The

The Tables of the Sun and Moons middle motions are neither made according to the usual Sexagenary Forms, nor according to the usual Degrees of a Circle and Decimal Parts, but according to a Circle divided into 100 Degrees and Parts, and this I thought good to do, to give the World a taste of the excellency of Decimal Numbers, which if a Canon of Sines and Tangents were fitted to it, would be found much better, as to the computing the Places of the Planets; but as to the Primum Mobile, *by reason of the general dividing a Circle into 360 Degrees, I should think such a Canon with the Decimal Parts most convenient, and in some cases the common Sexagenary Canon may be very useful, and indeed should wish and shall endeavour to have all printed together, one Table of Logarithms will serve them all, and two such Canons, one for the Study and another for the Pocket, would be sufficient for all Mathematical Books in that kind; and then men may use them all or either of them as they shall have occasion, or as every one is perswaded in his own mind.*

What I have done in this particular, as it was for mine own satisfaction, so I am apt to believe, that it will be pleasing to many others; and although I shall leave every one to abound in his own sense, yet I cannot think

that

that Custom should be such a Tyrant, as to force us always to use the Sexagenary form, if so, I wonder that men did not always use the natural Canon; if no alteration may be admitted, what reason can be given for the use of Logarithms; and if that be found more ready than the natural, in things of this kind, where none but particular Students are concerned, I should think it reasonable, to reduce all things hereafter, into that form, which shall be found most ready and exact; now the Part Proportional in the Artificial Sines and Tangents in the three first Degrees cannot be well taken by the common difference, and the way of finding them otherwise will not be so easie in the Sexagenary Canon, as in either of the other, and this me thinks, should render that Canon which divides each Degree into 100 Parts more acceptable; but thus to retain the use of Sines, Degrees, and Decimal Parts, doth not to me seem convenient, and to reckon up, a Planets middle motion, by whole Circles will sometimes cause a Division of Degrees by 60, which hath some trouble in it also, but if a Circle be divided into 100 Degrees, this inconvenience is avoided, and were there no other reason to be given, this me thinks should make such a Canon to be desirable; but till I can find an opportunity of publishing such an one, I shall forbear to

ſhew any further uſes of it, and for what is wanting here in this ſubject, I therefore refer thee to Mr. Street's Aſtronomia Caroli-na, *and the ſeveral Books written in Engliſh by Mr.* Wing.

The fourth Part of this Treatiſe is an Introduction unto Geography, *in which I have given general Directions, for the underſtanding how the habitable part of the World is divided in reſpect of Longitude and Latitude in reſpect of Climes and Parallels with ſuch other Particulars as will be found uſeful unto ſuch as ſhall be willing to underſtand Hiſtory; in which three things are required; The time when, and this depends upon* Aſtrono-my; *the place where, and this depends upon* Geography; *and the Perſon by whom any memorable Act was done, and this muſt be had from the Hiſtorical narration thereof; and he that reads Hiſtory without ſome know-ledge in* Aſtronomy *and* Geography *will find himſelf at a loſs, and be able to give but a lame account of what he reads; but after the learning of theſe Arts of* Grammar, (*I mean ſo much thereof, as tends to the un-derſtanding of every ones Native Language*) Arithmetick, Geometry *and* Aſtronomy; *a Child may proceed profitably to* Rhetorick *and* Logick, *the reading of Hiſtory, and the learning of the Tongues; and ſure there is*

no

no ſtudious and ingenious man, but will ſtand in need of ſome Recreation; and therefore if Muſick *in the Worſhip and Service of God be not Argument enough to allow that a place among the Arts, let that poor end of Delight and Pleaſure be her Advocate; and although that all men have not Voyces, yet I can hardly believe, that he expects any Melodious Harmony in Heaven, that will not allow Inſtrumental Muſick a place on Earth; and as for thoſe that have Voyces, ſurely the time of learning Vocal Muſick, muſt be in Youth, and I am perſwaded that the Arts and Sciences to ſome good degree may be learned by Children before they be full twelve years old, and would our Grammar Maſters leave off their horrible ſeverity, and apply themſelves to ſuch ways of teaching Youth, as the World is not now unacquainted with, I am perſwaded that it is no difficult matter, in four years time more to fit Children in ſome good meaſure for the Univerſity.*

The great Obſtruction in this Work, is the general Ignorance of Teachers, who being unacquainted with this Learning, cannot teach others what they know not themſelves. I could propound a remedy for this, Sed Cynthius aurem vellit; *Therefore I will forbear and leave what I have written, to be peruſed and cenſured as thou ſhalt think fit.*

John Newton.

Practical Geometry

OR, THE ART of SURVEYING.

CHAP. I.

Of the Definition and Division of Geometry.

GEometry is a Science explaining the kinds and properties of continued quantity or magnitude.

2. There are three Kinds or Species of Magnitude or continued Quantity, Lines, Superficies and Solids.

3. A Line is a Magnitude consisting only of length without either breadth or thickness.

4. In a Line two things are to be considered, the Terms or Limits, and the several Kinds.

5. The term or limit of a Line is a Point.

6. A Point is an indivisible Sign in Magnitude which cannot be comprehended by sense, but must be conceived by the Mind.

7. The kinds of Lines are two, Right and Oblique.

8. A Right Line is that which lieth between his Points, without any going up or going down on either side. As the Line *AB* lieth streight and equally between the Points *A* and *B*. *Fig.* 1.

9. An Oblique Line is that which doth not lie equally between its Points, but goeth up and down sometimes on the one side and sometimes on the other. And this is either simple or various.

10. A simple Oblique Line, is that which is exactly Oblique, as the Arch of a Circle; of Various Oblique Lines there is but little use in Geometry.

11. Thus are Lines to be considered in themselves, they may be also considered as compared to one another, and that either in respect of their distances, or in respect of their meetings.

12. In respect of their distances, they may be either equally distant, or unequally.

13. Lines equally distant are two or more, which by an equal space are distant from one another, and these are called Parallels; and these though infinitely extended will never concur.

14. Lines unequally distant, are such as do more or less incline to one another, and these being extended will at last concur.

15. Concurring Lines are either perpendicular or not perpendicular.

16. A Perpendicular Line, is a Right Line falling directly upon another Right Line, not declining or inclining to one side more than another; as the Line *AB* in *Fig.* 1.

17. A Perpendicular Line is twofold, to wit, either falling exactly in the middle of another Line, or upon some other Point which is not the middle.

18. A Line exactly Perpendicular, may be drawn in the same manner, as any Right Line may

may be divided into two equal Parts; the which may thus be done. If from the two Terms or Points of the Right Line given, there shall be described two Arches crossing one another above and below, a Line drawn through the Intersections of those Arches, shall be exactly Perpendicular, and also divide the Right Line given into her equal Parts. *Fig.* 1.

For Example; Let C D *be the Right Line given, and let it be required, to bisect this Line, and to erect a Perpendicular in the middle thereof.* 1. *Then setting one of your Compasses in the Point* C, *draw the Arches* E *and* F. 2. *Setting one Foot of your Compasses in* D, *draw the Arches* G *and* H, *and from the Intersections of these Arches draw the Right Line* K L, *so shall the Right Line* K L *be Perpendicular to the Right Line* C D, *and the Right Line* C D *also divided into two equal Parts, in the Point* A.

19. A Line Perpendicular to any other Point than the middle is twofold: for it is either drawn from some Point given in the Line; or from some Point given without the Line.

20. From a Point given in the Line, a Perpendicular may thus be drawn. *In Fig.* 2. *Let the given Line be* C D, *and let it be required to draw a Perpendicular Line to the Point* C, *your Compasses being opened to any reasonable distance, set one Foot in the Point* C, *and the other in any place on either side the Line* C D, *suppose at* A, *then describe the Arch* E C F, *this done draw the Line* E A, *and where that Line being extended shall cut the Arch* E C F, *a Right Line drawn from* C *to that Intersection shall be Perpendicular to the Point* C *in the Line* C D, *as was required.*

21. From a Point given without the Line, a Perpendicular may be drawn in this manner.

In Fig. 2. Let the given Line be C D, *and let it be required to draw another Line Perpendicular thereunto, from the Point* F *without the Line. From the Point* F *draw a ſtreight Line to ſome part of the Line* C D *at pleaſure, as* F E, *which being biſected, the Point of Biſection will be* A, *if therefore at the diſtance of* A F, *you draw the Arch* E C F, *the Right Line* C F *ſhall be Perpendicular to the Line* C D, *as was required.*

22. Hitherto concerning a Perpendicular Line. A Right Line not Perpendicular, is a Right Line falling indirectly upon another Right Line, inclining thereto on the one ſide more, and on the other leſs.

23. Lines unequally diſtant, and at laſt concurring, do by their meeting make an Angle.

24. An Angle therefore is nothing elſe, then the place, where two Lines do meet or touch one another, and the two Lines which conſtitute the Angle, are in Geometry called the ſides of the Angle.

25. Every Angle is either *Heterogeneous*, or *Homogeneous*: that is called an *Heterogeneous* Angle, which is made by the meeting of one Right Line, and another that is Oblique and Crooked; and that is called an *Homogeneous* Angle, which is made by the meeting of two Lines of the ſame kind, that is, of two Right Lines, or of two curved or Circular Lines.

26. An *Homogeneous* Angle made of two curved or Circular Lines, is to be conſidered in Geometry as in Spherical Triangles, but the other which is made of Right Lines, is in all the Parts of Geometry of more frequent uſe.

27. Right lined Angles are either Right or Oblique.

28. A

28. A Right Angle is that whose legs or sides are Perpendicular to one another, making the comprehended space on both sides equal. *Thus in Fig.* 1. *the Line* A K *is Perpendicular to the Line* C D, *and the Angles* K A C *and* K A D, *are right and equal to one another.*

29. An Oblique Angle is that, whose sides are not Perpendicular to one another.

30. An Oblique Angle is either acute or obtuse.

31. An Acute Angle is that which is less than a Right.

32. An Obtuse Angle, is that which is greater than a Right. *Thus in Fig.* 1. *The Angle* B A C *is an Acute Angle because less than the Right Angle* C A K. *And the Angle* B A D *is an Obtuse Angle being greater than the Right Angle* D A K.

The Geometrical Propositions concerning Lines and Angles are very many, but these following we think sufficient for our present purpose.

Proposition I.

To divide a Right Line given into any Number of equal Parts.

Let it be required to divide the Right Line *A B* into five equal Parts. From the extream Points of the given Line *A* and *B*, let there be drawn two Parallel Lines, then from the Point *A* at any distance of the Compasses, set off as many equal Parts wanting one, as the given Line is to be divided into, which in our Example is four, and are noted thus, 1. 2. 3. 4. and from the Point *B* set off the like Parts in the Line *B C*, and let them be

noted likewise thus, 1. 2. 3. 4. then shall the Parallel Lines, 14. 23. 32. and 41. divide the Right Line *A B* into 5 equal Parts, as was required.

Proposition II.

Two Right Lines being given, to find a Mean proportional between them.

Let the two Right Lines given be *D B* and *C B*, which let be made into one Line as *C D*, which being bisected the Point of bisection is *A*, from which as from a Centre describe the Arch *C E D*, and from the Point *B* erect the Perpendicular *B E*, so shall *B E*, be the Mean proportional required; for, *B C. B E* :: *B E. B D.*

Proposition III.

Three Right Lines being given, to find a fourth proportional.

Let the three given Lines be *A B. B C.* and *A D. Fig.* 5. to which a fourth proportional is required: draw *A E* at any Acute Angle, to the Line *A D* in the Point *A*; and make *D E* parallel to *B C*, so shall *A E* be the fourth proportional required; for, *A B. B C* :: *A D. A E.*

Proposition IV.

Upon a Right Line given, to make a right-lined Angle, equal to an Angle given.

Let it be required upon the Line *C D* in *Fig.* 6.

to

to make an Angle, equal to the Angle *D A E* in *Fig.* 5. From the Point *A* as a Center, at any extent of the Compasses describe the Arch *B G*, between the sides of the Angle given, and with the same extent describe the Arch *H L* from the Point *D*, and then make *H L* equal to *B G*, then draw the Line *D L*, so shall the Angle *C D D* be equal to the Angle *D A E* given, as was required.

CHAP. II.

Of Figures in the general, more particularly of a Circle and the affections thereof.

Hitherto we have spoken of the first kind of Magnitude, that is, of Lines, as they are considered of themselves, or amongst themselves.

2. The second kind of Magnitude is that which is made of Lines, that is, a Figure consisting of breadth as well as length, and this is otherwise called a Superficies.

3. And in a Superficies there are three things to be considered. 1. The Term or Limit. 2. The middle of the Term. 3. The Thing or Figure made by the Term or Limit.

4. The Term or Limit is that which comprehendeth and boundeth the Figure, it is commonly called the Perimeter or Circumference.

5. The Term of a Figure is either Simple or various.

6. A Simple Term is that which doth consist of a Simple Line, and is properly called a Cir-

cumference or Periphery: A Periphery therefore is the Term of a Circle or moſt Simple Figure.

7. A various Term is that which hath bending or crooked Lines, making Angles, and may therefore be called Angular.

8. The middle of Term is that which is the Center of the Figure; for every Figure, whether Triangular, Quadrangular, or Multangular, hath a Center as well as the Circular, differing in in this, that the Lines in a Circle drawn from the Center to the Circumference are all equal, but in other Figures they are not equal.

9. The Thing or Figure made by the Term or Limit, is all that *Area* or ſpace which is included by the Term or Terms. *And here it is to be obſerved, that the Term of a Figure is one thing, and the Figure it ſelf another; for Example, A Periphery is the Term of a Circle, but the Circle it ſelf is not properly the Periphery, but all that* Area *or ſpace which is included by the Periphery, for a Periphery is nothing but a Line, but the Circle is that which is included by that Line.*

10. As the Term of a Figure is either Simple or Various; ſo the Figure it ſelf is either Simple and Round, or Various and Angular.

11. A Simple Figure is that which is contained by a Simple or Round Line, and is either a Circle or an Ellipſis.

12. A Circle therefore is ſuch a Figure which is made by a Line ſo drawn into it ſelf, as that it is every where equally diſtant from the middle or Center.

13. An *Ellipſis* is an oblong Circle.

14. In a Circle we are to conſider the affections which are as it were the Parts or Sections thereof,

thereof, as they are made by the various applications of Right Lines.

15. And Right Lines may be applied unto a Circle, either by drawing them within, or without the Circle.

16. Right Lines inscribed within a Circle, are either such as do cut the Circle into two equal or unequal Parts, as the Diameter and lesser Chords, or such as do cut the Diameter and lesser Chords into two equal or unequal Parts, as the Right and versed Sines.

17. A Diameter is a Right Line drawn through the Center from one side of the Circumference to the other, and divideth the Circle into two equal Parts, *As in Fig.* 7. *The Right Line* G D *drawn through the Center* B *is the Diameter of the Circle* G E D L *dividing the same into the two equal Parts* G E D, *and* G L D: *and this is also called the greatest Chord or Subtense.*

18. A Chord or Subtense is a Right Line inscribed in a Circle, dividing the same into two equal or unequal Parts; if it divide the Circle into two equal Parts, it is the same with the Diameter, but if it divide the Circle into two unequal Parts it is less than the Diameter, and is the Chord or Subtense of an Arch less than a Semi-circle, and also of an Arch greater than a Semi-circle. *As in the former Figure, the Right Line* C A K *divideth the Circle into two unequal Parts, and is the Chord or Subtense of the Arch* C D K, *less than a Semi-circle, and of the Arch* C G K *greater than a Semi-circle: and these are the Lines which divide the Circle into two equal or unequal Parts. And as they divide the Circle into two equal Parts, so do they also divide one another; The lesser Chords when they are divided by*

the Diameter into two equal Parts, those Parts are called Right Sines, and the two Parts of the Diameter made by the intersection of the Chords are called versed Sines.

19. Sines are right or versed.

20. Right Sines are made by being besected, by the Diameter, and are twofold, *Sinus totus*, the whole Sine or Radius, and this is the one half of the Diameter, *as the Lines* BE *or* BD, *and all Lines drawn from the Center to the Circumference.*

21. *Sinus simpliter*, or the lesser Sines, are the one half of any Chord less than the Diameter, *as in the former Figure* CA *or* AK, *which are the equal Parts of the Chord* CAK, *are the Sines of the Arches* CD *and* DK *less than a Quadrant, and also the Sines of* CEG *and* KLG *greater than a Quadrant.*

22. Versed Sines are the Segments of the Diameter, made by the Chords intersecting it, at Right Angles, *as* AD *is the versed Sine of* CD *or* DG *and the other Segment* AG *is the versed Sine of the Arch* CEG *or* KLG.

23. The Right Lines drawn without the Circle are two, the one touching the Circle, and is called a Tangent, and the other cutting the Circle, and is called a Secant.

24. A Tangent is a Right Line touching the Circle, and drawn perpendicular to the Diameter, and extended to the Secant.

25. A Secant is a Right Line drawn from the Center through the Circumference, and extended to the Tangent. As in the former Figure, the Right Line *DF* is the Tangent of the Arch *CD*, and the Right Line *BF* is the Secant of the same Arch *CD*.

Proposition

Proposition I.

The Arch of a Circle being given to describe the whole Periphery.

Let *ABC* be an Arch given, and let the Circumference of that Circle be required. Let there be three Points taken in the given Arch at pleasure, as *A*, *B*, *C*; open your Compasses to more than half the distance of *A*, *B*, and setting one Foot in *A* describe the Arch of a Circle, and the Compasses remaining at the same distance, setting one Foot in *B*, describe another Arch so as it may cut the former in two Points, suppose *G*, and *H*, and draw the Line *HG* towards that Part on which you suppose the Center of the Center of the Circle will fall.

In like manner, opening your Compasses to more than half your distance of *B*, *C*, describe two other Arches from the Points *E* and *C*, cutting each other in *E* and *F*, then draw the Line *EF* till it intersect the former Line *HG*, so shall the Point of Intersection be the Center of the Circumference or Circle required, as in *Fig.* may be seen.

Proposition II.

The Conjugate Diameters of an Ellipsis being given, to draw the Ellipsis.

Let the given Diameter in *Fig.* 24. be *LB* and *ED*, the greatest Diameter. *LB* being bisected in the Point of Bisection, erect the Perpendicular

AD. which let be half of the leſſer Diameter *ED*, then open your Compaſſes to the extent of *AB*, and ſetting one Foot in *D*, with the other make a mark at *M* and *N* in the Diameter *BL*, then cutting a thred to the length of *BL*, faſten the thred with your Compaſſes in the Points *NM*, and with your Pen in the inſide of the thred deſcribe the Arch *BFKL*, ſo ſhall you deſcribe the one half of the Ellipſis required, and turning the Thred on the other ſide of the Compaſſes, you may with your Pen in the like manner deſcribe the other half of the Ellipſis *GBHL*.

CHAP. III.

Of Triangles.

HItherto we have ſpoken of the moſt Simple Figure, a Circle. Come we now to thoſe Figures that are Various or Angular.

2. And an Angular Figure is that which doth conſiſt of three or more Angles.

3. An angular Figure conſiſting of three Angles, otherwiſe called a Triangle, is a Superficies or Figure comprehended by three Right Lines including three Angles.

4. A Triangle may be conſidered either in reſpect of its Sides, or of its Angles.

5. A Triangle in reſpect of its Sides, is either *Iſopleuron*, *Iſoſceles*, or *Scalenum*.

6. An *Iſopleuron* Triangle, is that which hath three equal ſides. An *Iſoſceles* hath two equal Sides. And a *Scalenum* hath all the three Sides unequal.

7. A Triangle in reſpect of its Angles is Right or Oblique. 8. A

8. A Right angled Triangle is that which hath one Right Angle and two Acute.

9. An Oblique angled Triangle, is either Acute or Obtuse.

10. An Oblique acute angled Triangle, is that which hath all the three Angles Acute.

11. An Oblique obtuse angled Triangle, is that which hath one Angle Obtuse, and the other two Acute.

Proposition I.

Upon a Right Line given to make an Isopleuron *or an Equilateral Triangle.*

In *Fig.* 8. let it be required to make an Equilateral Triangle upon the Right Line *AB*. Open your Compasses to the extent of the Line given, and setting one Foot of your Compasses in *A*, make an Arch of a Circle above or beneath the Line given, then setting one Foot of your Compasses in *B*, they being full opened to the same extent, with the other foot draw another Arch of a Circle crossing the former, and from the Intersection of those Arches draw the Lines *AC* and *AB*, so shall the Triangle *ACB* be Equilateral as was desired.

Proposition II.

Upon a Right Line given to make an Isosceles *Triangle, or a Triangle having two Sides equal.*

In *Fig.* 8. let *AB* be the Right Line given, from the Points *A* and *B* as from two Centers, but at a lesser extent of the Compasses than *AB*;

if

if you would have *A B* the greatest Side, at a greater extent; if you would have it to be the least Side, describe two Arches cutting one another, as at *F*, and from the Intersection draw the Lines *AF*, and *F B*, so shall the Triangle *AF B* have two equal Sides, as was required.

Proposition 3.

To make a Scalenum *Triangle, or a Triangle, whose three Sides are unequal.*

In *Fig.* 9. let the three unequal Sides be *E F G* make *A B* equal to one of the given Lines, suppose *G*, and from *A* as a Center, at the extent of *E* describe the Arch of a Circle; in like manner from *B* at the extent of *F* describe another Arch intersecting the former, then shall the Right Lines *AC*, *CB* and *B A* comprehend a Triangle, whose three sides shall be unequal, as was required.

CHAP. IV.

Of Quadrangular and Multangular Figures.

WE have spoken of Triangles or Figures consisting of three Angles, come we now to those that have more Angles than three, as the *Quadrangle*, *Quinquangle*, *Sexangle*, *&c.*

1. A *Quadrangle* is a Figure or Superficies, which is bounded with four Right Lines.

3. A *Quadrangle* is either a *Parallelogram* or a *Trapezium*.

4. A *Parallelogram* is a *Quadrangle* whose opposite

site Sides are parallel having equal distances from one another in all Places.

5. A *Parallelogram* is either Right angled or Oblique.

6. A Right angled *Parallelogram*, is a *Quadrangle* whose four Angles are all Right, and is either Square or Oblong.

7. A Square *Parallelogram* doth consist of four equal Lines. The Parts of a Square are, the Sides of which the Square is made, and the Diagonal or Line drawn from one opposite Angle to another through the middle of the Square.

8. An Oblong is a Right angled *Parallelogram*, having two longer and two shorter Sides.

9. An Oblique angled *Parallelogram*, is that whose Angles are all Oblique, and is either a *Rhombus* or a *Rhomboides*.

10. A *Rhombus* is an Oblique angled and equilateral *Parallelogram*.

11. A *Rhomboides* is an Oblique angled and inequilateral *Parallelogram*.

12. A *Trapezium* is a *Quadrangular* Figure whose Sides are not all parallel; it is either Right angled or Oblique.

13. A Right angled *Trapezium* hath two opposite Sides parallel, but unequal, and the Side between them perpendicular.

14. An Oblique angled *Trapezium* is a *Quadrangle*, but not a *Parallelogram*; having at least two Angles Oblique, and none of the Sides parallel.

15. Thus much concerning *Quadrangles* or four sided Figures. Figures consisting of more than four Angles are almost infinite, but are reducible unto two sorts, Ordinate and Regular, or Inordinate and Irregular. 16. Or-

16. Ordinate and Regular *Polygons* are such, as are contained by equal Sides and Angles, as the *Pentagon*, *Hexagon*, and such like.

17. Inordinate or irregular *Polygons*, are such as are contained by unequal Sides and Angles. The construction of these Quadrangular and Multangular Figures is explained in the Propositions following.

Proposition I.

Upon a Right Line given to describe a Right angled Parallelogram, whether Square or Oblong.

In *Fig.* 10. let the given Line be *AB*, upon the Point *A* erect the Perpendicular *AD* equal to *AB* if you intend to make a Square, but longer or shorter, if you intend an oblong, and upon the Points *D* and *B* at the distance of *AB* and *AD* describe two Arches intersecting one another, and from the Intersection draw the Lines *ED* and *EB*, so shall the Right angled Figure *AE* be a Square, if *AB* and *AD* be equal, otherwise an Oblong, as was desired.

Proposition II.

To describe a Rhombus *or* Rhomboides.

In *Fig.* 11. To the Right Line *AB* draw the Line *AD* at any Acute Angle at pleasure, equal to *AB* if you intend a *Rhombus*, longer or shorter if you intend a *Rhomboides*, then upon your Compasses to the extent of *AD* and upon *B* as a Center describe an Arch; in like manner, at the extent of *AB* upon *D* as a Center describe another

other Arch interſecting the former, then draw the Lines *E D* and *E B*, ſo ſhall *A E* be the *Rhombus* or *Rhomboides*, as was required.

Propoſition III.

Upon a Right Line given to make a Regular Pentagon, *or five ſided Figure.*

In *Fig.* 12. Let the given Line be *A B*, upon A and *B* as two Centers deſcribe the Circles *E B G H* and *C A G K*, then open your Compaſſes to the extent of *B C*, and making *G* the Center, deſcribe the Arch *H A F K*, then draw the Lines *K F E* and *H F C*: ſo ſhall *A E* and *B C* be two ſides of the *Pentagon* deſired, and opening your Compaſſes to the extent of *A B*, upon *E* and *C* as two Centers deſcribe two Arches interſecting one another, and from the Point of Interſection draw the Lines *E D* and *D C*, ſo ſhall the Figure *A B* and *D E* be the *Pentagon* required.

Propoſition IV.

To make a Regular Pentagon *and* Decagon *in a given Circle.*

In *Fig.* 13. upon the Diameter *C A B* deſcribe the Circle *C D B L*, from the Center *A* erect the Perpendicular *A D*, and let the Semidiameter *A C* be biſected, the Point of Biſection is *E*, ſet the diſtance *E D* from *E* to *G*, and draw the Line *G D*, which is the ſide of a *Pentagon*, and *A G* the ſide of a *Decagon* inſcribed in the ſame Circle.

Proposition V.

In a Circle given to describe a Regular Hexagon.

The side of a *Hexagon* is equal to the *Radius* of a Circle, the *Radius* of a Circle therefore being six times applied to the Circumference, will give you six Points, to which Lines being drawn from Point to Point, will constitute a Regular *Hexagon*, as was desired.

Proposition VI.

In a Circle given to describe a Regular Heptagon *or Figure consisting of seven equal sides.*

The side of a *Heptagon* is equal to half the side of a Triangle inscribed in a Circle, having therefore drawn an *Hexagon* in a Circle, the Chord Line subtending two sides of the *Hexagon* lying together, is the side of a Triangle inscribed in that Circle, and half that Chord applied seven times to the Circumference, will give seven Points, to which Lines being drawn from that Point, will constitute a Regular *Heptagon*, as in *Fig.* 14. is plainly shewed.

CHAP. V.

Of Solid Bodies.

HAving ſpoken of the two firſt kinds of Magnitude, Lines and Superficies, come we now to the third, a Body or Solid.

2. A Body or Solid is a Magnitude conſiſting of length, breadth and thickneſs.

3. A Solid is either regular or irregular.

4. That is called a regular Solid, whoſe Baſes, Sides and Angles are equal and like.

5. And this either Simple or Compound.

6. A ſimple regular Solid, is that whith doth conſiſt of one only kind of Superficies.

7. And this is either a Sphere or Globe, or a plain Body.

8. A Globe is a Solid included by one round and convex Superficies, in the middle whereof there is a Point, from whence all Lines drawn to the Circumference are equal.

9. A ſimple plain Solid, is that which doth conſiſt of plain Superficies.

10. A plain Solid is either a Pyramid, a Priſm, or a mixt Solid.

11. A Pyramid is a Solid, Figure or Body, contained by ſeveral Plains ſet upon one right lin'd Baſe, and meeting in one Point.

12. Of all the ſeveral ſorts of Pyramids, there is but one that is Regular, to wit a *Tetrahedron*, or a Pyramid conſiſting of four regular or equilateral Triangles; the form whereof (as it may be cut in Paſtboard) may be conceived by *Figure* 15.

13. A Prism is a Solid contained by several Plains, of which those two which are opposite, are equal, like and parallel, and all others are *Paralellogram*.

14. A Prism is either a *Pentahedron*, a *Hexahedron*, or a *Polyhedron*.

15. A *Pentahedron* Prism, is a Solid comprehended of five Sides, and the Base a Triangle, as *Fig.* 16.

16. An *Hexahedron* Prism, is a Solid comprehended of six Sides, and the Base a *Quadrangle*, as *Fig.* 17.

17. An *Hexahedron* Prism, is distinguished into a *Parallelipipedon* and a *Trapezium*.

18. An *Hexahedron* Prism called a *Trapezium* is a Solid, whose opposites Plains or Sides, are neither opposite nor equal.

19. A *Parallelipipedon* is either right angled or oblique.

20. A right angled *Parallelipipedon* is an *Hexahedron* Prism, comprehended of right angled Plains or Sides; and it is either a Cube or an Oblong.

21. A Cube is a right angled *Parallelipipedon* comprehended of six equal Plains or Sides.

22. An Oblong *Parallelipipedon*, is an *Hexahedron* Prism, comprehended by unequal Plains or Sides.

23. An Oblique angled *Parallelipipedon*, is an *Hexahedron* Prism, comprehended of Oblique Sides.

24. A *Polyhedron* Prism, is a Solid comprehended by more than six Sides, and hath a multangled Base, as a *Quincangle*, *Sexangle*, *&c.*

25. A regular compound or mixt Solid, is such a Solid

a Solid as hath its Vertex in the Center, and the ſeveral Sides expoſed to view, and of this ſort there are only three; the *Octohedron*, the *Icoſahedron*, of both which the Baſe is a Triangle; and the *Dodecahedron*, whoſe Baſe is a *Quincangle*.

26. An *Octohedron* is a Solid Figure which is contained by eight equal and equilateral Triangles, as in *Fig.* 18.

27. An *Icoſahedron* is a Solid, which is contained by twenty equal and equilateral Triangles, as *Fig.* 19.

28. A *Dodecahedron* is a Solid, which is contained by twelve equal *Pentagons*, equilateral and equiangled, as in *Fig.* 20.

29. A regular compound Solid, is ſuch a Solid as is Comprehended both by plain and circular Superficies, and this is either a Cone or a Cylinder.

30. A Cone is a Pyramidical Body, whoſe Baſe is a Circle, or it may be called a round *Pyramis*, as *Fig.* 21.

31. A Cylinder is a round Column every where comprehended by equal Circles, as *Fig.* 22.

32. Irregular Solids are ſuch, which come not within theſe defined varieties, as Ovals, *Fruſtums* of Cones, Pyramids, and ſuch like.

And thus much concerning the deſcription of the ſeveral ſorts of continued Quantity, Lines, Plains and Solids; we will in the next place conſider the wayes and means by which the Dimentions of them may be taken and determined, and firſt we will ſhew the meaſuring of Lines.

CHAP. VI.

Of the Measuring of Lines both Right and Circular.

EVery Magnitude must be measured by some known kind of Measure; as Lines by Lines, Superficies by Superficies, and Solids by Solids, as if I were to measure the breadth of a River, or height of a Turret, this must be done by a Right Line, which being applied to the breadth or height desired to be measured, shall shew the Perches, Feet or Inches, or by some other known measure the breadth or height desired: but if the quantity of some Field or Meadow, or any other Plain be desired, the number of square Perches must be enquired; and lastly, in measuring of Solids, we must use the Cube of the measure used, that we discover the number of those Cubes that are contained in the Body or Solid to be measured. First, therefore we will speak of the several kinds of measure, and the making of such Instruments, by which the quantity of any Magnitude may be known.

2. Now for the measuring of Lines and Superficies, the Measures in use with us, are Inches, Feet, Yards, Ells and Perches.

3. An Inch is three Barley Corns in length, and is either divided into halves and quarters, which is amongst Artificers most usual, or into ten equal Parts, which is in measuring the most useful way of Division.

4. A Foot containeth twelve Inches in length, and is commonly so divided; but as for such things as are to be measured by the Foot, it is far better

better for use, when divided into ten equal Parts, and each tenth into ten more.

5. A Yard containeth three Foot, and is commonly divided into halves and quarters, the which for the measuring of such things as are usually sold in Shops doth well enough, but in the measuring of any Superficies, it were much better to be divided into 10 or 100 equal Parts.

6. An Ell containeth three Foot nine Inches, and is usually divided into halves and quarters, and needs not be otherwise divided, because we have no use for this Measure, but in Shop Commodities.

7. A Pole or Perch cotaineth five Yards and an half, and hath been commonly divided into Feet and half Feet. Forty Poles in length do make one Furlong, and eight Furlongs in length do make an English Mile, and for these kinds of of lengths, a Chain containing four Pole, divided by Links of a Foot long, or a Chain of fifty Foot, or what other length you please, is well enough, but in the measuring of Land, in which the number of square Perches is required; the Chain called Mr. *Gunters*, being four Pole in length divided into 100 Links, is not without just reason reputed the most useful.

8. The making of these several Measures is not difficult, a Foot may be made, by repeating an Inch upon a Ruler twelve times, a Yard is eight Foot, and so of the rest; the Subdivision of a Foot or Inch into halves and quarters, may be performed by the seventeenth of the first, and into ten or any other Parts by the first Proposition of the first Chapter, and all Scales of equal Parts, of what scantling you do desire. And this I

think is as much as needs to be ſaid concerning the dividing of ſuch Inſtruments as are uſeful in the meaſuring Right Lines.

9. The next thing to be conſidered is the meaſuring of Circular Lines, or Perfect Circles.

10. And every Circle is ſuppoſed to be divided into 360 Parts called Degrees, every Degree into 60 Minutes, every Minute into 60 Seconds, and ſo forward this diviſion of the Circle into 360 Parts is generally retained, but the Subdiviſion of thoſe Parts, ſome would have be thus and 100, but as to our preſent purpoſe either may be uſed, moſt Inſtruments not exceeding the fourth part of a Degree.

11. Now then a Circle may be divided into 360 Parts in this manner, Having drawn a Diameter through the Center of the Circle dividing the Circle into two equal Parts, croſs that Diameter with another at Right Angles through the Center of the Circle alſo, ſo ſhall the Circle be divided into four equal Parts or Quadrants, each Quadrant containing 90 Degrees, as in *Fig.* 7. *GE. ED. DL* and *LG*, are each of them 90 Degrees; and the *Radius* of a Circle being equal to the Chord of the ſixth Part thereof, that is to the Chord of 60 Degrees, as in *Fig.* 14. if you ſet the *Radius GB* from *L* towards *G*, and alſo from *G* towards *L*, the Quadrant *GL* will be ſubdivided into three equal Parts, each Part containing 30 Degrees, *GM.* 30. *MH* 30 and *HL* 30, the like may be done in the other Quadrants alſo; ſo will the whole Circle be divided into twelve Parts, each Part containing 30 Degrees.

And becauſe the ſide of a *Pentagon* inſcribed in a Circle is equal to the Chord of 72 Degrees, or the

the firſt Part of 360, as in *Fig.* 13. therefore if you ſet the Chord of the firſt Part of the Circle given from *G* to *L* or *L* to *G*, in *Fig.* 7. you will have the Chord of 72 Degrees, and the difference between *G P* 72 and *G H* 60 is *H P* 12, which being biſected, will give the Arch of 6 Degrees, and the half of ſix will give three, and ſo the Circle will be divided into 120 Parts, each Part containing three Degrees, to which the Chord Line being divided into three Parts, the Arch by thoſe equal Diviſions may be alſo divided, and ſo the whole Circle will be divided into 360, as was deſired.

12. A Circle being thus divided into 360 Parts, the Lines of Chords, Sines, Tangents and Secants, are ſo eaſily made (if what hath been ſaid of them in the Second Chapter be but conſidered) that I think it needleſs to ſay any more concerning their Conſtruction, but ſhall rather proceed unto their Uſe.

13. And the uſe of theſe Lines and other Lines of equal Parts we will now ſhew in circular and right lined Figures; and firſt in the meaſuring of a Circle and Circular Figures.

CHAP.

CHAP. VII.

Of the Measuring of a Circle.

THe squaring of a Circle, or the finding of a Square exactly equal to a Circle given, is that which many have endeavoured, but none as yet have attained: Yet *Archimedes* that Famous Mathematician hath sufficiently proved, That the *Area* of a Circle is equal to a Rectangle made of the *Radius* and half the Circumference: Or thus, The *Area* of a Circle is equal to a Rectangle made of the Diameter and the fourth part of the Circumference. For Example, let the Diameter of a Circle be 14 and the Circumference 44; if you multiply half the Circumference 22 by 7 half the Diameter, the Product is 154; or if you multiply 11 the fourth part of the Circumference, by 14 the whole Diameter, the Product will still be 154. And hence the Superficies of any Circle may be found though not exactly, yet near enough for any use.

2. But *Ludolphus Van Culen* finds the Circumference of a Circle whose Diameter is 1.00 to be 3.14159 the half whereof 1.57095 being multiplied by half the Diameter 50, *&c.* the Product is 7.85395 which is the *Area* of that Circle, and from these given Numbers, the *Area*, Circumference and Diameter of any other Circle may be found by the Proportions in the Propositions following.

Proposition

Propoſition I.

The Diameter of a Circle being given to find the Circumference.

As 1. to 3. 14159: ſo is the Diameter to the Circumference. Example. In *Fig.* 13. Let the Diameter *I B* be 13. 25. I ſay as 1. to 3. 14159, ſo *I B.* 13. 25 to 41, 626 the Circumference of that Circle.

Propoſition II.

The Diameter of a Circle being given to find the Superficial Content.

As 1. to 78539; ſo is the Square of the Diameter given, to the Superficial Content required. *Example,* Let the Diameter given be as before *I B* 13. 25 the Square thereof is 175. 5625 therefore, As 1. to 78539: ſo 175. 5625 to 137, 88 the Superficial Content of that Circle.

Propoſition III.

The Circumference of a Circle being given, to find the Diameter.

This is but the Converſe of the firſt Propoſition: Therefore as 3. 14159 is to 1: ſo is the Circumference to the Diameter; and making the Circumference an Unite, it is. 3. 14159. 1 : : 1. 318308, and ſo an Unite may be brought into the firſt place. *Example*, Let the given Circumference

cumference be 41. 626. I ſay,

As 1. to 318308: ſo 41. 626 to 13. 25. the Diameter required.

Propoſition IV.

The Circumference of a Circle being given to find the Superficial Content.

As the Square of the Circumference of a Circle given is to the Superficial Content of that Circle: ſo is the Square of the Circumference of another Circle given to the Superficial Content required. *Example*, As the Square of 3. 14159 is to 7853938: ſo is 1. the Square of another Circle to 079578 the Superficial Content required, and ſo an Unite for the moſt eaſie working may be brought into the firſt place: Thus the given Circumference being 41. 626. I ſay,

As 1. to 0. 79578: ſo is the Square of 41. 626 to 137. 88 the Superficial Content required.

Propoſition V.

The Superficial Content of a Circle being given, to find the Diameter.

This is the Converſe of the ſecond Propoſition, therefore as 78539 is to 1. ſo is the Superficial Content given, to the Square of the Diameter required. And to bring an Unite in the firſt place: I ſay.

As 7853978. 1 :: 1. 1. 27324, and therefore if the Superficial Content given be 137. 88, to find the Diameter: I ſay,

As

As 1. to 1.27324: ſo 137.88 to 175.5625 whoſe Square Root is 13.25, the Diameter ſought.

Propoſition VI.

The Superficial Content of a Circle being given, to find the Circumference.

This is the Converſe of the Fourth Propoſition, and therefore as 079578 is to 1: ſo is the Superficial Content given, to the Square of the Circumference required, and to bring an Unite in the firſt place: I ſay,

As 079578. 1 :: 1. 12.5664, and therefore if the Superficial Content given be 137.88, to find that Circumference: I ſay,

As 1. to 12.5664: ſo is the 137.88 to 1732.7 whoſe Square Root is 626 the Circumference.

Propoſition VII.

The Diameter of a Circle being given to find the Side of the Square, which may be inſcribed within the ſame Circle.

The Chord or Subtenſe of the Fourth Part of a Circle, whoſe Diameter is an Unite, is 7071067, and therefore, as 1. to 7071067: ſo is the Diameter of another Circle, to the Side required. *Example*, let the Diameter given be 13.25 to find the ſide of a Square which may be inſcribed in that Circle: I ſay,

As 1. to 7071067: ſo is 13.25 to 9.3691 the ſide required.

Propoſition

Propoſition VIII.

The Circumference of a Circle being given, to find the Side of the Square which may be inſcribed in the ſame Circle.

As the Circumference of a Circle whoſe Diameter is an Unite, is to the ſide inſcribed in that Circle; ſo is the Circumference of any other Circle, to the ſide of the Square that may be inſcribed therein. Therefore an Unite being made the Circumference of a Circle.

As 3.14159 to 7071067: ſo 1. to 225078.

And therefore the Circumference of a Circle being as before 41.626, to find the ſide of the Square that may be inſcribed: I ſay,

As 1. to 225078. ſo is 41.626 to 9.3691 the ſide inquired.

Propoſition IX.

The Axis *of a Sphere or Globe being given, to find the Superficial Content.*

As the Square of the Diameter of a Circle, which is Unity, is to 3.14159 the Superficial Content, ſo is the Square of any other *Axis* given, to the Superficial Content required. *Example*, Let 13.25 be the Diameter given, to find the Content of ſuch a Globe: I ſay,

As 1. to 3.14159: ſo is the Square of 13.25 to 551.54 the Superficial Content required.

Propoſition

Propoſition X.

To find the Area *of an* Ellipſis.

As the Square of the Diameter of a Circle, is to the Superficial Content of that Circle; ſo is the Rectangle made of the Conjugate Diameters in an *Ellipſis*, to the *Area* of that *Ellipſis*; And the Diameter of a Circle being one, the *Area* is 7853975, therefore in *Fig.* 26. the Diameters *AC* 8 and *BD* 5 being given, the *Area* of the *Ellipſis ABCD* may thus be found.

As 1. to 7853975: ſo is the Rectangle *AC* in *BD* 40 to 3. 1415900, the *Area* of the *Ellipſis* required.

CHAP. VIII.

Of the Meaſuring of Plain Triangles.

HAving ſhewed the meaſuring of a Circle, and *Ellipſis*, we come now to Right lined Figures, as the Triangle, Quadrangle, and Multangled Figures, and firſt of the meaſuring of the plain Triangles.

2. And the meaſuring of Plain Triangles is either in the meaſuring of the Sides and Angles, or of their *Area* and Superficial Content.

3. Plain Triangles in reſpect of their Sides and Angles are to be meaſured by two ſorts of Lines, the one is a Line of equal Parts, and by that the Sides muſt be meaſured, the other is a Line of Chords, the Conſtruction whereof hath been

been shewed in the sixth Chapter, and by that the Angles must be measured, the Angles may indeed be measured by the Lines of Sines, Tangents or Secants, but the Line of Chords being not only sufficient, but most ready, it shall suffice to shew how any Angle may be protracted by a Line of Chords, or the Quantity of any Angle found, which is protracted.

4. And first to protract or lay down an Angle to the Quantity or Number of Degrees proposed, do thus, draw a Line at pleasure as *AD* in *Figure* 5, then open your Compasses to the Number of 60 Degrees in the Line of Chords, and setting one Foot in *A*, with the other describe the Arch *BG*, and from the Point *A* let it be required to make an Angle of 36 Degrees: open your Compasses to that extent in the Line of Chords, and setting one Foot in *B*, with the other make a mark at *G*, and draw the Line *AG*, so shall the Angle *BAG* contain 36 Degrees, as was required.

5. If the Quantity of an Angle were required, as suppose the Angle *BAG*, open your Compasses in the Line of Chords to the extent of 60 Degrees, and setting one Foot in *A*, with the other draw the Arch *BG*, then take in your Compasses the distance of *BG*, and apply that extent to the Line of Chords, and it will shew the Number of Degrees contained in that Angle, which in our Example is 36 Degrees.

6. In every Plain Triangle, the three Angles are equal to two right or 180 Degrees, therefore one Angle being given, the sum of the other two is also given, and two Angles being given, the third is given also.

7. Plain

7. Plain Triangles are either Right Angled or Oblique.

8. In a Right Angled Plain Triangle, one of the Acute Angles is the Complement of the other to a Quadrant or 90 Degrees.

9. In Right Angled Plain Triangles, the Side subtending the Right Angle we call the *Hypotenuse*, and the other two Sides the Legs, thus in *Fig.* 5 *AE* is the *Hypotenuse*, and *AD* and *ED* are the Legs; these things premised, the several cases in Right Angled and Oblique Angled Plain Triangles may be resolved, by the Propositions following.

Proposition I.

In a Right Angled Plain Triangle, the Angles of one Leg being given, to find the Hypotenuse *and the other Leg.*

In the Right Angled Plain Triangle *ADE* in *Fig.* 5. Let the given Angles be *DAE* 36, and *DEA* 54, and let the given Leg be *AD* 476 to find the *Hypotenuse AE*, and the other Leg *ED*.

Draw a Line at pleasure, as *AD*, and by your Scale of equal Parts set from *A* to *D* 476 the Quantity of the Leg given, then erect a Perpendicular upon the Point *D*, and upon the Point *A* lay down your given Angle *DAE* 36 by the fourth hereof, and draw the Line *AE* till it cut the Perpendicular *DE*, then measure the Lines *AE* and *DE* upon your Scale of Equal Parts, so shall *AE* 588.3 be the *Hypotenuse*, and *DE* 343.8 the other Leg.

Proposition II.

The Hypotenuse *and Oblique Angles given, to find the Legs.*

Let the given *Hypotenuse* be 588, and one of the Angles 36 degrees, the other will then be 54 degrees. Draw a Line at pleasure, as *AD*, and upon the Point *A* by the fourth hereof lay down one of the given Angles (suppose the less) and draw the Line *AC*, and from your Scale of equal Parts, set off your *Hypotenuse* 588 from *A* to *E*, and from the Point *E* to the Line *AD* let fall the Perpendicular *ED*, then shall *AD* being measured upon the Scale be 476 for one Leg, and *ED* 345.8 the other.

Proposition III.

The Hypotenuse *and one Leg given to find the Angles and the other Leg.*

Let the given *Hypotenuse* be 588. and the given Leg 476. Draw a Line at pleasure as *AD*, upon which set the given Leg from *A* to *D*. 476, and upon the Point *D*, erect the Perpendicular *DE*, then open your Compasses in the Scale of Equal Parts to the Extent of your given *Hypotenuse* 588, and setting one Foot of that Extent in *A*, move the other till it touch the Perpendicular *DE*, then and there draw *AE*, so shall *ED* be 345.8 the Leg inquired, and the Angle *DAE*, will be found by the Line of Chords to be 36. whose Complement is the Angle *DEA*. 54.

Proposition IV.

The Legs given to find the Hypotenuse, *and the Oblique Angles.*

Let one of the given Legs be 476, and the other 345. 8, Draw the Line *AD* to the extent of 476, and upon the Point *D*, erect the Perpendicular *DE* to the extent of 345. 8, and draw the Line *AE*, so shall *AE* be the *Hypotenuse* 588, and the Angle *DAE* will by the Line of Chords be found to be 36 Degrees, and the Angle *DEA* 54, as before.

Hitherto we have spoken of Right angled plain Triangles: the Propositions following concern such as are Oblique.

Proposition V.

Two Angles in an Oblique angled plain Triangle, being given, with any one of the three Sides; to find the other two Sides.

In any Oblique angled plain Triangle, let one of the given Angles be 26. 50 and the other 38. and let the given Side be 632, the Sum of the two given Angles being deducted from a Semicircle, leaveth for the third Angle 115. 50 Degrees, then draw the Line *BC* 632. and upon the Points *B* and *C* protract the given Angles, and draw the Lines *BD* and *CD*, which being measured upon your Scale of equal Parts *BD* will be found to be 312. 43, and *BD* 431. 69.

Proposition VI.

Two Sides in an Oblique Angled Triangle being given, with an Angle opposite to one of them, to find the other Angles and the third Side, if it be known whether the Angle Opposite to the other Side given be Acute or Obtuse.

In an Oblique Angled Plain Triangle, let the given Angle be 38 Degrees, and let the Side adjacent to that Angle be 632, and the Side opposite 431. 1. upon the Line *BC* in *Fig.* 25. protract the given Angle 38 Degrees upon the Point *C*, and draw the Line *DC*, then open your Compasses to the Extent of the other Side given 431. 1. and setting one Foot in *B*, turn the other about till it touch the Line *DC*, which will be in two places, in the Points *D* and *E*; if therefore the Angle at *B* be Acute the third Side of the Triangle will be *CE*, according therefore to the Species of that Angle, you must draw the Line *BD* or *BE* to compleat the Triangle, and then you may measure the other Angles, and the third Side as hath been shewed.

Proposition VII.

Two Sides of an Oblique Angled Plain Triangle being given, with the Angle comprehended by them to find the other Angles and the third Side.

Let one of the given Sides be 632, and the other 431. 1, and let the Angle comprehended by them be Deg. 26. 50, draw a Line at pleasure,

as *B C*, and by help of your Scale of Equal Parts set off one of your given Sides from *B* to *C* 632. then upon the Point *B* protract the given Angle 26. 50. and draw the Line *B D*, and from *B* to *D*, set off your other given Side 431. 1. and draw the Line *D C*, so have you constituted the Triangle *B D C*, in which you may measure the Angles and the third Side, as hath been shewed.

Proposition VIII.

The three Sides of an Oblique Angled Triangle being given, to find the Angles.

Let the length of one of the given Sides be 632, the length of another 431. 1, and the length of the third 312. 4, and Draw a Line at pleasure, as *B C* in *Fig.* 25, and by help of your Scale of Equal Parts, set off the greatest Side given 632 from *B* to *C*. then open your Compasses in the same Scale to the extent of either of the other Sides, and setting one Foot of your Compasses in *B*, with the other describe an occult Arch, then extend your Compasses in the same Scale to the length of the third Side, and setting one Foot in *C* with the other describe another Arch cutting the former, and from the Point of Intersection draw the Lines *B D* and *D C*. to constitute the Triangle *B D C*, whose Angles may be measured, as hath been shewed.

And thus may all the Cases of Plain Triangles be resolved by Scale and Compass, he that desires to resolve them Arithmetically, by my *Trigonometria Britannica*, or my little Geometrical Trigonometry; only one Case of Right Angled Plain

Triangles which I shall have occasion to use, in the finding of the *Area* of the Segment of a Circle I will here shew how, to resolve by Numbers.

Proposition IX.

In a Right Angled Plain Angle the Hypotenuse *and one Leg being given to find the other Leg.*

Take the Sums and difference of the *Hypotenuse* and Leg given, then multiply the Sum by the Difference, and of the Product extract the Square Root, which Square Root shall be the Leg inquired.

Example. In *Fig.* 5. Let the given *Hypotenuse* be *AE* 588. 3, and the given Leg *AD* 476, and let *DE* be the Leg inquired. The Sum of *AE* and *AD* is 1064. 3, and their Difference is 112. 3, now then if you multiply 1064. 3 by 112. 3, the Product will be 119520. 89, whose Square Root is the Leg *DE*. 345. 8.

Proposition X.

The Legs of a Right Angled Plain Triangle being given, to find the Area *or Superficial Content thereof.*

Multiply one Leg by the other, half the Product shall be the Content. *Example,* In the Right angled plain Triangle *ADE*, let the given Legs be *AD* 476, and *DE* 345, and let the *Area* of that Triangle be required, if you multiply 476 by 345 the Product will be 164220, and the half thereof 82110 is the *Area* or Superficial Content required.

Proposition

Proposition XI.

The Sides of an Oblique angled plain Triangle being given to find the Area *or Superficial Content thereof.*

Add the three Sides together, and from the half Sum ſubtract each Side, and note their Difference; then multiply the half Sum by the ſaid Differences continually, the Square Root of the laſt Product, ſhall be the Content required.

Example. In *Fig.* 9. Let the Sides of the Triangle *ABC* be *AB* 20. *AC* 13, and *BC* 11 the Sum of theſe three Sides is 44, the half Sum is 22, from whence ſubtracting *AB* 20, the Difference is 2, from whence alſo if you ſubtract *AC* 13, the Difference is 9, and laſtly, if you ſubtract *BC* 11 from the half Sum 22, the Difference will be 11. And the half Sum 22 being multiplied by the firſt Difference 2, the Product is 44, and 44 being multiplied by the Second Difference 9, the Product is 396, and 396 being multiplied by the third Difference 11, the Product is 4356, whoſe Square Root 66, is the Content required.

Or thus, from the Angle *C* let fall the Perpendicular *DC*, ſo is the Oblique angled Triangle *ABC*, turned into two right, now then if you meaſure *DC* upon your Scale of Equal Parts, the length thereof will be found to be 6. 6, by which if you multiply the Baſe *AB* 20, the Product will be 132. 0, whoſe half 66, is the *Area* of the Triangle, as before.

Proposition XII.

The Sides of any Oblique angled Quadrangle being given, to find the Area *or Superficial Content thereof.*

Let the Sides of the Oblique angled Quadrangle *ABED* in *Fig.* 11. be given, draw the Diagonal *AE*, and also the Perpendiculars *DC* and *BF*, then measuring *AE* upon the same Scale by which the Quadrangular Figure was protracted, suppose you find the length to be 632, the length of *DC* 112, and the length of *BF* 136, if you multiply *AE* 632 by the Half of *DC* 56, the Product will be 35392 the *Area* of *ACED*. In like manner if you multiply *AE* 632, by the half of *BF* 68, the Product will be 42976 the *Area* of *AGEB*, and the Sum of these two Products is the *Area* of *ABED* as was required.

Or thus, take the Sum of *DC* 112, and *BF* 136; the which is 248, and multiply *AE* 632 by half that Sum, that is by 124, the Product will be 78368 the *Area* of the Quadrangular Figure *ABED*, as before.

Proposition XIII.

The Sides of a plain irregular multangled Figure being given, to find the Content.

In *Fig.* 26. Let the Sides of the multangled Figure, *A.B.C.D.E.F.G.H.* be given, and let the *Area* thereof be required, by Diagonals drawn from the opposite Angles reduce the Figure given,

into

into Oblique angled plain Triangles, and those Oblique angled Triangles, into right by letting fall of Perpendiculars, then measure the Diagonals and Perpendiculars by the same Scale, by which the Figure it self was protracted, the Content of those Triangles being computed, as hath been shewed, shall be *AF* the Content required: thus by the Diagonals *AG*. *BE* and *EC* the multangled Figure propounded is converted into three Oblique angled quadrangular Figures, *AFGH*. *AFEB* and *BEDC*, and each of these are divided into four Right angled Triangles, whose several Contents may be thus computed. Let *GA* 94 be multiplied by half *HL* 27 more Half of *KF* 29, that is by 23, the Product will be 21, be the *Area* of *AHGF*. Secondly, *OB* is 11, and *FN* 13, their half Sum 12, by which if you multiply *AE* 132, the Product will be 1584 the *Area* of *AFEB*. Thirdly, let *Bp* be 18 *wD* 32, the half Sum is 25, by which if you multiply *AEC* 125 the Product will be 3125 the *Area* of *BEDC*, and the Sum of these Products is 6871 the *Area* of the whole irregular Figure. *ABCDEFGH*, as was required.

Proposition XIV.

The Number of Degrees in the Sector *of a Circle being given, to find the* Area *thereof.*

In *Fig.* 27. *ADEG* is the *Sector* of a Circle, in which the Arch *DEG*, is Degrees. 23. 50, and by 1. *Prop.* of *Archimed. de Dimensione Circuli*, the length of half the Arch is equal to the *Area* of the *Sector* of the double Arch, there the length of

of *DE* or *EG* is equal to the *Area* of the *Sector ADEG*: and the length or circumference of the whole Circle whose Diameter is 1 according to *Van Culen*, is. 3. 14159265358979, therefore the length of one Centesme of a Degree, is. 0. 01745329259. Now then to find the length of any Number of Degrees and Decimal Parts, you must multiply the aforesaid length of one Centesme by the Degrees and Parts given, and the Product shall be the length of those Degrees and Parts required, and the *Area* of a *Sector* containing twice those Degrees and Parts. *Example*, the half of *DEG* 23. 50 is *DE* or *EG* 11. 75, by which if you multiply 0. 01745329259, the Product will be 205076187932 5, the length of the Arch *DE*, and the *Area* of the *Sector ADEG*.

Proposition XV.

The Number of Degrees in the Segment of a Circle being given, to find the Area *of that Segment.*

In *Fig.* 27. Let the *Area* of the Segment *DEGK* be required, in which let the Arch *DEG* be Degrees 23. 50, then is the *Area* of the *Sector ADEG* 205076187932 5 by the last aforegoing, from which if you deduct the *Area* of the Triangle *ADG*, the remainer will be the *Area* of the Segment *DEGK*. And the *Area* of the Triangle *ADG* may thus be found. *DK* is the Sine of *DE* 11. 75, which being sought in *Gellibrand's* Decimal Canon is. 203641751 1, and *AK* is the Sine of *DH* 78. 25, or the Cosine of *DE*. 97904547 24, which being multiplied by the Sine of *DE*, the Product will be 1993745343, or if you multiply *AG* the

Fig. 1.

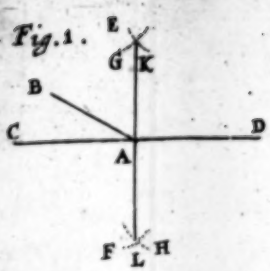


Fig. 2.

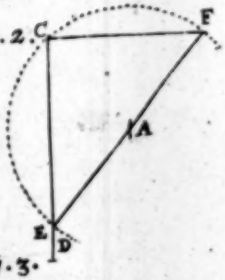


Fig. 3.

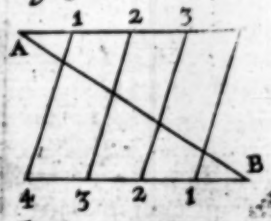


Fig. 4.

Fig. 5.

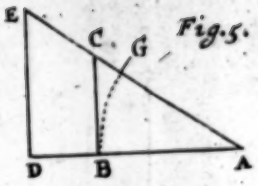


Fig. 6.

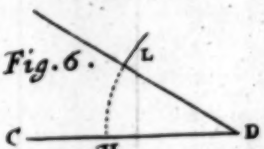


Fig. 7.

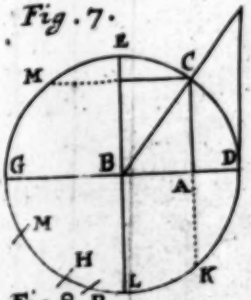


Fig. 8.

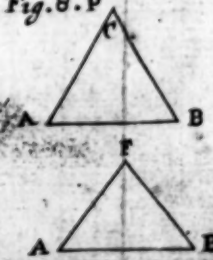


Fig. 9.

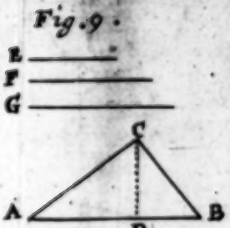


Fig. 10.

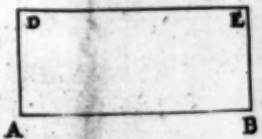


Fig. 11.

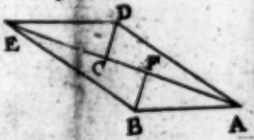


Fig. 12.

Fig. 13.

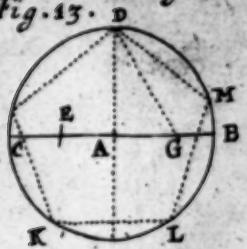
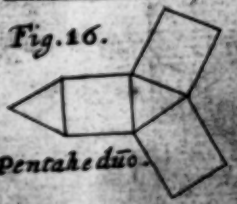


Fig. 14.

Fig. 15.

Fig. 16.



Pentahedūo.

A
B

the *Radius* by half *D F* the Sine of the double Arch *D E G*, the Product will be 19937453445 as before, and this Product being deducted from the *Area* of the *Sector A D E G* 20507618793 25, the remainer will be 570164348 75 the *Area* of the Segment *D E G L*, as was desired.

Proposition XVI.

The Diameter of a Circle being cut into any Number of Equal Parts, to find the Area *of any Segment made by the Chord Line drawn at Right Angles through any of those equal Parts of the Diameter.*

In *Fig.* 28. The *Radius A D* is cut into five Equal Parts, and the Segment *E D F L* is made by the Chord Line *E L F* at Right Angles to *A D* in the fourth Equal Part, or at eight tenths thereof: now then to find the *Area* of this Segment we have given *A E Radius*, and *A L* 8, and therefore by the ninth hereof *E L* will be found to be 6.6000, the Sine of *E D* 36. 87, by which if you multiply 0. 0174532, the Product is the *Area* of the *Sector A E D F* 64350286, and the *Area* of the Triangle *A E F* is 48, which being deducted from the *Area* of the *Sector*, the Remainer 16350286 is the *Area* of the *Sector E D P L*, as was required. And in this manner was that Table of Segments made by the Chord Lines cutting the *Radius* into 100 Equal Parts.

Another way.

In *Fig.* 28. Let the *Radius A D* be cut into 10. 100 or 1000 Equal Parts, and let the *Area* of the

the Segments made by the Chord Lines drawn at Right Angles through all those Parts be required: first find the Ordinates *G K* and *M P N E L*, the double of each Ordinate, will be the Chords of the several Arches, and the Sum of these Chords beginning with the least Ordinate, will orderly give you the *Area* of the several Segments made by those Chord Lines, but the Diameter must be be divided into 100000 Equal Parts, because of the unequal differences at the beginning of the Diameter: but taking the *Area* of the Circle to be 3.14159265 35, &c. as before, the *Area* of the Semicircle will be 1.57079632, from which if you deduct the Chord *G H* 1999999, the Chord answering to 999 Parts of the *Radius*, the remainer is. 1.56879632 the *Area* of the Segment *G D H*. And in this manner by a continual deduction of the Chord Lines from the *Area* of the Segment of the Circle given, was made that Table shewing the *Area* of the Segments of a Circle to the thousandth part of the *Radius*.

And because a Table shewing the *Area* of the Segments of a Circle to the thousandth part of the *Radius*, whose whole *Area* is Unity, is yet more useful in Common Practice, therefore from this Table, was that Table also made by this Proportion.

As the *Area* of the Circle whose Diameter is Unity, to wit 3.14149 is to the *Area* of any part of that Diameter, so is Unity the supposed *Area* of another Circle, to the like part of that Diameter. *Example*, the *Area* answering to 665 parts of the *Radius* of a Circle whose *Area* is 3.14159 is 0.91354794 therefore,

As 3.14159265 is to 0.91354794: So is 1.

to

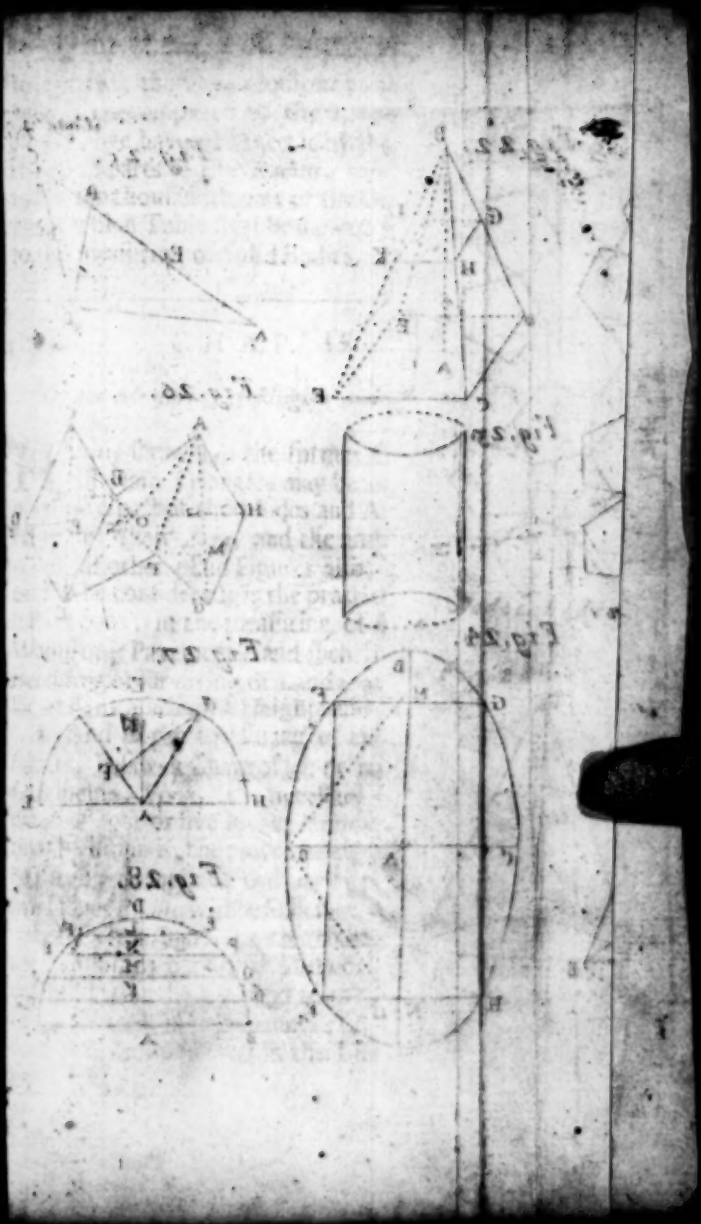


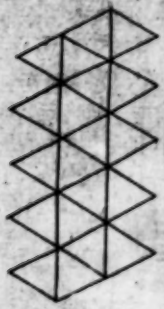
Fig. 17.

Cube.

Fig. 18.

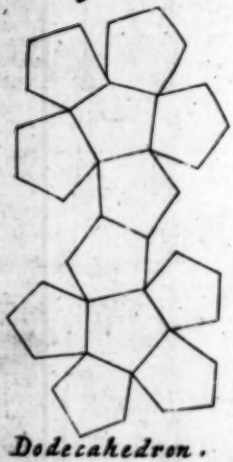
Octohedron.

Fig. 19.



Icosahedron.

Fig. 20.



Dodecahedron.

Fig. 21.

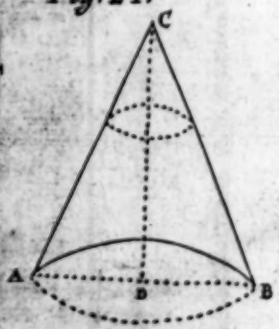


Fig. 22.

Fig. 23.

Fig. 24.

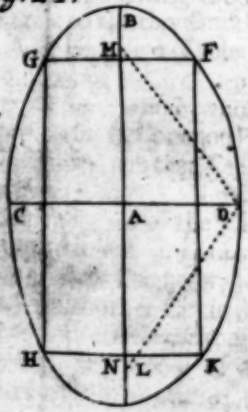


Fig. 25.

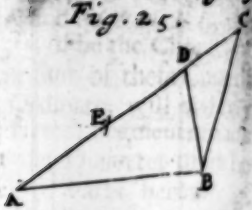


Fig. 26.

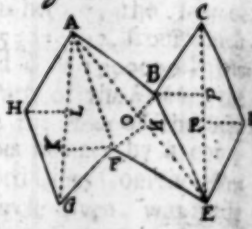


Fig. 27.

Fig. 28.

to 290791, the *Area* required; and the Table being thus computed to the 1000 parts of the *Radius*, we have enlarged it by the difference to the 5000 parts of the *Radius*, and consequently to the ten thousandth part of the Diameter: The use of which Table shall be shewed when we come to the measuring of Solid Bodies.

CHAP. IX.

Of the Measuring of Heights and Distances.

HAving shewed in the former Chapter, how all plain Triangles may be measured, not only in respect of their Sides and Angles, but in respect of their *Area*, and the finding of the *Area* of all other plain Figures also, that which is next to be considered, is the practical use of those Instructions, in the measuring of Board, Glass, Wainscot, Pavement, and such like, as also the measuring or surveying of Land; and first we will shew the measuring of Heights and Distances.

2. And in the measuring of Heights and Distances, besides a Chain of 50 or 100 Links, each Link being a Foot, it is necessary to have a Quadrant of four or five Inches *Radius*, and the larger the Quadrant is, the more exactly may the Angles be taken, though for ordinary Practice, four or five Inches *Radius* will be sufficient.

Let such a Quadrant therefore be divided on the Limb into 90 Equal Parts or Degrees, and numbred from the left hand to the right, at every tenth Degree, in this manner 10. 20. 30. 40. 50. 60. 70. 80. 90. and within the Limb of the Quadrant

drant draw another Arch, which being divided by help of the Limb into two Equal Parts, in the Point of Intersection set the Figure 1. representing the *Radius* or Tangent of 45 Degrees, and from thence both ways the Tangents of 63. 44 Deg. 71. 57 Deg. 75. 97 Deg. 78. 76 Deg. 80. 54 Deg. that is, 2. 3. 4. 5 and 6 being set also, your Quadrant will be fitted for the taking of Heights several ways, as shall be explained in the Propositions following.

Proposition I.

To find the Height of a Tower, Tree, or other Object at one Station.

At any convenient distance from the Foot of the Object to be measured, as suppose at *C* in *Fig.* 30. and there looking through the Sights of your Quadrant till you espie the top of the Object at *A*, observe what Degrees in the Limb are cut by the Thread, those Degrees from the left Side or Edge of the Quadrant to the Right, is the Quantity of the Angle *ACB*, which suppose 35 Degrees; then is the Angle *BAC* 55 Degrees, being the Complement of the former to 90 Degrees. This done with your Chain or otherwise measure the distance from *B* the Foot of the Object, to your Station at *C*, which suppose to be 125 Foot. Then as hath been shewed in the 1. *Prop. Chap.* 8. draw a Line at pleasure as *BC*, and by your Scale of Equal Parts, set off the distance measured from *B* to *C* 125 Foot, and upon the Point *C* lay down your Angle taken by observation 35 Degrees, then erect a Perpendicular upon

the

the Point *B*, and let it be extended till it cut the Hypothenusal Line *AC*, so shall *AB* measured on your Scale of Equal Parts, be 87.5 Foot for the Height of the Object above the Eye; to which the Height of the Eye from the Ground being added, their Sum is the Height required.

Another way.

Let *AB* represent a Tower whose Altitude you would take, go so far back from it, that looking through the Sights of your Quadrant, to the top of the Tower at *A* the Thread may cut just 45 Degrees in the Limb, then shall the distance from the Foot of the Tower, to your Station, be the Height of the Tower above the Eye.

Or if you remove your Station nearer and nearer to the Object, till your Thread hang over the Figures 2. 3. 4 or 5 in the Quadrant, the Height of the Tower at 2. will be twice as much as the distance from the Tower to the Station, at 3. it will be thrice as much, *&c.* As if removing my Station from *C* to *D*, the Thread should hang over 2 in the Quadrant, and the distance *BD* 62 Foot, then will 124 Foot be the Height of the Tower, above the Eye.

In like manner if you remove your Station backward till your Thread fall upon one of those Figures in the Quadrant, between 45 and 90 Degrees, the distance between the Foot of the Tower, and your Station will at 2. be twice as much as the Height, at 3. thrice as much, at 4. four times so much, and so of the rest.

A Third way by a Station at Random.

Take any Station at pleasure suppose at *C*, and looking through the Sights of your Quadrant, observe what Parts of the Quadrant the Thread falls upon, and then measure the distance between the Station, and the Foot of the Object, that distance being multiplied by the parts cut in the Quadrant, cutting off two Figures from the Product shall be the Height of the Object above the Eye.

Example, Suppose I standing at *C*, that the Thread hangs upon 36 Degrees, as also upon 72 in the Quadrant which is the Tangent of the said Arch, and let the measured distance be *CB* 125 Foot, which being multiplied by 72, the Product is 9000, from which cutting off his Figures because the *Radius* is supposed to be 100, the Height inquired will be 90 Foot, he that desires to perform this work with more exactness, must make use of the Table of Sines and Tangents Natural or Artificial, this we think sufficient for our present purpose.

Proposition II.

To find an inaccessible Height at two Stations.

Take any Station at pleasure as at *D*, and there looking through the Sights of your Quadrant to the top of the Object, observe what Degrees are cut by the Thread in the Limb, which admit to be 68 Degrees, then remove backward, till the Angle taken by the Quadrant, be but half so much

as

as the former, that is 34 Degrees, then is the distance between your two Stations equal to the Hypothenusal Line at your first Station, *viz. AD.* if the distance between your two Stations were 326 foot, then draw a Line at pleasure as *B D*, upon the Point *D* protract, the Angle *ADB* 68 Degrees, according to your first Observation, and from your Line of equal parts set off the Hypothenusal 326 Foot from *D* to *A*, and from the Point *A* let fall the Perpendicular *AB* which being measured in your Scale of Equal Parts, shall be the Altitude of the Object inquired.

Or working by the Table of Sines and Tangents, the Proportion is.

As the *Radius*, is to the measured distance or Hypothenusal Line *AD*; so is the Sine of the Angle *ADE*, to the height *AB* inquired.

Another more General way, by any two Stations taken at pleasure.

Admit the first Station to be as before at *D*, and the Angle by observation to be 68 Degrees, and from thence at pleasure I remove to *C*, where observing aim I find the Angle at *C* to be 32 Degrees, and the distance between the Stations 150 Foot. Draw a Line at pleasure as *B C*, and upon *C* lay down your last observed Angle 32 Degrees, and by help of your Scale of Equal Parts, set off your measured distance from *C* to *D* 150 Foot, then upon *D* lay down your Angle of 68 Degrees, according to your first Observation, and where the Lines *AD* and *AC* meet, let fall the Perpendicular *AB*, which being measured in your Scale of Equal Parts, shall be the height of the Object as before.

Or working by the Tables of Sines and Tangents, the Proportions.

1. As the Sine of *DAC* to the Distance *DC*. So the Sine of *ACD*, to the Side *AD*.

2. As the *Radius*, to the Side *AD*; so the Sine *ADB*, to the Perpendicular height *AB* inquired.

The taking of Distances is much after the same manner, but because there is required either some alteration in the sights of your Quadrant or some other kind of Instrument for the taking of Angles, we will particularly shew, how that may be also done several ways, in the next Chapter.

CHAP. X.

Of the taking of Distances.

For the taking of Distances some make use of a Semicircle, others of a whole Circle, with Ruler and Sights rather than a Quadrant, and although the matter is not much by which of these Instruments the Angles be taken, yet in all Cases the whole Circle is somewhat more ready, than either a Semicircle or Quadrant, the which with its Furniture is called the Theodolite.

2. A piece of Board or Brass then about twelve or fourteen Inches Diameter, being made Circular like a round Trencher, must be divided into four Quadrants, and each Quadrant divided into 90 Degrees, or the whole Circle into 360, and each Degree into as many other Equal Parts, as the largeness of the Degrees will well permit: let your Circle be numbred both ways to 360, that is from the right hand to the left, and from the left to the right.

3. Upon

3. Upon the backside of the Circle there must be a Socket made fast, that it may be set upon a three legged Staff, to bear it up in the Field.

4. You must also have a Ruler with Sights fixed at each end, for making of Observation, either fixed upon the Center of your Circle, or loose, as you shall think best; your Instrument being thus made, any distance whether accessible or inaccessible may thus be taken.

5. When you are in the Field, and see any Church, Tower, or other Object, whose Distance from you, you desire to know, choose out some other Station in the same Field, from whence you may also see the Object, and measure the distance between your Stations; then setting your Ruler upon the Diameter of your Circle, set your Instrument so, as that by the Sights on your Ruler, you may look to the other Station, this done turn your Ruler to that Object whose distance you desire to know, and observe how many Degrees of the Circle are cut by the Ruler, as suppose 38 Degrees, as the Angle *ACD* in *Fig.* 30. Then removing your Instrument to *D*, lay the Ruler on the Diameter thereof, and then turn the whole Instrument about till through your Sights you can espy the mark set up at your first Station at *C*, and there fix your Instrument, and then upon the Centre of your Circle turn your Ruler till through the Sights you can espy the Object whose distance is inquired, suppose at *A*; and observe the Degrees in the Circle cut by the Ruler, which let be 112, which is the Angle *ADC*, and let the distance between your two Stations be *DC* 326 Foot; so have you two Angles and the side between them, in a plain Triangle given, by which

to find the other ſides, the which by protraction may be done as hath been ſhewed, in the fifth Propoſition of Chapter 8. but by the Table of Sines and Tangents, the Proportion is.

As the Sine of *DAC*, is to *DC*; ſo is the Sine of *ACD* to the Side *AD*.

Or, as the Sine of *DAC*, is to the given Side *DC*.

So is the Sine of *ADC* to the Side *AC*.

6. There is another Inſtrument called the plain Table, which is nothing elſe, but a piece of Board, in the faſhion and bigneſs of an ordinary ſheet of paper, with a little frame, to faſten a ſheet of paper upon it, which being alſo ſet upon a Staff, you may by help of your Ruler, take a diſtance therewith in this manner.

Having meaſured the diſtance between your two Stations at *D* and *C*, draw upon your paper a Line, on which having ſet off your diſtance place your Inſtrument at your firſt Station *C*, and laying your Ruler upon the Line ſo drawn thereon, turn your Inſtrument till through the Sights you can eſpy the Station at *D*, then laying your Ruler upon the Point *C*, turn the ſame about till through the Sights you can eſpy the Object at *A*, and there draw a Line by the ſide of your Ruler, and remove your Inſtrument to *D*, and laying your Ruler upon the Line *DC*, turn the Inſtrument about, till through the Sight you can eſpy the Mark at *C*, and then laying your Ruler upon the Point *D*, turn the ſame, till through the Sights you can eſpy the Object at *A*, and by the ſide of your Ruler draw a Line, which muſt be extended till it meet with the Line *AC*, ſo ſhall the Line *AD* being meaſured upon your Scale of Equal Parts,

Parts, be the distance of the Object from *D*, and the Line *A C* shall be the distance thereof from *C*.

7. And in this manner may the distance of two, three or more Objects be taken, from any two Stations from whence the several Objects may be seen, and that either by the plain Table, or Theodolite.

CHAP. XI.

How to take the Plot of a Field at one Station, from whence the several Angles may be seen.

ALthough there are several Instruments by which the Plat of a Field may be taken, yet do I think it sufficient to shew the use of these two, the plain Table and Theodolite.

2. In the use of either of which the same chain which is used in taking of heights and distances, is not so proper. I rather commend that which is known by the Name of *Gunter*'s Chain, which is four Pole divided into 100 Links; being as I conceive much better for the casting up the Content of a Piece of Ground, than any other Chain that I have yet heard of, whose ease use shall be explained in its proper place.

3. When you are therefore entered the Field with your Instrument, whether plain Table, or Theodolite, having chosen out your Station, let visible Marks be set up in all the Corners thereof, and then if you use the plain Table, make a mark upon your paper, representing your Station, and laying your Ruler to this Point, direct

your Sights to the ſeveral Corners of the Field, where you have cauſed Marks to be ſet up, and draw Lines by the ſide of the Ruler upon the paper to the point repreſenting your ſtation, then meaſure the diſtance of every of theſe Marks from your Inſtrument, and by your Scale ſet thoſe diſtances upon the Lines drawn upon the paper, making ſmall marks at the end of every ſuch diſtance, Lines drawn from Point to Point, ſhall give you upon your paper, the Plot of the Field, by which Plot ſo taken the content of the Field may eaſily be computed.

Example. Let *Fig.* 31. repreſent a Field whoſe Plot is required; your Table being placed with a ſheet of paper thereupon, make a Mark about the middle of your Table, as at *A.* apply your Ruler from this Mark to *B* and draw the Line *A B*, then with your Chain meaſure the diſtance thereof which ſuppoſe to be 11 Chains 36 Links, then take 11 Chains 36 Links from your Scale, and ſet that diſtance from *A* to *B*, and at *B* make a mark.

Then directing the Sights to *C*, draw a Line by the ſide of your Ruler as before, and meaſure the diſtance *A C*, which ſuppoſe to be 7 Chains and 44 Links, this diſtance muſt be taken from your Scale, and ſet from *A* to *C* upon your paper.

And in this manner you muſt direct your Sights from Mark to Mark, until you have drawn the Lines and ſet down the diſtances, between all the Angles in the Field and your ſtation, which being done, you muſt draw the Lines from one Point to another, till you conclude where you firſt began, ſo will thoſe Lines *BC. CD. DE. FG.* and *G B*, give you the exact Figure of the Field.

4. To do this by the Theodolite, in ſtead of drawing

drawing Lines upon your paper in the Field: you muſt have a little Book, in which the Pages muſt be divided into five Columns, in the firſt Column whereof you muſt ſet ſeveral Letters to ſignifie the ſeveral Angles in the Field, from which Lines are to be drawn to your place of ſtanding, in the ſecond and third Columns the degrees and parts taken by your Inſtrument, and the fourth and fifth, to ſet down your diſtances Chains and Links, this being in readineſs, and have placed your Inſtrument direct your Sights to the firſt mark at *B*, and obſerve how many Degrees are comprehended between the Diameter of your Inſtrument, and the Ruler, and ſet them in the ſecond and third Columns of your Book againſt the Letter *B*, which ſtands for your firſt Mark, then meaſure the diſtance *AB* as before, and ſet that down, in the fourth and fifth Columns, and ſo proceed from Mark to Mark, until you have taken all the Angles and Diſtances in the Field, which ſuppoſe to be, as they are expreſſed in the following Table.

	Degr.	*Part*	*Chains*	*Links*
B	39	75	11	56
C	40	75	7	44
D	66	00	7	48
E	43	25	8	92
F	80	00	6	08
G	59	25	9	73

5. Having thus taken the Angles and Diſtances in the Field, to protract the ſame on Paper or

Parchment, cannot be difficult; for if you draw a Line at pleaſure as *E B* repreſenting the Diameter of your Inſtrument about the middle thereof, as at *A*, mark a Mark, and opening your Compaſſes to 60 Degrees in your Line of Chords, upon *A* as a Center deſcribe a Circle, then lay your Field book before you ſeeing that your firſt Obſervattion cut no Degrees, there are no Degrees to be marked out in the Circle, but the Degrees at *C* are 40. 75 which being taken from your Line of Chords, you muſt ſet them from *H* to *I*, and draw the Line *A I*. the Degrees at *D* are 96 which muſt in like manner be ſet from *I* to *K*, and ſo the reſt in order.

This done obſerve by your Field-book the length of every Line, as the Line *AB* at your firſt Obſervation was 11 Chains and 36 Links, which being by your Scale ſet from *A* will give the Point *B* in the Paper, the ſecond diſtance being ſet upon *AI* will give the Point *C*, and ſo proceeding with the reſt, you will have the Points *B C D E F* and *G*, by which draw the Lines *B C*. *C D*. *D E*; *E F*. *F G* and *G B*, and ſo at laſt you have the Figure of the Field upon your Paper, as was required.

And what is here done at one ſtation, may be done at two or more, by meaſuring one or two diſtances from your firſt ſtation, taking at every ſtation, the Degrees and diſtances to as many Angles, as are viſible at each ſtation.

And as for taking the the Plot of a Field by Interſection of Lines, he that doth but conſider how the diſtances of ſeveral Objects may be taken at two ſtations, will be able to do the other alſo, and therefore I think it needleſs, to make any illuſtration by example.

CHAP.

CHAP. XII.

How to take the Plot of a Wood, Park or other Champion Plain, by going round the same, and making Observation at every Angle.

BY these Directions which have been already given, may the Plot of any Field or Fields be taken, when the Angles may be seen alone or more stations within the Field, which though it is the case of some Grounds, it is not the case of all; now where observation of the Angles cannot be observed within, they must be observed without, and although this may be done by the plain Table, yet as I judge it may be more conveniently done by the Theodolite, in these cases thereof I chiefly commend that Instrument, I know some use a Mariners Compass, but the working with a Needle is not only troublesom, but many times uncertain, yet if a Needle be joyned with the Theodolite the joynt Observations of the Angles may serve to confirm one another.

2. Suppose the *Fig.* 32. to be a large Wood whose Plot you desire to take; Having placed your Instrument at the Angle *A*, lay your Ruler on the Diameter thereof, turning the whole Instrument till through the Sights you espy the Angle at *K*, then fasten it there, and turn your Ruler upon the Center, till through the Sights you espy your second Mark at *B*, the Degrees cut by the Ruler do give the quantity of that Angle *BAK*, suppose 125 Degrees, and the Line *AB* 6 Chains, 45 Links, which you must note in your Field-book, as was shewed before.

3. Then

3. Then remove your Instrument to *B*, and laying your Ruler upon the Diameter thereof, turn it about, till through the Sights you can espy your third mark at *C*, and there fasten your Instrument, then turn the Ruler backward till through the Sights you see the Angle at *A*, the Degrees cut by the Ruler being 106. 25 the quantity of the Angle *ABC*, and the Line *BC* containing 8 Chains and 30 Links, which note in your Field-book, as before.

4. Remove your Instrument unto *C*, and laying the Ruler on the Diameter thereof, turn the Instrument about till through the Sights you see the Angle at *D*, and fixing of it there, turn the Ruler upon the Center till you see your last station at *B*, and observe the Degrees cut thereby, which suppose to be 134 Degrees, and the Line *CD* 6 Chains 65 Links, which must be entered into your Field-book also, and because the Angle *BCD* is an inward Angle, note it with the Mark > for your better remembrance.

5. Remove your Instrument unto *D*, and laying the Ruler on the Diameter, turn the Instrument about, till through the Sights, you see the Angle at *E*, and there fixing your Instrument, turn your Ruler backward till you espy the Mark at *C*, where the Degrees cut are, suppose 68. 0 and the Line *DE* 8 Chains and 23 Links.

6. Remove your Instrument unto *E*, and laying the Ruler on the Diameter, turn the Instrument about, till through the Sights you see the Angle at *F*, and there fix it, then turn the Ruler backward till you see the Angle at *D*, where the Degrees cut by the Ruler suppose to be 125 and the Line *EF* 7 Chains and 45 Links.

7. Re-

7. Remove your Instrument unto *F*, and laying your Ruler upon the Diameter, turn the Instrument about, till through the Sights, you see the Angle at *G*, where fix the same, and turn the Ruler backward till you see the Angle at *E*, where the Degrees cut by the Ruler are 70, and the Line *F G* 4 Chains 15 Links, which must be set down with this > or the like Mark at the Angle.

8. Remove your Instrument unto *G*, and laying your Ruler upon the Diameter, turn the Instrument about, till through the Sights you see the Angle at *H*, where fix the same, and turn the Ruler backward till you see the Angle at *F*, where the Degrees cut by the Ruler are 65. 25, and the Line *G H* 5 Chains 50 Links.

9. Remove your Instrument in like manner to *H* and *K*, and take thereby the Angles and Distances as before, and having thus made observation at every Angle in the Field, set them down in your Field-book, as was before directed, the which in our present Example will be as followeth.

A	151.00	6.45
B	106.25	8.30
C >	134.00	6.65
D	68.00	8.23
E	125.00	7.45
F >	70.25	4.15
G	65.25	5.50
H	130.00	6.50
K	140.00	11.00

The taking of the inward Angles *B C D* and *E F G* was more for Conformity sake than any necessity,

neceſſity, you might have removed your Inſtrument from *B* to *D*, from *E* to *G*, the Length of the Lines *BC*. *CD*. *EF* and *G*, would have given by protraction the Plot of the Field without taking theſe Angles by obſervation; many other compendious ways of working there are, which I ſhall leave to the diſcretion of the Ingenious Practitioner.

10. The Angles and Sides of the Field being thus taken, to lay down the ſame upon Paper, Parchment, another Inſtrument called a Protractor is convenient, the which is ſo well known to Inſtrument-makers, that I ſhall not need here to deſcribe it, the chief uſe is to lay down Angles, and is much more ready for that purpoſe than a Line of Chords, though in effect it be the ſame.

11. Having then this Inſtrument in a readineſs draw upon your Paper or Parchment upon which you mean to lay down the Plot of that Field, a Line at pleaſure as *AB*. Then place the Center of your Protractor upon the Point *A*, and becauſe the Angle of your firſt obſervation at *A* was 115 Degrees 00 Parts, turn your Protractor about till the Line *AK* lie directly under the 115 Degree; and then at the beginning of your Protractor make a Mark, and draw the Line *AB*, ſetting off 6 Chains 45 Links from *A* to *B*.

12. Then lay the Center of your Protractor upon the Point *B*, and here turn your Protractor about, till the Line *AB* lie under 106 Degrees 25 Parts, and draw the Line *BC*, ſetting off the Diſtance 8 Chains, 30 Links from *B* to *C*.

13. Then lay the Center of your Protractor upon the Point *C*, and turn the ſame about till the Line *BC* lie under 134 Degrees, but remember-

to

to make it an inward Angle, as it is marked in your Field-Book, and there make a Mark, and draw the Line *C D*, ſetting off 6 Chains, 65 Links from *C* to *D*.

And thus muſt you do with the reſt of the Sides and Angles, till you come to protract your laſt Angle at *H*, which being laid down according to the former Directions the Line *HK* will cut the Line *AK* making *AK* 11 Chains and *HK* 6 Chains, 50 Links. This work may be alſo performed by protracting your laſt obſervation firſt; for having drawn the Line *AK*, you may lay the Center of your Protractor upon the Point *K*, and the Diameter upon the Line *AK*; and becauſe your Angle at *K* by obſervation was 140 Degrees, you muſt make a Mark by the Side of your Protractor at 140 Degrees; and draw the Line *K H*, ſetting off 6 Chains, 50 Links from *K* to *H*. And thus proceeding with the reſt of the Lines and Angles, you ſhall find the Plot of your Field at laſt to cloſe at *A*, as before it did at *K*.

CHAP. XIII.

The Plot of the Field being taken by any Inſtrument, how to compute the Content thereof in Acres, Roods, and Perches.

THe meaſuring of many ſided plain Figures hath been already ſhewed in the 13 *Propoſition* of the 8 *Chapter*, which being but well conſidered, to compute the Content of a Field cannot be difficult; It muſt be remembred indeed that 40 ſquare Pearches do make an Acre.

2. Now

2. Now then if the Plot be taken by a four Pole Chain divided into 100 Links, as 16 ſquare Poles are the tenth part of an Acre; ſo 10.000 ſquare Links of ſuch a Chain are equal to 16 ſquare Pole, or Perches; and by conſequence 100.000 ſquare Links are equal to an Acre, or the ſquare Pearches.

3. Having then converted your Plot into Triangles, you muſt caſt up the Content of each Triangle as hath been ſhewed, and then add the ſeveral Contents into one Sum, and from the aggregate cut off five Figures towards the right hand; the remainer of the Figures towards the left hand are Acres, and the five Figures ſo cut off towards the right hand are parts of an Acre, which being multiplied by four, if you cut off five Figures from the Product, the Figures remaining towards the left hand are Roods, and the five Figures cut off are the parts of a Rood, which being multiplied by forty, if you cut off five Figures from the Product, the Figures remaining towards the left hand are Perches, and the Figures cut off are the Parts of a Pearch.

Example. Let 258.94726 be the Sum of ſeveral Triangles, or the Content of a Field ready caſt up, the three Figures towards the left hand 258 are the Acres, and the other Figures towards the right hand 94726 are the Decimal Parts of an Acre, which being multiplied by 4, the Product is 3.78904, that is three Roods and 78904 Decimal Parts of a Rood, which being multiplied by 40, the Product is 31.56160, that is 31 Perches and 56160 Decimal Parts of a Perch; and therefore in ſuch a Field there are Acres 258, Roods 3, Pearches 31, and 56160 Decimal Parts of a Perch.

CHAP.

CHAP. XIV.

How to take the Plot of Mountainous and uneven Grounds, and how to find the Content.

WHen you are to take the Plot of any Mountainous or uneven piece of Ground, such as is that in *Figure* 33, you must first place your Instrument at *A*, and direct your Sights to *B*, measuring the Line *AB*, observing the Angle *GAB*, as was shewed before, and so proceed from *B* to *C*, and because there is an ascent from *C* to *D*, you must measure the true length thereof with your Chain, and set that down in your Book, but your Plot must he drawn according to the length of the Horizontal Line, which must be taken by computing the Base of a right angled Plain Triangle, as hath been shewed before, and so proceed from Angle to Angle until you have gone round the Field, and having drawn the Figure thereof upon your Paper, reduce into Triangles and *Trapezius*, as *ABC. CDE. ACEF* and *AFG.* then from the Angles *B. C. D. F* and *G*; let fall the Perpendiculars, *BK. CN. DL. FM.* and *GH.* This done you must measure the Field again from Angle to Angle, setting down the Distance taken in a straight Line over Hill and Dale, and so likewise the several Perpendiculars, which will be much longer than the streight Lines measured on your Scale, and by these Lines thus measured with your Chain cast up the Content; which will be much more than the Horizontal Content of that Field according to the Plot, but if it should be otherwise plotted than by the Horizontal

rizontal Lines, the Figure thereof could not be contained within its proper limits; but being laid down among other Grounds, would force ſome of them out of their places, and therefore ſuch Fields as theſe muſt be ſhadowed off with Hills, if it be but to ſhew that the Content thereof is computed according to the true length of the Lines from Corner to Corner, and not according to their Diſtance meaſured by Scale in the Plot.

CHAP. XV.

How to reduce Statute Meaſure into Cuſtomary, and the contrary.

WHereas an Acre of Ground by Statute Meaſure is to contain 160 ſquare Perches, meaſured by the Pole or Perch of ſixteen foot and a half: In many places of this Nation, the Pole or Perch doth by cuſtom contain 18 foot, in ſome 20. 24. 28 Foot; it will be therefore required to give the Content of a Field according to ſuch ſeveral quantities of the Pole or Perch.

2. To do this you muſt conſider how many ſquare Feet there is in a Pole according to theſe ſeveral Quantities.

In 16. 5 to the Pole, there are 272. 25 ſq. feet.
In 18 to the Pole there are 324 ſquare feet.
In 20 to the Pole there are 400 ſquare feet.
In 24 to the Pole there are 576 ſquare feet.
In 28 to the Pole there are 784 ſquare feet.

Now then if it were deſired to reduce 7 Acres, 3 Roods, 27 Perches, according to Statute Meaſure, into Perches of 18 Foot to the Perch; firſt reduce

duce your given quantity, 7 Acres. 3 Rods, 27 Poles into Perches, and they make 1267 Perches.

Then say, as 324. to 272. 25. so is 1267 to 1065.6. that is 1065 Perches, and 6 tenths of a Perch. But to reduce customary Measure into statute measure, say as 272.25. is to 324 so is 1267 Perches in customary measure, to 1507. 8 that is 1507 Perches and 8 tenths of a Perch in statute measure, the like may be done, with the customary measures of 20. 24 and 28 or any other measure that shall be propounded.

CHAP. XVI.

Of the Measuring of solid Bodies.

HAving shewed how the content of all plains may be computed, we are now come to the measuring of solid Bodies, as Prisms, Pyramids and Spheres, the which shall be explained in the Propositions following.

Proposition. I.

The base of a Prism or Cylinder being given, to find the solid content.

The base of *a Prism* is either Triangular, as the *Pentahedron*; Quadrangular, as the *Hexahedron*, or Multangular, or the *Polyhedron* Prism, all which must be computed as hath been shewed, which done if you multiply the base given by the altitude, the product shall be the solid content required.

Example. In an *Hexahedron* Prism, whose base

is quadrangular, one ſide of the Baſe being 65 foot and the other 43, the Superficies or Baſe will be 27.95. Which being multiplyed by the Altitude, ſuppoſe 12.5. the product. 359.375. is the ſolid content required.

In like manner the Baſe of a Cylinder being 45.6. and the altitude 15.4. the content will be 702.24.

And in this manner may Timber be meaſured whether round or ſquared, be the ſides of the ſquared Timber equal or unequal.

Example. Let the Diameter of a round piece of Timber be 2.75 foot. Then, As 1 it to 785397. ſo is the ſquare of the Diameter 2.75. to 5.9395 the Superficial content of that Circle.

Or if the circumference had been given 8.64. then, As 1 is to 079578, ſo is the ſquare of 8.64. to 5.9404 the ſuperficial content.

Now then if you multiply this Baſe 5.94. by the length, ſuppoſe 21 foot, the content will be 124.74.

If the ſide of a piece of Timber perfectly ſquare be 1.15 this ſide being multiplyed by it ſelf, the product will be 1.3225 the ſuperficial content, or content of the Baſe, which being multiplyed by 21 the length, the content will be 27.7745.

Or if a piece of Timber were in breadth 1.15. in depth 1.5 the content of the Baſe would be 1.725 which being multiplied by 21 the length, the content will be. 36.225.

Propoſition. II.

The Baſe and Altitude of a Pyramid or Cone being given, to find the ſolid content.

Multiply

Multiply the Altitude by a third part of the Base, or the whole Base by a third part of the Altitude, the Product shall be the solid content required.

Example. In a Pyramid having a Quadrangular Base as in *Fig.* 22. The side *CF* 17. *CD* 9. 5. the Product is the Base *CDEF*. 161. 5, which being multiplyed by 10.5 the third of the Altitude *AB* 31. 5 the Product is 1695. 75 the content. Or the third of the Base. *viz.* 53. & 3 being multiplied by the whole Altitude *AB* 31.5 the Product will be the content as before.

2. Example. In *Fig.* 21. Let there be given the Diameter of the Cone *AB* 3. 5. The Base will be 96.25. whose Altitude let be *CD* 16.92 the third part thereof is 5.64 & 96.25 being multiplied by 5.64, the Product 542.85 is the solid content required.

Proposition. III.

The Axis of a Sphere being given, to find the solid content.

If you multiply the Cube of the Axis given by 523598 the solid content of a Sphere whose Axis is an unite, the Product shall be the solid content required.

Example. Let the Axis given be 3, the Cube thereof is 27, by which if you multiply. 523598, the Product 14.137166 is the solid content required.

Proposition. IV.

The Basis and Altitude of the Frustum *of a Pyramid or Cone being given, to find the content.*

If the aggregate of both the Bases of the *Frustum* and the mean proportional between them, shall be multiplied by the third part of the Altitude, the Product shall be the solid content of the *Frustum*.

Example. In *Fig.* 22. Let *CDEF* represent the greater Base of a Pyramid, whose superficial content let be 1.92, and let the lesser Base be *HGLKO*. 85 the mean proportional between them is. 1.2775 and the aggregate of these three numbers is. 4.0475. Let the given Altitude be 15. the third part thereof is. 5 by which if you multiply 4.0475 the Product 20.2375 is the content of the *Frustum* Pyramid.

And to find the content of the *Frustum* Cone. I say.

As. 1. to 78539. so 20.23 to 15. 884397, the content of the Cone required.

But if the Bases of the *Frustum* Pyramid shall be square, you may find the content in this manner.

Multiply each Diameter by it self and by one another, and the aggregate of these Products, by the third part of the altitude, the last Product shall be the content of the *Frustum* Pyramid.

Example. Let the Diameter of the greater Base be 144, the Diameter of the lesser Base 108, and the altitude 60.

The

The Square of 144 is	20736
The Square of 108 is	11664
The Product of 144.108 is	15552
The Sum of these 3 Products is	47952

Which being multiplyed by 20 the third part of the Altitude, the Product 959040 is the content of the *Frustum* Pyramid.

And this content being multiplied by .785 39 the content of the *Frustum* Cone will be .753.228.

Another way.

Find the content of the whole Pyramid of the greater and lesser Diameter, the lesser content deducted from the greater, the remain shall be the content of the *Frustum*. To find the content of the whole Pyramid, you must first find their several Altitudes in this manner.

As the difference between the Diameters,
Is to the lesser Diameter.
So is the Altitude given, to the Altitude cut off.

Example. The difference between the former Diameter. 144. and 108 is 36, the Altitude 60. now then As 36. 108 :: 60. 108. the altitude cut off.

Now then if you multiply the lesser Base 11664 by 60 the third part of 180 the Product 699840 is the content of that Pyramid.

And adding 60 to 180 the Altitude of the greater Pyramid is 240, the third part whereof is 80, by which if you multiply the greater Base before found, 70736, the Product is the content of the

greater Pyramid. 1658880, from which if you deduct the leſſer 699840 the remainer 959040 is the content of the *Fruſtum* Pyramid as before.

And upon theſe grounds may the content of Taper Timber, whether round or ſquare, and of Brewers Tuns, whether Circular or Elliptical, be computed, as by the following Propoſitions ſhall be explained.

Propoſition. V.

The breadth and depth of a Taper piece of Squared Timber, both ends being given together with the length, to find the content.

Let the given Dimenſions.
At the Bottom be *A*.5.75 and *B* 2.34
At the Top. *C*.2.16 and *D*.1.83
And let the given length be 24 Foot.

According to the laſt Propoſition, find the Area or Superficial content of the Tree at both ends thus.

Multiply the breadth	3.75	0.574031
By the depth	2.34	0.369215
The Product	8.7750	0.943246
2. Multiply the breadth	2.16	0.334453
By the depth	1.82	0.262451
The Product is	3.9528	0.596904

3. Multi-

3. Multiply the 1. Content	8.7750	0.943246
by the second content.	3.9528	0.596904
And find the square root	5.8986	1.540150
		0.770075

The Sum of these 18.6264 being multiplyed by 8 one third of the length, the content will be found to be 149.0112. Thus by the Table of Logarithms the mean proportional between the two Bases is easily found, and without extracting the square Root, may by natural Arithmetick be found thus.

$A + \frac{1}{2} C$ x A half C multiplyed by B: And C more half A multiplyed by D being added together and multiplyed by 30, the length shall give the content. Example.

A 3.75		C 2.16	
$\frac{1}{2} C$ 1.08		$\frac{1}{2} A$ 1.875	
Sum 4.83	Sum.	4.035	
B 2.34	D.	1.83	
1932		12105	
1449		32280	
966		4035	
11.3022		7.38405	
		11.30220	
The sum of the Products		18.68625	

Being multiplyed by 8 the third of the length, the content will be. 149.49000. The like may be done for any other.

Propoſition. VI.

The Diameters of a piece of Timber being given at the Top and and Bottom, together with the length, to find the content.

The Propoſition may be reſolved either by the Squares of the Diameters, or by the Areas of the Circles anſwering to the Diameters given, for which purpoſe I have here annexed not only a Table of the Squares of all numbers under a thouſand, but a Table ſharing the third part of the Areas of Circles in full meaſure, to any Diameter given under 3 foot.

And therefore putting $S =$ The Sum of the Tabular numbers anſwering to the Diameters at each end.

$X =$ The difference between theſe Diameters.

$L =$ the length of the Timber, $C =$ The content.

Then $1\frac{1}{2} S = \frac{1}{4}\text{-}XX. \times L. = C.$

If you work by the Table of the ſquares of Numbers. you muſt multiply the leſs ſide of the Equation, by 0.26179 the third part of 0.78539 the Product being multiplyed by the length, will give the content.

But if you work by the Table of the third parts of the Areas of Circles in full meaſure, the tabular Numbers being multiplyed by the length will give the content. Only inſtead of the ſquare of the difference of the Diameter, you muſt take half the Tabular number anſwering to that Difference, and you ſhall have the content as before. Example.

Let

Let the greatest Diameter by 2.75, and the less 1.93.

Their difference is 0.83

The square of 2.75 is	7.5625
The square of 1.93 is	3.7249.
The Sum of the Squares	11.2874
The half Sum	5.6437
The Sum of them is	16.9311
Half the square of 0.82 deduct.	0.3362
The Difference is	16.5949
Which being multiplyed by	26179
	1493541
	1161643
	165949
	995694
	331898
The Product will be.	4.344378871

Or by the Table of Areas.

The Area of 2.75 is	1.979857
The Area of 1.93 is	0.975176
The Sum	2.955033
The half Sum	1.477516
The Sum of them	4.432549
Half the Area of 0.82 deduct	0.088016
The former Product	4.344533
Which being multiplyed by	24
	17378132
	8689066
The content is	104268792

But because that in measuring of round Timber the circumference is usually given and not the Diameter, I have added another Table by which the circumference being given, the Diameter may be found.

Example. Let the circumference of a piece of Timber be 8325220 looking this Number in the second column of that Table, I find the next less to be 8.168140 and thence proceeding in a streight Line, I find that in the seventh Column the Number given, and the Diameter answering thereunto to be 2.65. and thus may any other Diameter be found not exceeding the three foot. The Proportion by which the Table was made, is thus. As 1. to 3.14159 so is the Diameter given, to the circumference required.

Or

Fig. 29.

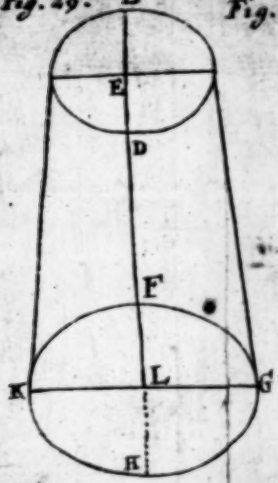


Fig. 30.

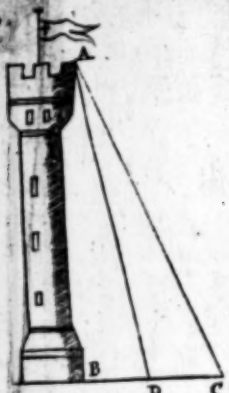


Fig. 32.

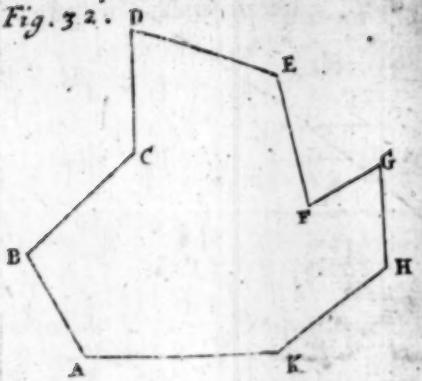


Fig. 31.

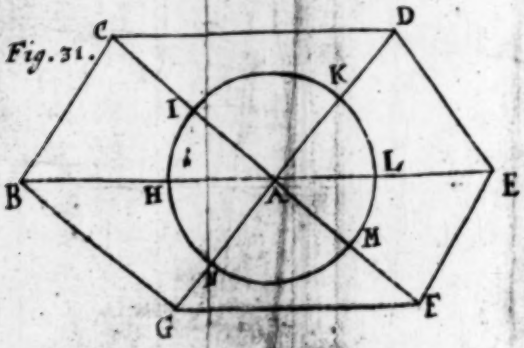
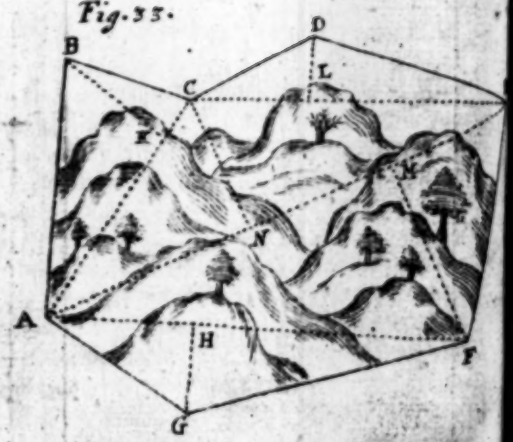


Fig. 33.



Or the Circumference being given, to find the Diameter, say: As. 1.10.0.3183, so is the Circumference given to the Diameter required.

And although by these two Tables all round Timber may be easily measured, yet it being more usual to take the Circumference of a Tree, then the Diameter, I have here added a third Table, shewing the third part of the Areas of Circles answering to any circumference under 10 foot, and that in Natural and Artificial numbers, the use of which Table shall be explained in the Proposition following.

Proposition. VII.

The Circumference of a piece of round Timber at both ends, with the length being given, to find the content.

The Circumference of a Circle being given, the Area thereof may be found as hath been shewed, in the 7 Chapter, Proposition 4. and by the first Proposition of this; and to find the third part of the Area, which is more convenient for our purpose I took a third part of the number given by which to find the whole, that is a third part of 0.07957747 that is 0.02652582 and having by the multiplying this number by the square of the Circumference computed three or four of the first numbers, the rest were found by the first and second differences.

The Artificial numbers were computed by adding the Logarithms of the Squares of the circumference, to 8.42966891 the Logarithm of 0.02652582.

And by these Natural and Artificial numbers

the

The content of round Timber may be found two ways

By the Natural numbers in the same manner as the content was computed, the Diameters being given, and by the Natural and Artificial numbers both, by finding a mean proportional between the two Areas at the top and bottom of the Tree, as by Example shall be explained.

Let the given Dimensions, or Circumferences be
At the Bottom 9.95
At the Top 3.75 Their difference is 6.20

The tabular Numbers.

	Natural	Artificial.
Answering to 9.95	2.626162	0.418931
And to 3.75	0.373019	9.571731
The Sum of the Logarith.		9.990662
The half Sum or Logarith. 989300		9.995331
The Sum of the Number is 3.988481		
The Sum of the Natural Numbers is		2.989181
The half Sum		1.499190
The Sum of them		4.498771
Half the number answer. to 6.20 is		0.509826
The remainer is		3.988945

Which being multiplyed by the length 24, the content will be 95.73468.

Mr. *Darling* in his Carpenters Rule made easie, doth propound a shorter way, but not so exact, which is by the Circumference given in the middle of the piece to find the side of the Square, namely

ly by multiplying the Circumference given by 28209, or 2821. which side of the Square being computed in Inches, and lookt in his Table of Timber measure, doth give the content of the Tree not exceeding 31 foot in length, the which way of measuring may be as easily performed by this Table. *Example.*

The circumference at the top and bottom of the Tree being given 9.95 and 3.75 the Sum is 13.70
The half thereof is the mean circumfer. 6.85
Which sought in the Table, the Numbers are.

The Natural number is 1.244657, which being multiplyed by 3 the Product is 3.733971, which multiplyed by the length 24, the content is 89.615304.

The Artificial number is	0.095049
The Logarithm of 24 is	1.380211
The Absolute Number 29.871	1.475260
Which multiplyed by 3, the Product is	89613

Proposition. VIII.

The Diameters of a Brewers Tun at top and bottom being given with the height thereof, to find the content.

In *Fig.* 29. Let the given Diameter.
At the top be *AC* 136 *BD* 128 Altit. 51 Inches.
At the bottom. *KG* 152 *HF* 144

The which by the 5 Proposition of this Chap. may thus be computed. *AC* 139 + *KG* 76 = 212 × *BD* 128 the Product is 27136.

And

And KG 1524 : AC 68 = 220 × HF 144 the Product is 31680. the Sum of these 2 Products is 58816 which being multiplyed by one third of 51, that is by 17, and that Product multiplyed by 26179 the third of 78539 will give the content.

The Logarithm of 58816. is	54-76949
The Logarithm of 17 is	1.230449
The Product	1.999944
The Logarithm of. 26179	9.417968
The content is. 261765	5.417912

Thus the content of a Tun may be found in Inches, which being divided 282 the number of Inches in an Ale Gallon, the quotient will be the content in Gallons.

Or thus; divide the former. 26179 by 282 the quotient will be. 00092836. by which the content may be found in Ale Gallons in this manner.

The former Product	5.999944
The Logarithm of 0.00092836	6.967719
The content in Gallons 928.24	2.967663

Proposition. IX.

The Diameters of a close Cask, at head and bung with the length given, to find the content.

In the resolving of this Proposition, we are to consider the several forms of Casks, as will as the kind of the Liquor, with which it is filled, for one and the same Rule will not find the content in all Cask. And

And a Coopers Cask is commonly taken, either for the middle *Fruſtum* of a Spheroid, the middle *Fruſtum* of a Parabolical Spindle, the middle *Fruſtum* of two Parabolick Conoids, or for the middle *Fruſtum* of two Cones abutting upon one common Baſe.

And the content of theſe ſeveral Casks may be found either by equating the Diameters, or by equating the Circles. for the one, a Table of Squares is neceſſary, and a Table ſhewing the third part of the Areas of a Circle to all Diameters. The making of the Table of Squares, every one knows, to be nothing elſe but the Product of a Number multiplyed, by it ſelf, thus the Square of 3 is 9. the Square of 8 is 64 and ſo of the reſt.

And the Area of a Circle to any given Diameter may be found, as hath been ſhewed, in Chap. 7 Propoſition 2. But here the Area of a Circle in Inches, will not ſuffice, it will be more fit for uſe, if the third part of the Area be found in Ale and Wine Gallons both, the which may indeed be done by dividing the whole Area in Inches by 3 and the quotient by 282 to make the Table for Ale-meaſure, and by 231 to make the Table for Wine-meaſure; but yet theſe Tables (*as I think*) may be more readily made in this manner.

The Square of any Diameter in Inches, being divided by 3.81972 will give the Area of the Circle in Inches: And this Diviſion being multiplyed by 282 will give you 1077.161 for a common Diviſion, by which to find the Area in Ale-Gallons, or being multiplyed by 231 the Product, 882.355 will be a common Diviſion by which to find the Area in Wine-Gallons.

But becauſe it is eaſier to multiply then divide:

If

If you multiply the ſeveral Squares by 26178 the third part of 78539 the Product will give the Area in Inches, or if you divide. 26179 by 282 the quotient will be .00092886 for a common Multiplicator, by which to find the Area in Ale-Gallons, or being divided by 231 the quotient will be 00113333 a common Multiplicator, by which to find the content in Wine-Gallons. An Example or two will be ſufficient for illuſtration. Let the Diameter given be 32 Inches, the Square thereof 1024 being divided by 3.81970 the quotient is 268.083, and the ſame Square 1024 being multiplyed by 261799, the Product will be 268.082.

Again if you divide 1024 by 1077.161 the quotient will be 9508, or being multiplied by 00092836, the Product will be 9508.

Laſtly if you divide 1024 by 882.755, the quotient will be 1.1605, or being multiplied by 00113333 the Product is 1.1605,

And in this manner may the Tables be made for Wine and Beer-meaſure, but the ſecond differences in theſe Numbers being equal, three or four Numbers in each Table being thus computed, the reſt may be found by Addition only.

Thus the Squares of 1. 2. 3. and 4 Inches are, 1. 4. 9 and 16 by which if you multiply 00113333, the ſeveral Products will be third part of the Area, of the Circles anſwering to thoſe Diameters in Wine-Gallons. Or 00092836 being multiplied by thoſe Squares, the ſeveral Products, will be the third part of the Areas of the Circles anſwering to thoſe Diameters in Ale-Gallons; the which with their firſt and ſecond differences are as followeth.

The

The Products or Areas in Wine-Gallons.

1.	00113333		
2.	00453332	339999	226666
3.	01019997	566665	226666
4.	01813328	796331	

The Products in Ale-Gallons.

1.	00092836		
2.	00371344	278508	185672
3.	00835524	464180	185672
4.	01485376	649852	

And by the continual addition of the second differences to the first, and the first differences to the products before found, the Table may be continued as far as you please.

The construction of the Tables being thus shewed: We will now shew their use in finding the content of any Cask.

Let S = the Sum of the Tabular Numbers answering to the Diameters at the Head and Bung. D = their difference. X = the difference of the Diameters themselves. L = the length of the Vessel, and C = the content thereof.

1. If a Cask be taken for the middle *Frustum* of a Spheroid, intercepted between two Planes parallel, cutting the Axis at right Angles: Then $1\frac{1}{3}S + \frac{1}{3}D \times L = C$.

2. If a Cask be taken for the middle *Frustum* of a parabolical Spindle, intercepted between two planes parallel cutting the Axis at right Angles. Then $1\frac{1}{2}S + \frac{1}{2}D \times L = C$.

3. If a Cask be taken for the middle *Frustum* of two Parabolick Conoids, abutting upon one common Base, intercepted between two Planes parallel, cutting the Axis at right Angle: Then $1\frac{1}{4}S : \times L = C$.

4. If a Cask be taken for the middle *Frustum* of two Cones, abutting upon one common base, intercepted between two Planes parallel cutting the Axis at Right Angles. Then $1\frac{2}{3}S - \frac{1}{3}XX. \times L = C$.

In all these four Equations, if you work by the Table of Squares of numbers, you must multiply the less side of the Equation by 262, if you would have the content in Cubical Inches; by 001133 if you would have the content in Wine-Gallons; and by 000928, if you would have the content in Ale-Gallons.

But if you work by the Tables of the third parts of the Areas Circle, the Tabular Numbers being multiplyed by the length only will give the content required, only in the fourth Equation instead of half the Square of the Difference of the Diameters, take half the Tabular Number answering to that difference, and you shall have the content required; as by the following Examples will better appear, then by many words.

Examples in Wine-measure by the Table of the Squares of Numbers.

The Diameter of a Vessel
At the Bung being 32 Inches.
At the Head 22 Inches.
The difference of the Diameters 10 Inches.
And the length of the Vessel 44 Inches.

Spheroid

Spheroid.	Parabolick Spindle.
1024	1024
484	484
1508	1508
754	754
270	540
2532	23160
2532	23160
7596	69480
7596	69480
7596	69480
28695156	262472280
44	44
114780624	104988912
114780624	104988912
126.2586864	115.4878032

Parabolick Conoid	Cone.
1024	1024
484	484
1508	1508
754	754
	50
2262	2212
2262	2212
6786	6636
6786	6636
6786	6636
25635246	25068596
44	44
102540984	100274384
102540984	100274384
112.79508241	110.30182224

This

This which hath been done by the Table of Squares may be more easily performed, by the Table of the third part of the Areas of Circles, ready reduced to Wine-Gallons.

Spheroid	Parabolick Spindle.
1.16053	1.16053
0.54853	0.54853
1.70906	1.70906
85453	85453
30600	61200
2.86959	2.624790
44	44
1147836	1049916
1147836	10499160
126.26196	115.490760

Parabolick Conoid	Cone.
1.16053	1.16053
0.54853	0.54853
1.70906	1.70906
85453	85453
	56666
2.56359	2.506924
44	44
1025436	10027696
1025436	10027696
112.79796	110.304656

Examples

Examples in Ale-measure by the Table of the Squares of Numbers.

Spheroid.	Parabolick Spindle.
1024	1024
484	484
1508	1508
754	754
270	540
2532	2316.0
00092836	.00092836
22758.	20844.
5064.	4632.
20256.	18528.
7596.	6948.
15192	138969
235660752	2.150081760
44	44
948623008	860032704
940643008	860032704
103.22673088	94.60359744

Parabolick Conoid	Cone.
1024	1024
484	484
1508	1508
754	754
	50
2262	2212
20358.	19909.
4524.	4424.
18096.	17696.
6786.	6636.
13527	13272
2.09995032	2.05423232
44	44
8.39980128	821692928
839980128	821692928
92.39781408	90.38622208

By

By the Areas of Circles.

Spheroid.	Parabolick Spindle.
0.95052	0.95052
0.44930	0.44930
1.39982	1.39982
.69991	69991
.25061	050122
2.35034	2.149852
34	44
940136	8599408
940136	8599408
103.41496	94.593488

Parabolick

Parabolick Conoid.	Cone.
0.95052	0.95052
0.44930	0.44930
1.39982	1.39982
69991	.69991
	46425
209973	2.053305
44	44
839892	8213220
839892	8213220
90.345420	90.345420

And here for the Singularity of the Example, I will set the Dimensions of a Cask lately made in *Herefordshire*, for that excellent Liquor of Red streak Cyder, the like whereof either for the largeness of the Cask, or incomparable goodness of that kind of Drink, is not to be found in all *England*, nay and perhaps not in the World.

The length of the Cask is 104 Inches.
The Diameter at the Bung 92 Inches.
And the Diameter at the Head 74 Inches.

The

The Numbers in the Table of Ale-Gallons answering to these Dimensions are.

	Spheroid	Parabolick Spindle.
Bung.92	7.859639	7.859639
Head.74	5.083699	5.083699
	12.941338	12.941338
	6.470669	6.470569
	1.386770	.277394
	20.798777	19.689401
	104	104
	83195108	78.757604
	20798777	19689401
Con.	2163.072808	2047.697704

Parabolick

Parabolick Conoid.	Cone.
7.857639	7.857639
5.083699	5.083699
12.941338	12.941338
6.470669	6.470669
	0.150394
19.412007	19.261613
104	104
77648028	77046452
19412007	19261613
201.8.848728	2003.207752

And thus you have the content of this Cask by four ſeveral Ways of Gauging, but that which doth beſt agree with the true content, found by theſe that filled the ſame is the ſecond way or that which takes a Cask to be the middle *Fruſtum* of a Parabolick Spindle, according to which the content is 2047 Gallons. That is allowing 64 Gallons to the Hogſhead. 32 Hogſheads very near.

Propoſition,

Proposition. X.

If a Cask be not full, to find the quantity of Liquor contained in it, the Axis being posited parallel to the Horizon.

To resolve this Proposition, there must be given the whole content of the Cask, the Diameter at the Bung, and the wet Portion thereof, then by help of the Table of Segments, whose Area is unity, and the Diameter divided into 10.000 equal parts, the content may thus be found.

As the whole Diameter, is to its wet Portion.

So is the Diameter in the Table. 10.000 to its like Portion, which being sought in the Table of Segments, gives you a Segment, by which if you multiply the whole content of the Cask, the Product is the content of the Liquor remaining in the Cask.

But in the Table of Segments in this Book, you have the Area, to the equal parts of one half of the Diameter only, when the Cask therefore is more then half full, you must make use of the dry part of the Diameter instead of the wet, so shall you find what quantity of Liquor is wanting to fill up the Cask, which being deducted from the whole content of the Cask; the remainer is the quantity of Liquor yet remaining, an Example in each will be sufficient, to explane the use of this Table.

1. *Example,* In a Wine Cask not half full, let the great Diameter be as before 32 Inches, the content

content 126.25 Gallons, and let the wet part of the Diameter be 12 Inches, First I say.

As the whole Diameter 32. is to the wet part 12. so is 10.000 to 3750, which being sought in the Table, I find, the Area of that Segment to be.342518 which being multiplyed by the whole content of the Cask 126.25, the Product is 43.2428750 and therefore there is remaining in the Cask 43 & ½ *ferè*.

2. Example. In the same Cask let the wet part of the Diameter be 18 Inches. I say.

As 32.18 :: 10000.5625 whose Complement to 10000 is 4375 which being sought in the Table, I find the Area answering thereto to be 420630; now then I say.

As the whole Area of the Circle 1000000 is to the whole content of the Cask 126.25.

So is the Area of the Segment sought.420630, to the content 53.1044375 which is in this case the content of the Liquor that is wanting, this therefore being deducted from the content of the whole Cask, 136.25. the part remaining in the Vessel is.73.1455625.

Thus may Casks be gauged in whole or in part, in which a Table of Squares is sometimes necessary, as being the Foundation, from whom the other Tables are deduced; such a Table therefore is here exhibited, for all Numbers under 1000, by help whereof the Square of any Number under 10.000 may easily be found in this manner.

The Rectangle made of the Sum and Difference of any two Numbers, is equal to the Difference of the Squares of these Numbers.

Example, Let the given Numbers be 36 and 85 their

their Sum is 121, their difference 49, by which if you multiply 121, the Product will be 5929. The Square of 36 is 1296, and the Square of 85 is 7225, the difference between which Squares is 5929 as before.

And hence the Square of any Number under 10.000 may thus be found, the Squares of all Numbers under 1000 being given.

Example. Let the Square of 5715 be required. The Square of 571 by the Table is 326041, therefore the Square of 5710 is 32604100: the Sum of 5710 and 5715 is 11425, and the difference 5, by which if you multiple 11425, the Product is 52125 which being added unto 32604100 the Sum 32656325 is the Square of 5715. The like may be done for any other.

TABLES

TABLES

FOR THE

Measuring

OF

TIMBER,

AND THE

GAUGING

OF

CASKS

AND

Brevvers Tuns.

LONDON,
Printed for *Thomas Passinger* at the three Bibles on *London-Bridge*, 1679.

TABLES

FOR THE

Measuring

OF

TIMBER,

AND

GAUGING

OF

CASKS

AND

Brewers Tuns

1	1	3	34	1156	69	67	4489	135
2	4	5	35	1225	71	68	4624	137
3	09	7	36	1296	73	69	4761	139
4	16	9	37	1369	75	70	4900	141
5	25	11	38	1444	77	71	5041	143
6	36	13	39	1521	79	72	5184	145
7	49	15	40	1600	81	73	5329	147
8	64	17	41	1681	83	74	5476	149
9	81	19	42	1764	85	75	5625	151
10	100	21	43	1841	87	76	5776	153
11	121	23	44	1936	89	77	5929	155
12	144	25	45	2025	91	78	6084	157
13	169	27	46	2116	93	79	6241	159
14	196	29	47	2209	95	80	6400	161
15	225	31	48	2304	97	81	6561	163
16	256	33	49	2401	99	82	6724	165
17	287	35	50	2500	101	83	6889	167
18	324	37	51	2601	103	84	7056	169
19	361	39	52	2704	105	85	7225	171
20	400	41	53	2809	107	86	7396	173
21	441	43	54	2916	109	87	7569	175
22	484	45	55	3025	111	88	7744	177
23	529	47	56	3136	113	89	7921	179
24	576	49	57	3249	115	90	8100	181
25	625	51	58	3364	117	91	8281	183
26	676	53	59	3481	119	92	8464	185
27	729	55	60	3600	121	93	8649	187
28	784	57	61	3721	123	94	8836	189
29	841	59	62	3844	125	95	9025	191
30	900	61	63	3969	127	96	9216	193
31	961	63	64	4096	129	97	9409	195
32	1024	65	65	4225	131	98	9604	197
33	1089	67	66	4356	133	99	9801	199
34	1156	67	67	4489	135	100	10000	201

101	10201	203	134	17956	269
102	10404	205	135	18225	271
103	10609	207	136	18496	273
104	10816	209	137	18769	275
105	11025	211	138	19044	277
106	11236	213	139	19321	279
107	11449	215	140	19600	281
108	11664	217	141	19881	283
109	11881	219	142	20164	285
110	12100	221	143	20449	287
111	12321	223	144	20736	289
112	12544	225	145	21025	291
113	12769	227	146	21316	293
114	12996	229	147	21609	295
115	13225	231	148	21904	297
116	13456	233	149	22201	299
117	13689	235	150	22500	301
118	13924	237	151	22801	303
119	14161	239	152	23104	305
120	14400	241	153	23409	307
121	14641	243	154	23716	309
122	14884	245	155	24025	311
123	15129	247	156	24336	313
124	15376	249	157	24649	315
125	15625	251	158	24964	317
126	15876	253	159	25281	319
127	16129	255	160	25600	321
128	16384	257	161	25921	323
129	16641	259	162	26244	325
130	16900	261	163	26569	327
131	17161	263	164	26896	329
132	17424	265	165	27225	331
133	17689	267	166	27556	333
134	17956	269	167	27889	335

167	27889	335
168	28224	337
169	28561	339
170	28900	341
171	29241	343
172	29584	345
173	29929	347
174	30276	349
175	30625	351
176	30976	353
177	31329	355
178	31684	357
179	32041	359
180	32400	361
181	32761	363
182	33124	365
183	33489	367
184	33856	369
185	34225	371
186	34596	373
187	34969	375
188	35344	377
189	35721	379
190	36100	381
191	36481	383
192	36864	385
193	37249	387
194	37636	389
195	38025	391
196	38416	393
197	38809	395
198	39204	397
199	39601	399
200	40000	401
201	40401	403
202	40804	405
203	41209	407
204	41616	409
205	42025	411
206	42436	413
207	42849	415
208	43264	417
209	43681	419
210	44100	421
211	44521	423
212	44944	425
213	45369	427
214	45796	429
215	46225	431
216	46656	433
217	47089	435
218	47524	437
219	47961	439
220	48400	441
221	48841	443
222	49284	445
223	49729	447
224	50176	449
225	50625	451
226	51076	453
227	51529	455
228	51984	457
229	52441	459
230	52900	461
231	53361	463
232	53824	465
233	54289	467
234	54756	469

234	54756	469
235	55225	471
236	55696	473
237	56169	475
238	56644	477
239	57121	479
240	57600	481
241	58081	483
242	58564	485
143	59049	487
244	59536	489
245	60025	491
246	60516	493
247	61009	495
248	61504	497
249	62001	499
250	62500	501
251	63001	503
252	63504	505
253	64009	507
254	64516	509
255	65025	511
256	65536	513
257	66049	515
258	66564	517
259	67071	519
260	67600	521
261	68121	523
262	68644	525
263	69169	527
264	69696	529
265	70225	531
266	70756	533
277	71289	535
267	71289	535
268	71824	537
269	72361	539
270	72900	541
271	73441	543
272	73984	545
273	74529	547
274	75076	549
275	75625	551
276	76176	553
277	76729	555
278	77284	557
279	77841	559
280	78400	561
281	78961	563
282	79524	565
283	80089	567
284	80616	569
285	81225	571
286	81796	573
287	82369	575
288	82944	577
289	83521	579
290	84100	581
291	84681	583
292	85264	585
293	85849	587
294	86436	589
295	87025	591
296	87616	593
297	88200	595
298	88804	597
299	89401	599
300	90000	601

301	090601	603
302	091204	605
303	091809	607
304	092416	609
305	093025	611
306	093636	613
307	094249	615
308	094864	617
309	095481	619
310	096109	621
311	096721	623
312	97344	625
313	97969	627
314	98596	629
315	99325	631
316	99856	633
317	100487	645
318	101124	637
319	101761	639
320	102400	641
321	103041	643
322	103684	645
323	104329	647
324	104976	649
325	105625	651
326	106276	653
327	106929	655
328	107584	657
329	108241	659
330	108900	661
331	109561	663
332	110224	665
333	110889	667
334	111556	669

334	111556	669
335	112225	671
336	112896	673
337	113569	675
338	114244	677
339	114921	679
340	115600	681
341	116281	683
342	116964	685
343	117649	687
344	118336	689
345	119025	691
346	119716	693
347	120409	695
348	121104	697
349	121801	699
350	122500	701
351	123201	703
352	123904	705
353	124609	707
354	125316	709
355	126025	711
356	126736	713
357	127449	715
358	128164	717
359	128881	719
360	129600	721
361	138321	723
362	131044	725
363	131769	727
364	132496	729
365	133225	731
366	133956	733
367	134689	735

367	134689	735	401	160801	803
368	135424	737	402	161604	805
369	136161	739	403	162409	807
370	136900	741	404	163216	809
371	137641	743	405	164025	811
372	138384	745	406	164836	813
373	139129	747	407	165649	815
374	139876	749	408	166464	817
375	140625	751	409	167281	819
376	141376	753	410	168100	821
377	142129	755	411	168921	823
378	142884	757	412	169744	825
379	143641	759	413	170569	827
380	144400	761	414	171396	829
381	145161	763	415	172225	831
382	145924	765	416	173056	833
383	146689	767	417	173889	835
384	147456	769	418	174724	837
385	148225	771	419	175561	839
386	148996	773	420	176400	841
387	149769	775	421	177241	843
388	150544	777	422	178084	845
389	151321	779	423	178929	847
390	152100	781	424	179776	849
391	152881	783	425	180625	851
392	153664	785	426	181476	853
393	154449	787	427	182329	855
394	155236	789	428	183184	857
395	156025	791	429	184041	859
396	156816	793	430	184900	861
397	157609	795	431	185761	863
398	158404	797	432	186624	865
399	159201	799	433	187489	867
400	160000	801	434	188356	869

434	188356	869
435	189225	871
436	190096	873
437	190969	875
438	191844	877
439	192721	879
440	193600	881
441	194481	883
442	195364	885
443	196249	887
444	197136	889
445	198025	891
446	198916	893
447	199809	895
448	200704	897
449	201601	899
450	202500	901
451	203401	903
452	204304	905
453	205209	907
454	206116	909
455	207025	911
456	207936	913
457	208849	915
458	209764	917
459	210681	919
460	211600	921
461	212521	923
462	213444	925
463	214369	927
464	215296	929
465	216225	931
466	217156	933
467	218089	935

467	218089	935
468	219024	937
469	219961	939
470	220900	941
471	221841	943
472	222784	945
473	223729	947
474	224676	949
475	225625	951
476	226576	953
477	227529	955
478	228484	957
479	229441	959
480	230400	961
481	231361	963
482	232324	965
483	233289	967
484	234256	969
485	235225	971
486	236196	973
487	237169	975
488	238144	977
489	239121	979
490	240100	981
491	241081	983
492	242064	985
493	243049	987
494	244036	989
495	245025	991
496	246016	993
497	247009	995
498	248004	997
499	249001	999
500	250000	1001

501	251001	1003
502	252004	1005
503	253009	1007
504	254016	1009
505	255025	1011
506	256036	1013
507	257049	1015
508	258064	1017
509	259081	1019
510	260100	1021
511	261121	1023
512	262144	1025
513	263169	1027
514	264196	1029
515	265225	1031
516	266256	1033
517	267289	1035
518	268324	1037
519	269361	1039
520	270400	1041
521	271441	1043
522	272484	1045
523	273529	1047
524	274576	1049
525	275625	1051
526	276676	1053
527	277729	1055
528	278784	1057
529	279841	1059
530	288900	1061
531	281961	1063
532	283024	1065
533	284089	1067
534	285156	1069

534	285156	1069
535	286225	1071
536	287296	1073
537	288369	1075
538	289444	1077
539	290521	1079
540	291600	1081
541	292681	1083
542	293764	1085
543	294849	1087
544	295936	1089
545	297025	1091
546	298116	1093
547	299209	1095
548	300324	1097
549	301401	1099
550	302500	1101
551	303601	1103
552	304704	1105
553	305809	1107
554	306916	1109
555	308025	1111
556	309136	1113
557	310249	1115
558	311364	1117
559	312481	1119
560	313600	1121
561	314721	1123
562	315844	1125
563	316969	1127
564	318096	1129
565	319225	1131
566	320356	1133
567	321489	1135

567	321489	1135	601	361201	1203
568	322624	1137	602	362404	1205
569	323761	1139	603	963609	1207
570	324900	1141	604	364816	1209
571	326041	1143	605	366025	1211
572	327184	1145	606	367236	1213
573	328329	1147	607	368449	1215
574	329476	1149	608	369664	1217
575	330625	1151	609	370881	1219
576	331776	1153	610	372100	1221
577	332929	1155	611	373321	1223
578	334084	1157	612	374544	1225
579	335241	1159	613	375769	1227
580	336400	1161	614	376996	1229
581	337561	1163	615	378225	1231
582	338724	1165	616	379456	1233
583	339889	1167	617	380689	1235
584	341056	1169	618	381924	1237
585	342225	1171	619	383161	1239
586	343396	1173	620	384400	1241
587	344569	1175	621	385641	1243
588	345744	1177	622	386834	1245
589	346921	1179	623	388129	1247
590	348100	1181	624	389376	1249
591	349281	1183	625	390625	1251
592	350464	1185	626	391876	1253
593	351649	1187	627	393129	1255
594	352836	1189	628	394385	1257
595	354025	1191	629	395641	1259
596	355216	1193	630	396900	1261
597	356409	1195	631	398161	1263
598	357604	1197	632	399424	1265
599	358801	1199	633	400689	1267
600	369000	1201	634	401956	1269

634	401956	1269
635	403225	1271
636	404496	1273
637	405769	1275
638	407044	1277
639	408321	1279
640	409600	1281
641	410881	1283
642	412164	1285
643	413449	1287
644	414736	1289
645	416025	1291
646	417316	1293
647	418609	1295
648	419904	1297
649	421201	1299
650	422500	1301
651	423801	1303
652	425104	1305
653	426409	1307
654	427716	1309
655	429025	1311
656	430336	1313
657	431649	1315
658	432964	1317
659	434281	1319
660	435600	1321
661	436921	1323
662	438244	1325
663	439569	1327
664	440896	1329
665	442225	1331
666	443556	1333
667	444889	1335

667	444889	1335
668	446224	1337
669	447561	1339
670	448900	1341
671	450241	1343
672	451584	1345
673	452929	1347
674	454276	1349
675	455625	1351
676	456976	1353
677	458329	1355
678	459684	1357
679	461041	1359
680	462400	1361
681	463761	1363
682	465124	1365
683	466489	1367
684	467856	1369
685	469225	1371
686	470596	1373
687	471969	1375
688	473344	1377
689	474721	1379
690	476100	1381
691	477481	1383
692	478864	1385
693	480249	1387
694	481636	1389
695	483025	1391
696	484416	1393
697	485809	1395
698	487204	1397
699	488601	1399
700	490000	1401

701	491401	1403
702	492804	1405
703	494209	1407
704	495616	1409
705	497025	1411
706	498436	1413
707	499849	1415
708	501264	1417
709	502681	1419
710	504100	1421
711	505521	1423
712	506944	1425
713	508369	1427
714	509796	1429
715	511225	1431
716	512656	1433
717	514089	1435
718	515524	1437
719	516961	1439
720	518400	1441
721	519841	1443
722	521284	1445
723	522729	1447
724	524176	1449
725	525625	1451
726	527076	1453
727	528529	1455
728	529984	1457
729	531441	1459
730	532900	1461
731	534361	1463
732	535824	1465
733	537289	1467
734	538756	1469

734	538756	1469
735	540225	1471
736	541696	1473
737	543169	1475
738	544644	1477
739	546121	1479
740	547600	1481
741	549081	1483
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317731	321175	324638	328119	331619	
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429134	433135	437155	441193	445250	
469982	474169	478374	482598	486841	
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37	1, 270924	277803	284701	291617	298852
38	1. 340551	347616	354700	361802	368921
39	1. 412035	419286	426555	433842	441148
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43	1, 716537	724530	732542	740572	748621
44	1. 797394	805483	813681	821897	830131
45	1, 879929	888293	896676	905078	913498
46	1. 964409	972959	981528	990116	998721
47	2, 050747	059483	068237	077010	085801
48	2. 138941	147862	156803	165761	174739
49	2. 228992	238099	247225	256369	265532
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55	2. 808289	818510	828750	839008	849285
56	2. 911336	921743	932169	942613	953076
57	3. 016241	026834	037445	048075	058723
58	3. 123003	133781	144578	155393	166227
59	3. 231621	242585	253567	264568	275588
60	3. 342096	353245	364413	375600	386805
61	3. 454427	465762	477116	488489	499880
62	3. 568615	580136	591676	603234	614811
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65	3. 922321	934398	946495	958610	970744
66	4. 043936	056199	068481	080782	093102
67	4. 167408	179857	192325	204811	217316

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76	5. 362107	376327	390466	404624	418800
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78	5. 648142	662633	677144	691673.	706220
79	5. 793894	808572	823268	837982	852715
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81	6. 090969	106018	121085	136171	151276
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84	6. 550508	566113	581738	597381	613042
85	6. 707401	723192	739002	754830	770678
86	6. 866150	882727	898123	914137	930170
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93	8. 029385	046662	063957	081271	098604
94	8. 202988	220451	237932	255432	272950
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116	12. 492012	513559	535125	556709	578312
117	12. 708320	730052	751804	773574	795363
118	12. 926484	948403	970340	992296	014276
119	13. 146505	168610	190733	212874	235034
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121	13. 592118	614594	637088	659601	682138
122	13. 817710	840371	863051	885749	908466
123	14. 045158	068005	090870	113754	136657
124	14. 274463	297495	320547	343616	366703
125	14. 505625	528843	552080	575335	598609
126	14. 738643	761047	785469	808910	832370
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128	15. 210250	234025	257819	281631	305462
129	15. 448838	472799	496779	520777	544794
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143	18. 984033	010059	037172	063770	090386
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146	19. 788921	816039	843175	870329	897502
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158	23. 175579	204924	234288	263671	293072
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163	24. 665596	695870	726163	756474	786803
164	24. 969170	999630	030108	060604	091119
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175	28. 431015	463516	496047	528586	561141
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186	32. 117542	152086	186649	221231	255831
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1.4	0.513126	520483	527892	535353	542867
1.5	0.589048	596928	604861	612846	620883
1.6	0.670206	678610	687066	695574	704135
1.7	0.756600	765527	774507	783539	792623
1.8	0.848230	857680	867184	876739	886348
1.9	0.945097	955070	965097	975176	985308
2.0	1.047197	057695	068246	078849	089504
2.1	1.154535	165557	176631	187757	198936
2.2	1.267109	278654	290252	301902	313604
2.3	1.384918	396987	409109	421282	433508
2.4	1.507964	520557	533201	545899	558648
2.5	1.636246	649362	662530	675751	689024
2.6	1.769763	783403	797095	810840	824637
2.7	1.908517	921680	036896	951164	965485
2.8	2.052507	067194	081933	096725	111569
2.9	2.201732	216943	232206	247521	262889
3.0	2.356194				

5	6	7	8	9	
005890	006702	007566	008482	009450	
016362	017697	019085	020525	022017	
032070	033929	035840	037803	039819	
053014	055396	057831	060318	062858	
079194	082100	085058	088069	091132	
110610	114039	117521	121055	124642	
147262	151215	155220	159278	163388	
189149	193626	198155	202737	207371	
236273	241274	246326	251432	256589	
288633	294157	299734	305362	311043	
346219	352277	358377	364529	370734	
409061	415632	422256	482932	435660	
477129	484224	491371	498570	505822	
550433	558051	565722	573445	581220	
628973	637114	645309	653555	661855	
712748	721414	730132	738902	747725	
801760	810949	820191	829485	838831	
896008	905721	915486	925303	935173	
995492	005728	016017	026358	036751	
100211	110971	121784	132648	143565	
210167	221451	232787	244175	255616	
325359	337166	349026	360937	372902	
445787	458117	470500	482936	495424	
571450	584305	597211	610170	623182	
702350	715728	729158	742641	756176	
838486	852387	866341	880347	894406	
979857	994283	008760	023290	037872	
126465	141414	156415	171468	186574	
278309	293781	309306	324883	340512	

	0	1	2	3	4
10	0. 314159	345575	376991	408407	439821
20	0. 628318	659734	691150	722566	753982
30	0. 942477	974893	006309	037725	069148
40	1. 256637	288052	319468	350884	382300
50	1. 570796	602212	633628	665044	696460
60	1. 884955	916371	947787	979203	010619
70	2. 199114	230530	261946	293362	324778
80	2. 513274	544690	576105	607521	638937
90	2. 827433	858849	890265	921681	953097
1.00	3. 141592	173008	204424	235840	267256
10	3. 455751	487167	518583	549999	581415
20	3. 769911	801327	832743	864158	895574
30	4. 084070	115486	146902	178318	209734
40	4. 398229	429645	461061	492477	523893
50	4. 712388	743804	775220	806636	838051
60	5. 026548	057964	089380	120796	152211
70	5. 340707	372123	403539	434945	466371
80	5. 654866	686282	717698	749114	780530
90	5. 969026	000441	031857	063273	094689
2.00	6. 283105	314601	346017	377433	408849
10	6. 597344	628760	660176	691592	723008
20	6. 911503	942919	974336	005751	037167
30	7. 225663	257079	288494	319910	351326
40	7. 539822	571238	602654	634070	665486
50	7. 853981	885397	916813	948229	979645
60	8. 168140	199556	230972	262388	293804
70	8. 482300	513716	545132	576547	607963
80	8. 796458	827875	859291	890707	922123
90	9. 110618	142034	173450	204866	236282
3.00	9. 424777				

5	6	7	8	9	
471238	501654	534070	565486	596902	
785398	816814	848230	879645	911061	
100557	131973	162389	193805	225221	
413716	445132	476548	507964	539380	
727875	759291	790707	822123	853539	
042035	073451	104867	136283	167698	
356194	387610	419026	450442	481858	
670353	701769	733185	764601	796017	
984513	015928	047344	078760	110176	
298672	330088	361504	392920	424335	
613831	644247	675663	707079	738495	
926990	958406	989822	021238	052654	
241150	272566	303981	335397	366813	
555309	586725	618141	649557	680973	
869468	900884	932300	963716	995132	
183627	215043	246459	277875	309291	
497787	529203	560618	592034	623450	
811946	843362	874778	906194	937610	
126105	157521	188937	220353	251769	
440264	471680	503096	534512	565928	
754424	785840	817256	848671	880087	
068583	099999	131415	162831	194247	
382742	414158	445574	476990	508406	
696901	728317	759733	791149	822565	
011061	042477	073893	105309	136724	
325220	356636	388052	419468	450884	
639379	670795	702211	733627	765043	
953539	984954	016370	047786	079202	
267798	299114	330530	361946	393362	

	Natural	Artificial
0.01	0.000002	4.423668
.02	0.000010	5.015728
.03	0.000023	5.377911
.04	0.000042	5.627788
0.05	0.000066	5.821608
.06	0.000092	5.979971
.07	0.000129	6.113864
.08	0.000169	6.229848
.09	0.000214	6.332153
0.10	0.000265	6.423668
.11	0.000320	6.506454
.12	0.000382	6.582031
.13	0.000448	6.651555
.14	0.000519	6.715924
0.15	0.000596	6.775851
.16	0.000679	6.831908
.17	0.000766	6.884566
.18	0.000859	6.934213
.19	0.000957	6.981176
0.20	0.001061	7.025728
.21	0.001169	7.068107
.22	0.001283	7.108514
.23	0.001403	7.147124
.24	0.001527	7.184091
0.25	0.001657	7.219548
.26	0.001793	7.253615
.27	0.001933	7.286396
.28	0.002012	7.317984
.29	0.002230	7.348464
0.30	0.002353	7.377911
.31	0.002549	7.406392
.32	0.002716	7.433968
.3	0.002888	7.460695
.3	0.003066	7.486626
0.35	0.003249	7.511804

	Natural	Artificial
0.36	0.003437	7.536273
0.37	0.003631	7.560071
0.38	0.003830	7.583236
0.39	0.004034	7.605798
0.40	0.004244	7.627788
0.41	0.004458	7.649236
0.42	0.004679	7.670167
0.43	0.004904	7.690605
0.44	0.005135	7.710574
0.45	0.005371	7.730093
0.46	0.005612	7.749184
0.47	0.005892	7.767864
0.48	0.006111	7.786151
0.49	0.006368	7.804060
0.50	0.006631	7.821608
0.51	0.006899	7.838809
0.52	0.007172	7.855675
0.53	0.007451	7.872220
0.54	0.007734	7.888456
0.55	0.008024	7.905394
0.56	0.008318	7.920044
0.57	0.008618	7.935418
0.58	0.008923	7.950524
0.59	0.009233	7.965372
0.60	0.009549	7.979971
0.61	0.009870	7.994328
0.62	0.010196	8.008452
0.63	0.010528	8.022358
0.64	0.010864	8.036028
0.65	0.011207	8.049495
0.66	0.011554	8.062756
0.67	0.011907	8.075818
0.68	0.012265	8.088686
0.69	0.012628	8.101367
0.70	0.012997	8.113864

	Natural	Artificial
0.71	0.013371	8.126175
0.72	0.013750	8.138333
0.73	0.014135	8.150314
0.74	0.014525	8.162132
0.75	0.014920	8.173791
0.76	0.015321	8.185296
0.77	0.015727	8.190991
0.78	0.016138	8.207858
0.79	0.016554	8.218927
0.80	0.016976	8.229848
0.81	0.017403	8.240638
0.82	0.017835	8.251296
0.83	0.018273	7.261825
0.84	0.018716	8.272227
0.85	0.019164	8.282506
0.86	0.019618	8.292665
0.87	0.020077	8.302707
0.88	0.020541	8.312634
0.89	0.021011	8.322448
0.90	0.021485	8.332153
0.91	0.021999	8.341751
0.92	0.022451	8.351244
0.93	0.022942	8.360634
0.94	0.023438	8.369924
0.95	0.023939	8.379116
0.96	0.024446	8.388211
0.97	0.024958	8.397212
0.98	0.025475	8.406121
0.99	0.025997	8.414939
1.00	0.026525	8.423668
1.01	0.027058	8.432311
1.02	0.027597	8.442369
1.03	0.028141	8.449343
1.04	0.028690	8.457735
1.05	0.029244	8.466047
1.06	0.029804	8.474280
1.07	0.030369	8.482437
1.08	0.030939	8.490516
1.09	0.031515	8.498521
1.10	0.032096	8.506454
1.11	0.032682	8.514314
1.12	0.033273	8.522104
1.13	0.033870	8.528825
1.14	0.034472	8.537478
1.15	0.035080	8.545064
1.16	0.035697	8.552584
1.17	0.036324	8.560040
1.18	0.036934	8.567432
1.19	0.037563	8.574762
1.20	0.038197	8.582031
1.21	0.038836	8.589239
1.22	0.039481	8.596388
1.23	0.040130	8.603471
1.24	0.040786	8.610512
1.25	0.041446	8.617488
1.26	0.042112	8.624410
1.27	0.042783	8.631276
1.28	0.043459	8.638088
1.29	0.044141	8.644848
1.30	0.044828	8.651555
1.31	0.045520	8.658211
1.31	0.046218	8.664816
1.33	0.046921	8.671372
1.34	0.047629	8.677878
1.35	0.048343	8.684336
1.36	0.049062	8.689746
1.37	0.049786	8.697110
1.38	0.050515	8.703427
1.39	0.051250	8.709698
1.40	0.051990	8.715924

	Natural	Artificial
1.41	0.052735	8.722107
1.42	0.053453	8.728245
1.43	0.054242	8.734340
1.44	0.054007	8.740393
1.45	0.055770	8.746404
1.46	0.056542	8.752374
1.47	0.057319	8.758303
1.48	0.058103	8.764192
1.49	0.058889	8.770071
1.50	0.059687	8.775851
1.51	0.060481	8.781622
1.52	0.061285	8.793356
1.53	0.062094	8.793051
1.54	0.062908	8.798710
1.55	0.063728	8.804332
1.56	0.064556	8.809918
1.57	0.065383	8.815468
1.58	0.066219	8.820983
1.59	0.067059	8.826463
1.60	0.067906	8.831908
1.61	0.068754	8.837320
1.62	0.069614	8.842698
1.63	0.070476	8.848044
1.64	0.071343	8.853356
1.65	0.072216	8.858636
1.66	0.073091	8.863885
1.67	0.073944	8.869101
1.68	2.074533	8.874595
1.69	0.075760	8.879442
1.70	0.076659	8.884566
1.71	0.077564	8.889661
1.72	0.078440	8.894725
1.73	0.079355	8.899761
1.74	0.080309	8.904767
1.75	0.081235	8.909745

	Natural	Artificial
1.76	0.082166	8.914694
1.77	0.083102	8.919615
1.78	0.084044	8.924508
1.79	0.084991	8.929374
1.80	0.085943	8.934213
1.81	0.086901	8.939026
1.82	0.087864	8.943811
1.83	0.088832	8.948571
1.84	0.089805	8.953291
1.85	0.090784	8.958011
1.86	0.091768	8.962694
1.87	0.092758	8.967352
1.88	0.093752	8.971984
1.89	0.094418	8.976592
1.90	0.095748	8.981176
1.91	0.096768	8.985735
1.92	0.097784	8.990271
1.93	0.098806	8.994783
1.94	0.099832	8.999272
1.95	0.100864	9.003738
1.96	0.101901	9.008181
1.97	0.102944	9.012601
1.98	0.103981	9.016999
1.99	0.105044	9.021375
2.00	0.106103	9.025728
2.01	0.107166	9.030061
2.02	0.108235	9.034371
2.03	0.109310	9.038660
2.04	0.110389	9.042929
2.05	0.111441	9.047176
2.06	0.112564	9.051403
2.07	0.113660	9.055609
2.08	0.114427	9.059795
2.09	0.115867	9.063961
2.10	0.116978	9.068127

	Natural	Artificial
2,11	0.118095	9.072233
2,12	0.119210	9.076340
2.13	0.120345	9.080428
2.14	0.121477	9.084496
2.15	0.122614	9.088545
2.16	0.123758	9.092576
2.17	0.124907	9.096588
2.18	0.126061	9.100581
2.19	0.127220	9.104557
2.20	0.128384	9.108514
2.21	0.129554	9.112453
2.22	0.130729	9.116374
2.23	0.131910	9.120278
2.24	0.133095	9.124164
2.25	0.134286	9.128033
2.26	0.135483	9.131885
2.27	0.136684	9.135720
2.28	0.137891	9.139538
2.29	0.139104	9.143339
2.30	0.140321	9.147124
2.31	0.141544	9.150892
2.32	0.142772	9.154644
2.33	0.144006	9.158380
2.34	0.145244	9.162100
2.35	0.146488	9.165804
2.36	0.147738	9.169492
2.37	0.148992	9.173165
2.38	0.150252	9.176821
2.39	0.151518	9.180464
2.40	0.152788	9.184091
2.41	0.154064	9.187702
2.42	0.155345	9.191299
2.43	0.156632	9.194881
2.44	0.157924	9.198448
2.45	0.159221	9.202001
2.46	0.160523	9.205539
2.47	0.161831	9.209062
2.48	0.163144	9.212571
2.49	0.164462	9.216067
2.50	0.165786	9.219548
2.51	0.167115	9.222016
2.52	0.168449	9.226469
2.53	0.169789	9.229909
2.54	0.171133	9.233336
2.55	0.172484	9.236749
2.56	0.173839	9.240148
2.57	0.175200	9.243535
2.58	0.176566	9.246908
2.59	0.177937	9.250268
2.60	0.179314	9.253615
2.61	0.180685	9.256949
2.62	0.182083	9.260271
2.63	0.183476	9.263580
2.64	0.184874	9.266876
2.65	0.186277	9.270160
2.66	0.187686	9.273432
2.67	0.189099	9.276691
2.68	0.190519	9.279938
2.69	0.191943	9.283173
2.70	0.193373	9.286396
2.71	0.194808	9.289607
2.72	0.196248	9.292806
2.73	0.197694	9.295994
2.74	0.199145	9.299170
2.75	0.200601	9.302334
2.76	0.202630	9.305486
2.77	0.203525	9.308628
2.78	0.205002	9.311758
2.79	0.206475	9.314877
2.80	0.207961	9.317984

	Natural	Artificial
2.81	0.209450	9.321081
2.82	0.210943	9.324167
2.83	0.212442	9.327241
2.84	0.213946	9.330306
2.85	0.215455	9.333358
2.86	0.216970	9.336400
2.87	0.218490	9.339442
2.88	0.220015	9.342453
2.89	0.221546	9.344464
2.90	0.223082	9.348464
2.91	0.224623	9.351454
2.92	0.226169	9.354434
2.93	0.227721	9.357404
2.94	0.229278	9.360363
2.95	0.230840	9.363312
2.96	0.232408	9.366252
2.97	0.233981	9.369181
2.98	0.235559	9.372101
2.99	0.237143	9.375011
3.00	0.238732	9.377911
3.01	0.240326	9.380801
3.02	0.241926	9.383682
3.03	0.243530	9.386554
3.04	0.245141	9.389416
3.05	0.246756	9.392268
3.06	0.248377	9.395111
3.07	0.249003	9.397945
3.08	0.251634	9.400770
3.09	0.253271	9.403585
3.10	0.254913	9.406392
3.11	0.256627	9.409189
3.12	0.258212	9.411578
3.13	0.259870	9.414757
3.14	0.261534	9.417528
3.15	0.263201	9.420290

	Natural	Artificial
3.16	0.264876	9.423043
3.17	0.266555	9.425787
3.18	0.268239	9.428523
3.19	0.269926	9.431250
3.20	0.271624	9.433968
3.21	0.273324	9.436678
3.22	0.275030	9.439380
3.23	0.276741	9.442073
3.24	0.278457	9.444758
3.25	0.280179	9.447435
3.26	0.281905	9.450104
3.27	0.283604	9.452764
3.28	0.285375	9.455416
3.29	0.287124	9.458060
3.30	0.288866	9.460695
3.31	0.290619	9.463324
3.32	0.292378	9.465945
3.33	0.294142	9.468557
3.34	0.295911	9.471161
3.35	0.297686	9.473758
3.36	0.299465	9.476347
3.37	0.301251	9.478928
3.38	0.303041	9.481501
3.39	0.304504	9.484068
3.40	0.306638	9.486616
3.41	0.308444	9.489177
3.42	0.310256	9.491721
3.43	0.312073	9.494257
3.44	0.313895	9.496785
3.45	0.315726	9.499307
3.46	0.317556	9.501821
3.47	0.319394	9.504327
3.48	0.321238	9.506781
3.49	0.323087	9.509319
3.50	0.324941	9.511804

	Natural	Artificial
3.51	0.326800	9.514283
3.52	0.328665	9.516754
3.53	0.330502	9.519218
3.54	0.332411	9.521675
3.55	0.334291	9.524125
3.56	0.336177	9.526568
3.57	0.338068	9.528005
3.58	0.339965	9.531434
3.59	0.341867	9.533857
3.60	0.343774	9.536273
3.61	0.345687	9.538683
3.62	0.347604	9.541086
3.63	0.349528	9.543482
3.64	0.351456	9.545871
3.65	0 353390	9.547254
3.66	0.355329	9.550631
3.67	0.357273	9.553001
3.68	0.359223	9.555364
3.65	0.361178	9.557721
3.70	0.363138	9.560071
3.71	0.365104	9.562416
3.72	0.367074	9.564754
3.73	0.369051	9.567086
3.74	0.371032	9.569412
3.75	0.373019	9.571731
3.76	0.375011	9.574044
3.77	0.377008	9.576351
3.78	0.379011	9.578652
3.79	0 381015	9.580947
3.80	0.383032	9.583236
3.81	0.385051	9.585518
3.82	0.387075	9.587795
3.83	0.389104	9.590066
3.8	9.391139	9.592331
3.85	9.393179	9.594590

	Natural	Artificial
3.86	0.395224	9.596743
3.87	0.397274	9.599090
3.88	0.399330	9.601332
3.89	0.401391	9.603568
3.90	0.403457	9.605798
3.91	0.405529	9.608023
3.92	0.407606	9.610241
3.93	0.409688	9.612454
3.94	0.411776	9.614661
3.95	0.413869	9.616863
3.96	0.415967	9.619059
3.97	0.418070	9.621249
3.98	0.420179	9.623435
3.99	0.422293	9.625614
4.00	0.424413	9.627788
4.01	0.426504	9.629957
4.02	0.428667	9.632121
4.03	0.430803	9.634279
4.04	0.432943	9.636431
4.05	0.435089	9.638578
4.06	0.437241	9.640710
4.07	0.439357	9.642857
4.08	0.441559	9.644989
4.09	0.443726	9.647115
4.10	0.445899	9.649236
4.11	0.448076	9.651352
4.12	0.450259	9.653463
4.13	0.452448	9.655569
4.14	0.454642	9.657669
4.15	0.456840	9.659765
4.16	0.459045	9.661855
4.17	0.461254	9.663941
4.18	0.463465	9.666021
4.19	0.465690	9.668056
4.20	0.467915	9.670167

	Natural	Artificial
4.21	0.470146	9.672233
4.22	0.472382	9.674293
4.23	0.474623	9.676349
4.24	0.476870	9.678400
4.25	0.479122	9.680446
4.26	0.481380	9.682488
4.27	0.483642	9.684524
4.28	0.485910	9.686556
4.29	0.488183	9.688583
4.30	0.490462	9.690605
4.31	0.492746	9.692623
4.32	0.495032	9.694636
4.33	0.497330	9.696634
4.34	0.499629	9.698648
4.35	0.501934	9.700647
4.36	0.504245	9.702641
4.37	0.506560	9.704631
4.38	0.508882	9.706617
4.39	0.511208	9.708597
4.40	0.513539	9.710574
4.41	0.515876	9.712546
4.42	0.518219	9.714513
4.43	0.520566	9.716476
4.44	0.522919	9.718434
4.45	0.527277	9.720388
4.46	0.527641	9.722338
4.47	0.530009	9.724283
4.48	0.532383	9.726224
4.49	0.534429	9.728161
4.50	0.537147	9.730093
4.51	0.539537	9.732021
4.52	0.541933	9.733945
4.53	0.544333	9.735865
4.54	0.546739	9.737780
4.55	0.549150	9.739691

	Natural	Artificial
4.56	0.551567	9.741598
4.57	0.553989	9.743501
4.58	0.556416	9.745399
4.59	0.558848	9.747294
4.60	0.561286	9.749184
4.61	0.563729	9.751070
4.62	0.566177	9.752952
4.63	0.568631	9.754830
4.64	0.571090	9.756704
4.65	0.573554	9.758574
4.66	0.576024	9.760440
4.67	0.578499	9.762302
4.68	0.580979	9.764160
4.69	0.583464	9.766014
4.70	0.585955	9.767864
4.71	0.588451	9.769710
4.72	0.590952	9.771552
4.73	0.593459	9.773411
4.74	0.595971	9.775225
4.75	0.598488	9.777056
4.76	0.601011	9.778882
4.77	0.603539	9.780705
4.78	0.606072	9.782524
4.79	0.608611	9.784339
4.80	0.611154	9.786151
4.81	0.613704	9.787959
4.82	0.616258	9.789762
4.83	0.618818	9.791563
4.84	0.621383	9.793359
4.85	0.623953	9.795152
4.86	0.626529	9.796941
4.87	0.629110	9.798726
4.88	0.631696	9.800508
4.89	0.634288	9.802286
4.90	0.636885	9.804060

	Natural	Artificial
4.92	0.639487	9.805831
4.92	0.642094	9.807599
4.93	0.644707	9.809362
4.94	0.647325	9.811122
4.95	0.649948	9.812879
4.96	0.652577	9.814632
4.97	0.655211	9.815638
4.98	0.657851	9.818127
4.99	0.660495	9.819869
5.00	0.663145	9.821608
5.01	0.665800	9.823344
5.02	0.668461	9.825076
5.03	0.671127	9.826804
5.04	0.673798	9.828529
5.05	0.676474	9.830451
5.06	0.679156	9.831979
5.07	0.681843	9.833684
5.08	0.684536	9.835396
5.09	0.687267	9.837104
5.10	0.689936	9.838809
5.11	0.692644	9.840516
5.12	0.695358	9.842208
5.13	0.698077	9.843903
5.14	0.700801	9.845595
5.15	0.703531	9.847283
5.16	0.706265	9.848968
5.17	0.709006	9.850649
5.18	0.711751	9.852328
5.19	0.71450:	9.854003
5.20	0.717258	9.855675
5.21	0.720019	9.857344
5.22	0.722786	9.859009
5.23	0.725558	9.860672
5.24	.728335	9.862331
5.25	0.731118	9.863987

	Natural	Artificial
5.26	0.733905	9.865640
5.27	0.736699	9.867290
5.28	0.739497	9.868936
5.29	0.742301	9.870580
5.30	0.745110	9.872220
5.31	0.747924	9.873857
5.32	0.750744	9.875492
5.33	0.753569	9.877123
5.34	0.756399	9.878751
5.35	0.759235	9.880376
5.36	0.762076	9.881998
5.37	0.764922	9.883617
5.38	0.767774	9.885233
5.39	0.770630	9.886846
5.40	0.773493	9.888456
5.41	0.776360	9.890063
5.42	0.779233	9.891667
5.43	0.782111	9.893268
5.44	0.784994	9.894866
5.45	0.78788.	9.896461
5.46	0.790777	9.898054
5.47	0.793676	9.899643
5.48	0.796581	9.901230
5.49	0.799490	9.902813
5.50	0.802406	9.905394
5.51	0.805326	9.905972
5.52	0.808252	9.907547
5.53	0.811183	9 909119
5.54	0.814129	9.910688
5.55	0.817061	9.912254
5.56	0.820008	9.913818
5.57	0.822994	9.915379
5.58	0.825918	9.916937
5.59	0.828881	9.918492
5.60	0.831849	9.920044

	Natural	Artificial		Natural	Artificial
5.61	0.834823	9.921594	5.96	0.942239	9.974161
5.62	0.837801	9.923141	5.97	0.945404	9.975617
5.63	0.840786	9.924685	5.98	0.948574	9.977071
5.64	0.843775	9.926227	5.99	0.951749	9.978522
5.65	0.846770	9.927765	6.00	0.954929	9.979971
5.66	0.849770	9.929301	6.01	0.958115	9.981417
5.67	0.852776	9.930835	6.02	0.961306	9.982861
5.68	0.855786	9.932365	6.03	0.964502	9.984303
5.69	0.858801	9.933893	6.04	0.967704	9.985742
5.70	0.861824	9.935418	6.05	0.970911	9.987179
5.71	0.864850	9.936941	6.06	0.974123	9.988614
5.72	0.867882	9.938460	6.07	0.977341	9.990046
5.73	0.870919	9.939978	6.08	0.980564	9.991476
5.74	0.873963	9.941492	6.09	0.983792	9.992903
5.75	0.877010	9.943004	6.10	0.987025	9.994328
5.76	0.880063	9.944513	6.11	0.990264	9.995751
5.77	0.883121	9.946020	6.12	0.993508	9.997171
5.78	0.886185	9.947524	6.13	0.996758	9.998589
5.79	0.889254	9.949026	6.14	1.000012	0.000005
5.80	0.892328	9.950524	6.15	1.003272	0.001419
5.81	0.895408	9.952021	6.16	1.006538	0.002830
5.82	0.898493	9.953514	6.17	1.009808	0.004239
5.83	0.901586	9.955006	6.18	1.013084	0.005645
5.84	0.904679	9.956494	6.19	1.016366	0.007050
5.85	0.907779	9.957980	6.20	1.019652	0.008452
5.86	0.910886	9.959464	6.21	1.022944	0.009852
5.87	0.913997	9.960945	6.22	1.026241	0.011249
5.88	0.917114	9.962423	6.23	1.029544	0.012645
5.89	0.920236	9.963908	6.24	1.032851	0.014038
5.90	0.923363	9.965372	6.25	1.036164	0.015428
5.91	0.926496	9.966843	6.26	1.039450	0.016817
5.92	0.929967	9.968312	6.27	1.042807	0.018203
5.93	0.932777	9.969778	6.28	1.046136	0.019588
5.94	0.935926	9.971241	6.29	1.049470	0.020970
5.95	0.939080	9.972702	6.30	1.052809	0.022350

	Natural	Artificial
6.31	1.056154	0.023717
6.32	1.059505	0.025103
6.33	1.062860	0.026476
6.34	1.066331	0.027847
6.35	1.069587	0.029216
6.36	1.072958	0.030583
6.37	1.076335	0.031947
6.38	1.079717	0.033310
6.39	1.083105	0.034670
6.40	1.086497	0.036028
6.41	1.089899	0.037384
6.42	1.093298	0.038738
6.43	1.096707	0.040090
6.44	1.100121	0.041440
6.45	1.103540	0.042788
6.46	1.106965	0.044133
6.47	1.110394	0.045477
6.48	1.113829	0.046818
6.49	1.117270	0.048158
6.50	1.120716	0.049495
6.51	1.124167	0.050830
6.52	1.127623	0.052164
6.53	1.131084	0.053495
6.54	1.134551	0.054824
6.55	1.138024	0.056150
6.56	1.141501	0.057476
6.57	1.144984	0.058799
6.58	1.148472	0.060120
6.59	1.152966	0.061439
6.60	1.155464	0.062756
6.61	1.158968	0.064071
6.62	1.162478	0.065438
6.63	1.165992	0.066895
6.64	1.166951	0.068005
6.65	1.173038	0 069312

	Natural	Artificial
6.66	1.176568	0.070617
6.67	1.180104	0.071920
6.68	1.183645	0.073221
6.69	1.187192	0.074521
6.70	1.190744	0.075815
6.71	1.194301	0.077113
6.72	1.197863	0.078407
6.73	1.201431	0.079699
6.74	1.205004	0.080988
6.75	1.208582	0.082276
6.76	1.212166	0.083562
6.77	1.215755	0.084846
6.78	1.219349	0.086118
6.79	1.222949	0.087408
6.80	1.226554	0.088686
6.81	1.230164	0.089963
6.82	1.233779	0.091237
6.83	1.237073	0.092510
6.83	1.241026	0.093781
6.85	1.244657	0.095049
6.86	1.248294	0.096317
6.87	1.251936	0.097582
6.88	1.255587	0.098845
6.85	1.259236	0.100107
6.90	1.262894	0.101367
6.91	1.266557	0.102525
6.92	1.270226	0.103881
6.93	1.274900	0.105135
6.94	1.27757	0.106387
6.95	1.28126,	0.107638
6.96	1.284951	0.108877
6.97	1.288648	0.110134
6.98	1.292348	0.111379
6.99	1.296054	0.112623
7.00	1.299765	0.113864

	Natural	Artificial		Natural	Artificial
7.01	1.306481	0.115104	7.36	1.436893	0.157424
7.02	1.307203	0.116343	7.37	1.440800	0.158603
7.03	1.310930	0.117579	7.38	1.444713	0.159781
7.04	1.314662	0.118814	7.39	1.448630	0.160957
7.05	1.318399	0.120047	7.40	1.452554	0.162132
7.06	1.322141	0.121278	7.41	1.456482	0.163305
7.07	1.325890	0.122507	7.42	1.460416	0.164476
7.08	1.329644	0.123735	7.43	1.464355	0.165646
7.09	1.333402	0.124961	7.44	1.468299	0.166814
7.10	1.337166	0.126175	7.45	1.472249	0.167981
7.11	1.340936	0.127408	7.46	1.476204	0.169146
7.12	1.344710	0.128628	7.47	1.480164	0.170310
7.13	1.348490	0.129747	7.48	1.484130	0.171472
7.14	1.352275	0.131065	7.49	1.488101	0.172632
7.15	1.356066	0.132280	7.50	1.492077	0.173791
7.16	1.359862	0.133494	7.51	1.496059	0.174948
7.17	1.363667	0.134707	7.52	1.500045	0.176104
7.18	1.367469	0.135917	7.53	1.504038	0.177258
7.19	1.371281	0.137126	7.54	1.508035	0.178411
7.20	1.375098	0.138333	7.55	1.512038	0.179562
7.21	1.378921	0.139539	7.56	1.516046	0.180712
7.22	1.382748	0.140743	7.57	1.520059	0.181860
7.23	1.386581	0.141946	7.58	1.524078	0.183007
7.24	1.390419	0.143146	7.59	1.528102	0.184151
7.25	1.394263	0.144344	7.60	1.532131	0.185296
7.26	1.398112	0.145542	7.6	1.536166	0.186438
7.27	1.401966	0.146737	7.61	1.540160	0.187578
7.28	1.405826	0.147931	7.63	1.544251	0.188717
7.29	1.409691	0.149123	7.64	1.548301	0.189855
7.30	1.413561	0.150314	7.65	1.552359	0.190991
7.31	1.417436	0.151503	7.66	1.556418	0.192126
7.32	1.421317	0.152691	7.67	1.560418	0.193259
7.33	1.425203	0.153876	7.68	1.564556	0.194391
7.34	1.429094	0.155060	7.69	1.568633	0.195521
7.35	1.432991	0.156243	7.70	1.572716	0.196650

	Natural	Artificial		Natural	Artificial
7.71	1.576803	0.197777	8.06	1.723212	0.236338
7.72	1.580896	0.198903	8.07	1.727491	0.237415
7.73	1.584994	0.200027	8.08	1.731775	0.238491
7.74	1.589098	0.201150	8.09	1.736064	0.239565
7.75	1.593207	0.202272	8.10	1.740359	0.240638
7.78	1.597321	0.203402	8.11	1.744659	0.241710
7.77	1.601440	0.204500	8.12	1.748964	0.242780
7.78	1.605565	0.205628	8.13	1.753274	0.243850
7.79	1.609695	0.206744	8.14	1.757590	0.244917
7.80	1.613831	0.207858	8.15	1.761911	0.245984
7.81	1.617971	0.208970	8.16	1.766237	0.247049
7.82	1.622117	0.210082	8.17	1.770569	0.248113
7.83	1.626269	0.211192	8.18	1.774906	0.249175
7.84	1.630425	0.212311	8.19	1.779248	0.250236
7.85	1.634587	0.213408	8.20	1.783596	0.251296
7.86	1.638754	0.214514	8.21	1.787949	0.252355
7.87	1.642927	0.215618	8.22	1.792307	0.253412
7.88	1.647105	0.216721	8.23	1.796670	0.254468
7.89	1.651288	0.217822	8.24	1.801039	0.255523
7.90	1.655476	0.218923	8.25	1.805413	0.256576
7.91	1.659670	0.220021	8.26	1.809793	0.257629
7.92	1.663869	0.221119	8.27	1.814177	0.258679
7.93	1.668073	0.222215	8.28	1.818568	0.259729
7.94	1.672283	0.223309	8.29	1.822963	0.260777
7.95	1.676498	0.224403	8.30	1.827363	0.261825
7.96	1.680718	0.225495	8.31	1.831769	0.262870
7.97	1.684944	0.226585	8.32	1.836181	0.263915
7.98	1.689175	0.227674	8.33	1.840597	0.264958
7.99	1.693411	0.228762	8.34	1.845019	0.266001
8.00	1.697652	0.229848	8.35	1.849480	0.267041
8.01	1.701899	0.230933	8.36	1.853879	0.268081
8.02	1.706151	0.232017	8.37	1.858316	0.269119
8.03	1.710408	0.233100	8.38	1.862760	0.270156
8.04	1.714671	0.234181	8.39	1.867208	0.271192
8.05	1.718939	0.235260	8.40	1.871662	0.272227

	Natural	Artificial
8.41	1.876121	0.273250
8.42	1.880585	0.274293
8.43	1.885054	0.275324
8.44	1.889529	0.276353
8.45	1.894010	0.277382
8.46	1.898495	0.278409
8.47	1.902886	0.279435
8.48	1.907482	0.280450
8.49	1.911983	0.281484
8.50	1.916490	0.282506
8.51	1.921002	0.283528
8.52	1.925520	0.283748
8.53	1.930042	0.285566
8.54	1.934570	0.286584
8.55	1.939104	0.287601
8.56	1.943642	0.288616
8.57	1.948186	0.289630
8.58	1.952735	0.290643
8.59	1.957290	0.291655
8.60	1.961849	0.292665
8.61	1.966414	0.293675
8.62	1.970985	0.294683
8.63	1.975561	0.295690
8.64	1.980142	0.296696
8.65	1.984728	0.297701
8.66	1.989320	0.298704
8.67	1.993916	0.299707
8.68	1.998519	0.300708
8.69	2.003126	0.301708
8.70	2.007406	0.302707
8.71	2.012357	0.303705
8.72	2.016981	0.304701
8.73	2.021609	0.305697
8.74	2.026243	0.306691
8.75	2.030883	0.307685

	Natural	Artificial
8.76	2.035528	0.308677
8.77	2.040177	0.309668
8.78	2.044833	0.310657
8.79	2.049493	0.311646
8.80	2.054159	0.311634
8.81	2.058830	0.313620
8.82	2.063507	0.314606
8.83	2.068189	0.315590
8.84	2.072876	0.316573
8.85	2.077568	5.317555
8.86	2.082266	0.318536
8.87	2.086969	0.319516
8.88	2.091677	0.320494
8.89	2.096391	0.321472
8.90	2.101110	0.322448
8.91	2.105834	0.323424
8.92	2.110564	0.324498
8.93	2.115299	0.325371
8.94	2.120039	0.326343
8.95	2.124784	0.327314
8.96	2.129535	0.328284
8.97	2.134291	0.329253
8.98	2.139053	0.330221
8.99	2.143819	0.331178
9.00	2.148591	0.332153
9.01	2.153368	2.333118
9.02	2.158151	0.334081
9.03	2.162939	0.335044
9.04	2.167732	0.336005
9.05	2.172531	0.337966
9.06	2.177335	0.338925
9.07	2.182144	0.338883
9.08	2.186958	0.339840
9.09	2.191778	0.340796
9.10	2.196603	0.341751

	Natural	Artificial
9.11	2.201433	0.341705
9.12	2.206269	0.343658
9.13	2.211110	0.344610
9.14	2.215956	0.345561
9.15	2.220808	0.346511
9.16	2.225665	0.347459
9.17	2.230527	0.348407
9.18	2.235394	0.349354
9.19	2.240267	0.350299
9.20	2.245145	0.351244
9.21	2.250029	0.352188
9.22	2.254917	0.353130
9.23	2.259811	0.354072
9.24	2.264711	0.355012
9.25	2.269615	0.355872
9.26	2.274525	0.356890
9.27	2.279440	0.357828
9.28	2.284361	0.358764
9.29	2.289287	0.359700
9.30	2.294218	0.360634
9.31	2.299154	0.361568
9.32	2.304096	0.362500
9.33	2.309043	0.363432
9.34	2.313996	0.364361
9.35	2.318953	0.365292
9.36	2.323916	0.366220
9.37	2.328885	0.367148
9.38	2.333858	0.368074
9.39	2.338847	0.369000
9.40	2 343821	0.369924
9.41	2.348144	0.370848
9.42	2.353806	0.371770
9.43	2.358806	0.372692
9.44	2.363478	0.373612
9.45	2.368489	0.374532

	Natural	Artificial
9.46	2.373838	0.375451
9.48	2.378893	0.376368
9.48	2.383886	0.377285
9.49	2.388918	0.378201
9.50	2.393955	0.379116
9.51	2.398998	0.380029
9.52	2.404045	0.380942
9.53	2.409099	0.381854
9.54	2.414157	0.382765
9.55	2.419221	0.383675
9.56	2.424290	0.384584
9.57	2.429364	0.385492
9.58	2.434444	0.386399
9.59	2.439529	0.387306
9.60	2.444629	0.388211
9.61	2.449715	0.389115
9.62	2.454816	0.390019
9.63	2.459922	0.390921
9.64	2.465034	0.391822
9.65	2.470150	0.392723
9.66	2.475273	0.393623
9.67	2.480400	0.394521
9.68	2.485533	0.395419
9.69	2.490671	0.396316
9.70	2.495814	0.397212
9.71	2.500963	0.398107
9.72	2.506117	0.399001
9.73	2.511243	0.399894
9.74	2.516441	0.400786
9.75	2.521611	0.401678
9.76	2.526786	0.402568
9.77	2.531966	0.403458
9.78	2.537152	0.404346
9.79	2.542343	0.405234
9.80	2.547540	0.406121

	Natural	Artificial
9.81	2.552745	0.407906
9.82	2.557948	0.407891
9.83	2.563164	0.408775
9.84	2.568378	0.409659
9.85	2.573601	0.410541
9.86	2.578829	0.411422
9.87	2.584063	0.412303
9.88	2.589302	0.413182
9.89	2.594546	0.414061
9.90	2.599795	0.414939

	Natural	Artificial
9.91	2.605050	0.415816
9.92	2.610310	0.416692
9.93	2.615573	0.417567
9.94	2.620846	0.418441
9.95	2.626122	0.419315
9.96	2.631404	0.420187
9.97	2.636690	0.421059
9.98	2.641981	0.421929
9.99	2.647279	0.422789
10.00	2.652582	0.423668

A Table

A Table for the ſpeedy finding of the Length or Circumference anſwering to any Arch in Degrees and Decimal Parts.

A Table for the ſpeedy finding of the Length or Circumference anſwering to any Arch, in Degrees and Decimal Parts.

1	0.	0174	5329	2519	26	0.	4537	8560	5495
2	0.	0349	0658	5038	27	0.	4712	3889	8013
3	0.	0523	5987	7557	28	0.	4886	8219	0532
4	0.	0698	1317	0076	29	0.	5061	4548	3051
5	0.	0872	6646	2595	30	0.	5235	4877	5570
6	0.	1047	1975	5114	31	0.	5410	5206	8089
7	0.	1221	7304	7633	32	0.	5585	0536	0608
8	0.	1396	2634	0152	33	0.	5759	5865	3127
9	0.	1570	7963	2671	34	0.	5934	1194	5646
10	0.	1745	3292	5190	35	0.	6108	6523	8165
11	0.	1919	8621	7709	36	0.	6283	1853	0684
12	0.	2094	3951	0228	37	0.	6457	7128	3203
13	0.	2268	9280	2747	38	0.	6632	2511	5722
14	0.	2443	4609	5266	39	0.	6806	7840	8241
15	0.	2617	9938	7785	40	0.	6981	3170	0760
16	0.	2792	5268	0304	41	0.	7155	8499	3279
17	0.	2967	0597	2823	42	0.	7330	3828	5798
18	0.	3141	5926	5342	43	0.	7504	9157	8317
19	0.	3316	1255	7861	44	0.	7679	4487	0836
20	0.	3490	6585	0380	45	0.	7853	9816	3355
21	0.	3665	1914	2899	46	0.	8028	3145	5874
22	0.	3839	7243	5418	47	0.	8203	0474	8393
23	0.	4014	2572	7937	48	0.	8377	5804	0912
24	0.	4188	7902	0456	49	0.	8552	1133	3431
25	0.	4363	3231	2975	50	0.	8726	6462	4950

A Table for the ſpeedy finding of the Length or Circumference anſwering to any Arch, in Degrees and Decimal Parts.

51	0.8901	1791	8469	
52	0.9075	7121	0988	
53	0.9250	3450	3507	
54	0.9424	7779	6026	
55	0.9599	3108	8545	
56	0.9773	8438	1064	
57	0.9948	3767	3583	
58	1.0122	9096	6102	
59	1.0297	4425	8621	
60	1.0471	9755	1140	
61	1.0646	5084	3659	
62	1.0821	0413	6178	
63	1.0995	5742	8697	
64	1.1170	1072	1216	
65	1.1344	6401	3735	
66	1.1519	1730	6254	
67	1.1693	7059	8773	
68	1.1868	2389	1292	
69	1.2042	7718	3811	
70	1.2217	3047	6330	
71	1.2391	8376	8849	
72	1.2566	3706	1368	
73	1.2740	9035	3887	
74	1.2915	4364	6406	
75	1.3089	9693	8925	
76	1.3264	5023	1444	
77	1.3439	0352	3963	
78	1.3613	5681	6482	
79	1.3788	1010	9001	
80	1.3962	6340	1520	
81	1.4137	1669	4039	
82	1.4311	6998	6558	
83	1.4486	2327	9057	
84	1.4660	7657	1596	
85	1.4835	2986	4115	
86	1.5009	8315	6634	
87	1.5184	3644	9153	
88	1.5358	8974	1572	
89	1.5533	4303	4191	
90	1.5707	9632	6710	
91	1.5882	4961	9229	
92	1.6057	0291	1748	
93	1.6231	5620	4267	
94	1.6406	0949	6786	
95	1.6580	6278	9305	
96	1.6755	1608	1824	
97	1.6929	6937	4343	
98	1.7104	2266	6862	
99	1.7278	7595	9381	
100	1.7453	2925	1900	

A Common Divisor for the speedy converting of the Table, shewing the *Area* of the Segments of a Circle whose Diameter is 1.0000 &c. into a Table shewing the *Area* of the Segment of any Circle whose *Area* is given.

1	0031	4159	2653
2	0062	8318	5306
3	0094	2477	7959
4	0125	6637	0612
5	0157	0796	3265
6	0188	4955	5918
7	0219	9114	8571
8	0251	3274	1224
9	0282	7433	3877
10	0314	1592	6530
11	0345	5751	9183
12	0376	9911	1836
13	0408	4070	4489
14	0439	8229	7142
15	0471	2388	9795
16	0502	6548	2448
17	0534	0707	5101
18	0565	4866	7754
19	0596	9026	0407
20	0628	3185	3060
21	0659	7344	5713
22	0691	1503	8366
23	0722	5663	1019
24	0753	9822	3672
25	0785	3981	6325
26	0816	8140	8978
27	0848	2300	1631
28	0889	6459	4284
29	0911	0618	6937
30	0942	4777	9590
31	0973	8937	2243
32	1005	3096	4896
33	1036	7255	7549
34	1068	1415	0202
35	1099	5574	2855
36	1130	9733	5508
37	1162	3892	8161
38	1193	8052	0814
39	1225	2211	3467
40	1256	6370	6120
41	1288	0529	8773
42	1319	4689	1426
43	1350	8848	4079
44	1382	3007	6732
45	1413	7166	9385
46	1445	1326	2038
47	1476	5485	4691
48	1507	9644	7344
49	1539	3803	9997
50	1570	7963	2650

A Common Diviſor for the ſpeedy converting of the Table, ſhewing the *Area* of the Segments of a Circle whoſe Diameter is 2. 0000 *&c.* into a Table ſhewing the *Area* of the Segment of any other Circle whoſe *Area* is given.

51	1602	2122	5303	76	2387	6104	1618
52	1633	6281	7956	77	2419	0263	4281
53	1665	0441	0609	78	2450	4422	6934
54	1696	4600	3262	79	2482	8581	9587
55	1727	8759	5915	80	2513	2741	2240
56	1759	2918	6568	81	2544	6900	4893
57	1790	7078	1221	82	2576	1059	7546
58	1822	1237	3874	83	2607	5219	0199
59	1853	5396	6527	84	2638	9378	2852
60	1884	9555	9180	85	2670	3537	5505
61	1916	3715	1833	86	2701	7696	8158
62	1947	7874	4486	87	2733	1856	0811
63	1979	2033	7139	88	2764	6015	3464
64	2010	6192	9792	89	2796	0174	6117
65	2042	0352	2445	90	2827	4333	8770
66	2073	4511	5098	91	2858	8493	1423
67	2104	8670	7751	92	2890	2652	4076
68	2136	2830	0404	93	2921	6811	6729
69	2167	6989	3057	94	2953	0970	9382
70	2199	1148	5710	95	2984	5130	2035
71	2230	5307	8363	96	3015	9289	4688
72	2261	9467	1016	97	3047	3448	7341
73	2293	3626	3669	98	3078	7607	9994
74	2324	7785	6322	99	3110	1767	2647
75	2356	1944	8975	100	3141	5926	5300

A Table ſhewing the Ordinates, Arches and Areas of the Segments of a Circle, whoſe Diameter is 2000, &c. to every Hundredth Part of the *Radius*.

Ordinates

	Ordinates	Deg. & Dec. p.	Areas	
100	10000000000	90.00000000	1.57079632	
99	9999499771	89.42704196	1.55079682	
98	9997999799	88.85400799	1.53079890	
97	9995498987	88.28987110	1.51080538	
96	9991999774	87.70756124	1.49081774	
95	9987492177	87.13402020	1.47083808	
94	9981983770	86.56018749	1.45086837	
93	9975469913	85.98601581	1.43091081	
92	9967948635	85.41143529	1.41096718	
91	9959417653	84.83639513	1.39103966	
90	9949874371	84.26083018	1.37113017	
89	9939315871	83.68468641	1.35124084	
88	9927738916	83.10789860	1.33137360	
87	9915139938	82.53040793	1.31153053	
86	9901515035	81.95215479	1.29171372	
85	9886859966	81.37307468	1.27192518	
84	9871170138	80.79310474	1.25216697	
83	9854440623	80.21218180	1.23244118	
82	9836666101	79.63024030	1.21274989	
81	9817840903	79.04721672	1.19309522	
80	9797958971	78.46304188	1.17347924	
79	9777013859	77.87761112	1.15390361	
78	9754998718	77.29096735	1.13437189	
77	9731906288	76.70191903	1.11488481	
76	9707718879	76.11243681	1.09544458	
75	9682458365	75.52148845	1.07605462	
74	9656086163	74.92996014	1.05671627	
73	9628603221	74.33573392	1.03743102	
72	9600000000	73.73979456	1.01810110	
71	9570266454	73.14202474	0.99903143	
70	9539392014	72.54239737	0.97992192	
69	950 365565	71.94076969	0.96087497	
68	9474175415	71.33707564	0.94189323	
67	9439809319	70.73122476	0.92297905	

	Ordinates	Deg. & Dec. p.	Areas
67	9439809319	70.73122476	0.92197905
66	9404254356	70.12312662	0.90413479
65	9367496997	69.51268521	0.88536283
64	9329523031	68.89980401	0.86666560
63	9290317540	68.28438326	0.84804557
62	9249864864	67.66631784	0.82950517
61	9208148564	67.04550117	0.81104695
60	9165151389	66.42182324	0.79267345
59	9120855212	65.79516567	0.77438721
58	9075241043	65.16541298	0.75619089
57	9028288874	64.53244020	0.73808713
56	8979977728	63.89612058	0.72007866
55	8930285549	63.25631645	0.70216884
54	8879189152	62.61289754	0.68435845
53	8826664149	61.96570387	0.66665234
52	8772684879	61.31459838	0.64905275
51	8717224755	60.65941181	0.63156249
50	8660254037	60.00000000	0.61418485
49	8601744009	59.33617061	0.59692260
48	8541662601	58.66774875	0.57977892
47	8479976415	57.99454553	0.56275702
46	8416650165	57.31636147	0.54586011
45	8351646544	56.63307065	0.52909299
44	8284926070	55.94420256	0.51245467
43	8216446926	55.24977433	0.49595300
42	8146264741	54.54945742	0.47959008
41	8074032449	53.84299205	0.46336957
40	8000000000	53.13010237	0.44725211
39	7924014134	52.41049708	0.43137885
38	7846018098	51.68386597	0.41560051
37	7765951325	50.94987748	0.39998218
36	7683749084	50.20810657	0.38453683
35	7599341076	49.45831012	0.36925312
34	7512655988	48.70012721	0.35414227

	Ordinates	Deg. & Dec. p.	Areas	
34	751126533988	48.70012721	0.35414217	
33	742361098 1	47.93293539	0.33920561	
32	7332121111	47.15635717	0.32444946	
31	7238093671	46.36989113	0.30987884	
30	7141428428	45.57299618	0.29549884	
29	7042016756	44.76508489	0.28131493	
28	6939740629	43.94551977	0.26733268	
27	6834471449	43.11360613	0.25355796	
26	6726068688	42.26858452	0.23999689	
25	6614378277	41.40962595	0.22665594	
24	6499230723	40.53580218	0.21354168	
23	6380438856	39.64611132	0.20066138	
22	6257795138	38.73942400	0.18802248	
21	6131068422	37.81443867	0.17563291	
20	6000000000	36.86989765	0.16350111	
19	5864298764	35.90406873	0.15163601	
18	5723635208	34.91520640	0.14004712	
17	5577633906	33.90125515	0.12874491	
16	5425863986	32.85988059	0.11774053	
15	5267826876	31.78833069	0.10704574	
14	5102940328	30.68341722	0.09667379	
13	4930517214	29.54136121	0.08663902	
12	4749736834	28.35773666	0.07695728	
11	4559605246	27.12675321	0.06764629	
10	4358898943	25.84193282	0.05872590	
09	4146082488	24.49464857	0.05021866	
08	3919183588	23.07391815	0.04215095	
07	3675595189	21.56518547	0.03455313	
06	3411744421	19.94844363	0.02746104	
05	3122498999	18.19487244	0.02092302	
04	2800000000	16.26020471	0.01499411	
03	2431049156	14.06986184	0.00975364	
02	1989974874	11.47834097	0.00551730	
01	1410673597	8.10961446	0.00188278	

	Ordinates	Deg. & Dec. p.	Areas	
010	1410673597	8.10961446	0.00188178	
009	1338618691	7.69281247	0.00160779	
008	1262378707	7.25214680	0.00134761	
007	1181143513	6.78328892	0.00110317	
006	1093800713	6.27948064	0.00087554	
005	0998749217	5.73196797	0.00066616	
004	0893532316	5.12640010	0.00047674	
003	0774015503	4.43921228	0.00030969	
002	0632139125	3.62430750	0.00016860	
001	0447101778	2.56255874	0.00005961	

The

	1. 57979832		983	1. 53679796	
	199999			199969	
999	1. 56879832		982	1. 53479827	
	199999			199965	
998	1. 56679832		981	1. 53279862	
	199999			199963	
997	1. 56479833		980	1. 53079899	
	199998			199957	
996	1. 56279834		979	1. 52879941	
	199997			199953	
995	1. 56079836		978	1. 52679988	
	199996			199949	
994	1. 55879839		977	1. 52480039	
	199995			199944	
993	1. 55679844		976	1. 52280095	
	199994			199939	
992	1. 55479849		975	1. 52080156	
	199992			199934	
991	1. 55299657		974	1. 51880222	
	199991			199929	
990	1. 55079666		973	1. 51680293	
	199988			199924	
989	1. 54879677		972	1. 51480369	
	199986			199918	
988	1. 54679690		971	1. 51280451	
	199984			199912	
987	1. 54479706		970	1. 51080539	
	199981			199906	
986	1. 54279724		969	1. 50880633	
	199978			199909	
985	1. 54079745		968	1. 50680733	
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493	0. 60208903		476	0. 57295539	
	172329			170281	
492	0. 60036574		475	0. 57125258	
	172211			170158	
491	0. 59864363		474	0. 56955100	
	172093			170034	
490	0. 59692160		473	0. 56785066	
	171975			169910	
489	0. 59520185		472	0. 56615156	
	171856			163786	
488	0. 59348419		471	0. 56445370	
	171736			169661	
487	0. 59176693		470	0. 56275702	
	171617			169536	
486	0. 59005076		469	0. 56106166	
	171498			169411	
485	0. 58833578		468	0. 55936755	
	171377			169285	
484	0. 58662201		467	0. 55767470	
	171256			169159	

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465	0. 55429278	
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464	0. 55260377	
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463	0. 55091598	
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462	0. 54922946	
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461	0. 54754422	
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460	0. 54586011	
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459	0. 54417743	
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457	0. 54081594	
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454	0. 53578341	
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453	0. 53410850	
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452	0. 53243490	
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444	0. 51909488	
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443	0. 51743320	
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441	0. 51411385	
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	164811	

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432	0.49924240			162278	
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431	0.49759705			162134	
	164397		414	0.46984058	
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	164259		413	0.46822069	
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	164120		412	0.46660225	
428	0.49266929			1616991	
	163980		411	0.46498526	
427	0.49102949			161570	
	163835		410	0.46336957	
426	0.48939114			1614101	
	163700		409	0.46175547	
425	0.48775414			161260	
	163560		408	0.46014287	
424	0.48611854			161113	
	163419		407	0.45853174	
423	0.48448435			160966	
	163277		406	0.45692208	
422	0.48285158			160818	
	163135		405	0.45531390	
421	0.48122023			160670	
	162998		404	0.45370720	
420	0.47959025			160522	
	162843		403	0.45210198	
419	0.47796165			160375	
	162708		402	0.45049825	
418	0.47633457			160223	
	162565		401	0.44889602	
417	0.47470892			160073	
	162421		400	0.44729529	

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	159923			157313	
399	0. 44569599		382	0. 41874213	
	155773			157156	
398	0. 44409826		381	0. 41717057	
	159613			156999	
397	0. 44250193		380	0. 41560058	
	159472			156841	
396	0. 44090731		379	0. 41403217	
	159320			156682	
395	0. 43931411		378	0. 41246535	
	159168			156522	
394	0. 43772243		377	0. 41090013	
	159016			156363	
393	0. 43613227		376	0. 40933650	
	158863			156204	
392	0. 43454364		375	0. 40777446	
	158710			156044	
391	0. 43295654		374	0. 40621402	
	158557			155883	
390	0. 43137086		373	0. 40465519	
	158403			155722	
389	0. 42978683		372	0. 40309797	
	158248			155561	
388	0. 42820435		371	0. 40154236	
	158093			155399	
387	0. 42662342		370	0. 39998818	
	157938			155238	
386	0. 42504404		369	0. 39843580	
	157781			155025	
385	0. 42346622		368	0. 39688555	
	157626			154911	
384	0. 42188996		367	0. 39533644	
	157470			154788	

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		153923
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355	0. 37687404	
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		152582
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		152443
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		151556
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		151038
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		150164
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338	0. 35114075	
		149811
337	0. 34964264	
		149634
336	0. 34814630	
		149457
335	0. 34665173	
		149279
334	0. 34515894	
		149100

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	148921		316	0. 31859869	
332	0. 34217873				145803
	148742		315	0. 31714066	
331	0. 34069131				145614
	148562		314	0. 31568452	
330	0. 33920561				145425
	148381		313	0. 31423027	
329	0. 33772180				145236
	148200		312	0. 31277791	
328	0. 33623980				145047
	148024		311	0. 31132744	
327	0. 33475956				144856
	147842		310	0. 30987884	
326	0. 33328114				144665
	147663		309	2. 30843219	
325	0. 33180451				144474
	147480		308	0. 30698745	
324	0. 33032971				144282
	147288		307	0. 30554463	
323	0. 32885683				144090
	147104		306	0. 30410373	
322	0. 32738579				143897
	146919		303	0. 30166476	
321	0. 32591660				143703
	146735		304	0. 30122773	
320	2. 32444946				143508
	146550		303	0. 29978265	
319	0. 32298396				143315
	146362		302	0. 29835950	
318	0. 32152034				143120
	145175		301	0. 29692830	
317	0. 32005859				142926
	145990		300	0. 29549904	

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	141730			139311	
299	0. 29407154		282	0. 27011313	
	142533			139105	
298	0. 29264621		281	0. 26872208	
	141335			138898	
297	0. 29122286		280	0. 26733268	
	141137			138690	
296	0. 28980149		279	0. 26594578	
	141939			138482	
295	0. 28838210		278	0. 26456096	
	141741			138273	
294	0. 28696469		277	0. 26317823	
	141460			138063	
293	0. 28555009		276	0. 26179760	
	141260			137853	
292	0. 28413749		275	0. 26041907	
	141191			137643	
291	0. 28272558		274	0. 25904264	
	140990			137432	
290	0. 28131493		273	0. 25766832	
	140730			137210	
289	0. 27990763		272	0. 25629612	
	140527			137008	
288	0. 27850236		271	0. 25492604	
	140331			136795	
287	0. 27709905		270	0. 25355796	
	140124			136583	
286	0. 27569781		265	0. 25219213	
	139910			136370	
285	0. 27429861		268	0. 25082843	
	139710			136153	
284	0. 27290141		267	0. 24946690	
	139517			135936	

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266	0. 24810754		250	0. 22665594	
		135720			132173
265	0. 24675934		249	0. 22533421	
		135504			131946
			248	0. 22401475	
264	0. 24539530				131718
		135287	247	0. 22269757	
263	0. 24404243				131488
		135069	246	0. 22138269	
262	0. 24269174				131259
		134850	245	0. 22007010	
261	0. 24134324				131029
		134553	244	0. 21875981	
260	0. 23999689				130799
		134333	243	0. 21745182	
259	0. 23865356				130567
		134189	242	0. 21614615	
258	0. 23731165				130334
		133968	241	0 21484281	
257	0. 23597197				130101
		133746	240	0. 21354168	
256	0. 23463451				129867
		133523	239	0. 21224301	
255	0. 23329928				129632
		133300	238	0. 21094669	
254	0. 23196628				129396
		133076	237	0. 20965273	
253	0. 23063552				129160
		132801	236	0. 20836113	
252	0. 22930751				128924
		132575	235	0. 20707189	
251	0. 22798176				128688
		132399	234	0. 20578501	
250	0. 22665777				128445

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	123968	
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	127729	
230	0.20066138	
	127488	
229	0.19938650	
	127245	
228	0.19811405	
	127002	
227	0.19684403	
	126758	
226	0.19557645	
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225	0.19431131	
	126269	
224	0.19304861	
	126023	
223	0.19178839	
	125776	
222	0.19053063	
	125528	
221	0.18927535	
	125279	
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218	0.18552444	
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217	0.18427915	
	124278	
	124028	
216	0.18303637	
	124025	
215	0.18179611	
	123771	
214	0.18055841	
	123517	
213	0.17932324	
	123262	
212	0.17809062	
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211	0.17686056	
	122749	
210	0.17563291	
	122490	
209	0.17440801	
	122232	
208	0.17318569	
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207	0.17196595	
	121713	
206	0.17074882	
	121451	
205	0.16953431	
	121189	
204	0.16832242	
	120926	
203	0.16711316	
	120663	
202	0.16590653	
	120399	
201	0.16470254	
	120133	
200	0.16350121	

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	119866			115084	
199	0. 16230245		182	0. 14234343	
	119598			114900	
198	0. 16110647		181	0. 14119343	
	119329			114615	
197	0. 15991318		180	0. 14004728	
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196	0. 15872259		179	0. 13890400	
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195	0. 15753470		178	0. 13776354	
	118518			113752	
194	0. 15634952		177	0. 13662602	
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193	0. 15516706		176	0. 13549140	
	117972			113164	
192	0. 15398733		175	0. 13435916	
	117698			112873	
191	0. 15281035		174	0. 13323103	
	117422			112587	
190	0. 15163596		173	0. 13210516	
	117146			112292	
189	0. 15046450		172	0. 13098214	
	116869			111996	
188	0. 14929581		171	0. 12986128	
	116591			111700	
187	0. 14812990		170	0. 12874498	
	116312			111403	
186	0. 14696678		169	0. 21763088	
	116032			111105	
185	0. 14580646		168	0. 12651983	
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184	0. 14464895		167	0. 12541178	
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165	0. 12320475	
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164	0. 12210579	
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163	0. 12100987	
	109287	
162	0. 11991700	
	108980	
161	0. 11882720	
	108671	
160	0. 11774053	
	108361	
159	0. 11665692	
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158	0. 11557645	
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157	0. 11449910	
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156	0. 11342485	
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155	0. 11235375	
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154	0. 11128581	
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153	0. 11022103	
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152	0. 10915944	
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151	0. 10810106	
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150	0. 10704589	
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149	0. 10599395	
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147	0. 10389980	
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146	0. 10285762	
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139	0. 09565503	
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138	0. 09463953	
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137	0. 09362743	
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136	0. 09261874	
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135	0. 09161348	
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126	0. 08272305	
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115	0. 07225434	
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109	0. 06673685	
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95	0. 05441920		78	0. 04058274	
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94	0. 05357053		77	0. 03981077	
	84440			76707	
93	0. 05272613		76	0. 03905370	
	84010			76214	
92	0. 05188603		75	0. 03829146	
	83666			75748	
91	0. 05104937		74	0. 03753398	
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90	0. 05121866		73	0. 03678140	
	82700			74764	
89	0. 04939166		72	0. 03603376	
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88	0. 04856907		71	0. 03529111	
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87	0. 04775093		70	0. 03455313	
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86	0. 04693727		69	0. 03382067	
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85	0. 04612811		68	0. 03309321	
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84	0. 04532349		67	0. 03237089	
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62	0.02883789	11
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61	0.02814735	11
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59	0.02678243	02
	67405	
58	0.02610838	03
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57	0.02543998	03
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56	0.02477725	03
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55	0.02412024	03
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54	0.02346901	04
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53	0.02282362	04
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52	0.02218412	04
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51	0.02155059	04
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50	0.02093309	04

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49	0.02030159	04
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47	0.01907725	05
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46	0.01847448	05
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45	0.01787808	05
	58996	
44	0.01728811	05
	58344	
43	0.01670468	06
	57683	
42	0.01611784	06
	57016	
41	0.01553768	06
	56340	
40	0.01499411	06
	55655	
39	0.01443756	07
	54960	
38	0.01388796	07
	54256	
37	0.01334540	08
	53540	
36	0.01281000	07
	52815	
35	0.01228185	08
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34	0.01176106	08
	51331	

33	0.01124776			36176	
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31	0.01024403			33929	
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29	0.00927147			31517	
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26	0.00787430			27442	
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25	0.00742556			25959	
	43997		8	0.00134877	
24	0.00698559			24434	
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23	0.00655457			22749	
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22	0.00613272			20925	
	41244		5	0.00066769	
21	0.00572028			18912	
	40273		4	0.00047847	
20	0.00531730			16675	
	39291		3	0.00031172	
19	0.00492439			14061	
	38297		2	0.00017111	
18	0.00454142			10792	
	37248		1	0.00006319	
17	0.00416894			6319	
	36176		00	0.00000000	

A TABLE SHEWING THE AREA OF THE SEGMENTS OF A CIRCLE WHOSE

Whole Area is Unity, to the ten Thousandth part of the Diameter.

	0	1	2	3	4
0	000000	000004	000007	000011	000014
1	000053	000062	000071	000080	000089
2	000151	000163	000175	000187	000200
3	000278	000292	000307	000322	000336
4	000428	000444	000461	000478	000494
5	000599	000617	000636	000654	000673
6	000787	000807	000827	000847	000867
7	000991	001012	001034	001056	001077
8	001211	001234	001257	001280	001304
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17	003743	003776	003809	003842	003876
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FINIS.

COSMOGRAPHIA,

THE

Second Part,

OR, THE

DOCTRINE

OF THE

PRIMUM MOBILE.

Astronomy.

AN INTRODUCTION TO Astronomy.

The First Part.

Of the *Primum Mobile*.

CHAP. I.

Of the General Subject of Astronomy.

1. Astronomy, Is a Science concerning the Measure and Motion of the Spheres and Stars.

2. Astronomy hath two parts, the first is Absolute, and the other Comparative.

3. The Absolute part of Astronomy is that which treateth of the Measure and Motion of the Orbs and Stars absolutely without respect to any distinction of Time.

4. The Comparative part of Astronomy is that, which treateth of the Motion of the Stars, in reference to some certain distinction of Time.

5. The Absolute part of Astronomy treateth of the *Primum Mobile*, or Diurnal Motion of all the Celestial Orbs or Spheres.

6. The *Primum Mobile*, or Diurnal Motion of the Heavens, is that Motion, by which the several Spheres are moved round the World in a Day and a Night, that is, in 24 hours, from East towards West, and so forward from West towards East, and so continually returning to the same point from whence they began their Motion.

7. This first and common Motion of the Heavens, will be best understood, by help of an Instrument called a Globe, which is an Artificial representation of the Heavens, or the Earth and Waters under that Form and Figure, or Roundness which they are supposed to have.

8. This Representation or Description of the Visible World is by Circles, great and small, some of which are expressed upon, and others are framed without the Globe.

9. The Circles without the Globe are chiefly two; the *Meridian* and the *Horizon*, the one of Brass, and the other of Wood: And these two Circles are variable or mutable; for although there is but one *Horizon* and one *Meridian* in respect of the whole World, or in respect of the whole Heaven and Earth, yet in respect of the particular parts of Heaven, or rather in respect of the diverse Provinces, Countries and Cities on the Earth, there are diverse both *Horizons* and *Meridians*.

10.

10. The Meridian then is a great Circle without the Globe, dividing the Globe, and consequently the Day and Night into two equal parts, from the North and South ends whereof a strong Wyre of Brass or Iron is drawn or supposed to be drawn through the Center of the Globe representing the Axis of the Earth, by means whereof the whole Globe turneth round within the said Circle, so that any part may be brought directly under this Brass Meridian at pleasure.

11. This Brass Meridian is divided into 4 equal parts or Quadrants, and each of them are subdivided into 90 Degrees, that is 360 for the whole Circle. The reason why this Circle is not divided in 380 Degrees throughout, but still stoping at 90, beginneth again with 10.20.30 &c. is, for that the use of this Meridian in reference to its Division in Degrees, requireth no more than that Number.

12. The Horizon is a great Circle without the Globe, which divides the upper part of Heaven from the lower, so that the one half is always above that Circle, and the other under it.

13. The Poles of this Circle are two, the one directly over our Heads, and is called the Zenith; the other is under feet, and is called the Nadir.

14. The Horizon is either Rational or Sensible.

15. The Rational Horizon is that, which divideth the Heavens and the Earth into two equal parts, which though it cannot be perceived and distinguished by the eye, yet may be conceived in our minds, in which respect all the Stars may be conceived to rise and set as in our view.

16. The Visible Horizon is that Circle which the eye doth make at its farthest extent of sight, when the body in any particular place doth turn it self round. Of these two Circles there needeth no more to be said at present, only we may observe, that it was ingeniously devised by those, who first thought upon it, to set one Meridian and one Horizon without the Globe, to avoid the confusion, if not the impossibility, of drawing a several Meridian and a several Horizon for every place, which must have been done, if this or the like device had not been thought upon.

17. Besides these two great Circles without the Globe, there are 4 other great Circles drawn upon the Globe it self besides the Meridian. 1. The Æquator or Equinoctial Circle. 2. The Zodiack. 3. The Æquinoctial Colure. 4. Solstitial Colure. And these four Circles are immovable, that is, in whatsoever part of the World you are, these Circles have no variation, as the other two have.

18. The Æquator is a great Circle drawn upon the Globe, in the middle between the two Poles, and plainly dividing the Globe into two equal parts.

19. The Æquator is the measure of the Motion of the *Primum Mobile*, for 15 Degrees of this Circle do always arise in an hours time; the which doth clearly shew, that the whole Heavens are turned round by equal intervals in the space of one day or 24 hours.

20. In this Circle the Declinations of the Stars are computed from the mid-Heaven towards the North or South.

21. This Circle gives denomination to the Æquinox, for the Sun doth twice in a Year and no more crois this Circle, to wit, when he enters the first points of *Aries*, and *Libra*, and then he maketh the Days and the Nights equal: His entrance into *Aries* is in *March*, and is called the Vernal Equinox; and his entrance into Libra, is in *September*, and is called the Autumnal Equinox.

22. And from one certain point in this Circle, the Longitude of Places upon the Earth are reckoned; and the Latitude of Places are reckoned from this Circle towards the North, or the South Poles.

23. The Zodiack is a great Circle drawn upon the Globe, cutting the Æquinoctial Points at Oblique Angles: for although it divides the whole World into two equal parts, in reference to its own Poles; yet in reference to the Poles of the World, it hath an Oblique Motion.

24. The Poles of this Circle are as far distant from the Poles of the World, as the greatest Obliquity thereof is from the Equinoctial, that is 23 Degrees, and 31 Minutes or thereabouts.

25. This Circle doth differ from all other Circles upon the Globe in this: other Circles (to speak properly) have Longitude assigned them, but no Latitude; but this hath both. Whereas other Circles are in reference to their Longitude or Rotundity only divided into 360 Degrees, this Circle in respect of its Latitude is supposed to be divided into 16 Degrees in Latitude.

26. The Zodiack then in respect of Longitude is commonly divided into 360 Degrees as o-

ther Circles are: but more peculiarly in respect of its self it is divided into 12 Parts called Signs, and each Sign into 30 Degrees, and 12 times 30 do make 360.

27. The 12 Signs into which the Zodiack is divided, have these Names and Characters. *Aries* ♈. *Taurus* ♉. *Gemini* ♊. *Cancer* ♋. *Leo* ♌. *Virgo* ♍. *Libra* ♎. *Scorpio* ♏. *Sagittarius* ♐. *Capricornus* ♑. *Aquarius* ♒. and *Pisces* ♓.

28. These two Circles of the Equator and Zodiack are crossed by two other great Circles, which are called Colures: They are drawn through the Poles of the World, and cut one another as well as the Equator at Right Angles. One of them passeth through the Intersections of the Equinoctial points, and is called the Equinoctial Colure. The other passeth through the points of the greatest distance of the Zodiack, from the Equator, and is called the Solstitial Colure.

29. The other great Circles described upon the Globe are the Meridians: Where we must not think much to hear of the Meridians again. That of Brass without the Globe is to serve all turns, and the Globe is framed to apply it self thereto. The Meridians upon the Globe, will easily be perceived to be of a new and another use.

30. The Meridians upon the Globe are either the great or the less: Not that the great are any greater than the less, for they have all one and the same center, and equally pass through the Poles of the Earth; But those which are called less, are of less use than that, which is called the great.

31.

31. The great is otherwise called the fixt and first Meridian, to which the less are second, and respectively moveable. The great Meridian is as it were the *Landmark* of the whole Sphere, from whence the Longitude of the Earth, or any part thereof is accounted. And this the only Circle which passing through the Poles is graduated or divided into Degrees, not the whole Circle but the half, because the Longitude is to be reckoned round about the Earth.

32. The lesser Meridians are those black lines, which you see to pass through the Poles and succeeding the great at 10 and 10 Degrees, as in most Globes; or at 15 and 15 Degrees difference, as in some. Every place never so little more East or West than another, hath properly a several Meridian, yet because of the huge distance of the Earth from the Heavens, there is no sensible difference between the Meridians of places that are less than one Degree of Longitude asunder, and therefore the Grographers as well as the Astronomers allow a new Meridian to every Degree of the Equator; which would be 180 in all: but except the Globes were made of an extream and an unusual Diameter, so many would stand too thick for the Description. Therefore most commonly they put down but 18, that is, at 10 Degrees distance from one another; the special use of the lesser Meridians being to make a quicker dispatch, in the account of the Longitudes. Others set down but 12, at 15 Degrees difference; aiming at this, That the Meridians might be distant from one another a full part of time, or an hour: for seeing that the Sun is carried 15 Degrees of the Equinoctial every hour, the Meridians set at that

stance must make an hours difference in the rising or setting of the Sun in those places which differ 15 Degrees in Longitude.

And to this purpose also upon the North end of the Globe, without the Brass Meridian, there is a small Circle of Brass set, and divided into two equal parts, and each of them into twelve, that is, twenty four all; to shew the hour of the Day and Night, in any place where the Day and Night exceed not 24 hours; for which purpose it hath a little Brass Pin turning about upon the Pole, and pointing to the several hours, which is therefore the *Index Horarius*, or Hour *Index*.

33. Having described the great Circles framed without and drawn upon the Globe, we will now describe the lesser Circles also; And these lesser Circles are called Parallels, that is, such as are in all places equally distant from the Equator; and these Circles how little soever, are supposed to be divided into 360 Degrees: but these Degrees are not so large as in the great Circles, but do proportionably decrease according to the *Radius* by which they are drawn.

34. These lesser Circles are either the Tropicks or the Polar Circles.

35. The Tropicks are two small Circles drawn upon the Globe, one beyond the Equator towards the North Pole, and the other towards the South, Shewing the way which the Sun makes in his Diurnal Motion, when he is at his greatest distance from the Equator either North or South. These Circles are called Tropicks ἀπὸ τῆς τροπῆς, that is, from the Suns returning: for the Sun coming to these Circles, he is at his greatest distance from the Equator, and in the same Moment of time

sloping

sloping as it were his course, he returns nearer and nearer to the Equator again.

36. These Tropical Circles do shew the point of Heaven in which the Sun doth make either the longest Day, or the Shortest Day in the Year, according as he is in the Northern or the Southern Tropick: And are drawn at 23 Degrees and a half distant from the Equator.

37. The Polar Circles are two lesser Circles drawn upon the Globe at the Radius of 23 Degrees and a half distant from the Poles of the World, shewing thereby the Poles of the Zodiack, which is so many Degrees distant from the Equator on both sides thereof.

38. These Polar Circles are 66 Degrees and a half distant from the Equator, and 43 Degrees distant from his neatest Tropick. They are called the Arctick and Antarctick Circles.

39. The Arctick Circle is that which is described about the Arctick Pole, and passeth almost through the middle of the Head of the greater Bear. It is called the Arctick Circle ἀπὸ τῶν ἄρκτων· from the two conspicuous Stars towards the North, called the greater and the lesser Bear.

40. The Antarctick Circle is that which is described about the Antarctick or South Pole. It is so called ἀντὶ τῆς ἄρκτου that is, from being opposite to the greater and lesser Bear.

Having thus described the Globe or Astronomical Instrument by which the Frame of the World is represented to our view, I will proceed to shew the use for which it is intended.

CHAP.

CHAP. II.

Of the Distinctions and Affections of Spherical Lines or Arches.

THE uſes of the Globe as to practice, are either ſuch as concern the Heavens or the Earth, in either of which, if we ſhould deſcend unto particulars, the uſes would be more in number, than a ſhort Treatiſe will contain: Seeing therefore that all Problems which concern the Globe, may be beſt and moſt accurately reſolved by the Doctrine of Spherical Triangles, we willcontract theſe uſes of the Globe(which otherwiſe might prove infinite) to ſuch Problems as come within the compaſs of the 28 Caſes of Right and Oblique angled Spherical Triangles.

2. And that the nature of Spherical Triangles may be the better underſtood, and by which of the 28 Caſes the particular Problems may be beſt reſolved, I will ſet down ſome General Definitions and Affections, which do belong to ſuch Lines or Arches of which the Triangle muſt be framed, with the Parts and Affections of thoſe Triangles, and how the things given and required in them, may be repreſented and reſolved upon and by the Globe, as alſo how they may be repreſented and reſolved by the Projection of the Sphere, and by the Canon of Triangles.

3. A Spherical Triangle then is a Figure conſiſting of three Arches of the greateſt Circles upon the Superficies of a Sphere or Globe, every one being leſs than a ſemicircle,

4.

4. A great Circle is that which divideth the Sphere or Globe into two equal parts, and thus the Horizon, Equator, Zodiack and Meridians before described are all of them great Circles: And of these Circles or any other, there must be three Arches to make a Triangle, and every one of these Arches severally must be less than a semicircle: To make this plain,

In *Fig.* 1. The streight Line *H A R* doth represent the Horizon, *P R* the height of the Pole above the Horizon, *P M S* a Meridian, and these three Arches by their intersecting one another do visibly constitute four Spherical Triangles. 1. *P M R*, 2. *P M H*. 3. *S H M*. 4. *S M R*. And every Arch is less than a semicircle, as in the Triangle *P M R*, the Arch *P R* is less than the Semicircle *P R S*, the Arch *M R* is less than the Semicircle *A M R*, and the Arch *P M* is less than the Semicircle *P M S*, the like may be shewed in the other Triangles.

5. Spherical or circular Lines are Parallel or Angular.

6. Parallel Arches or Circles, are such as are drawn upon the same Center within, without, or equal to another Arch or Circle. Thus in *Fig.* 1. The Arches ♋ *M* ♋ and ♑ *o* ♑ are though lesser Circles, parallel to the Equinoctial Æ *A Q* and do in that Scheme represent the Tropicks of Cancer and Capricorn. The manner of describing them or any other Parallel Circle is thus, set off their distance from the great Circle, to which you are to draw a parallel with your Compasses, by help of your Line of Chords, which in this Example is 23 Degrees and a half from Æ to ♋, then draw the Line *A* ♋, and upon the point ♋ erect a Perpendi-

pendicular, where that Perpendicular ſhall cut the Axis *P Z* extended, is the Center of that Parallel.

7. A Spherical Angle, is that which is conteined by two Arches of the greateſt Circles upon the Superficies of the Globe interſecting one another: Angles made by the Interſection of two leſſe Circles, or of a little Circle with a great, we take no notice of in the Doctrine of Spherical Triangles.

8. A Spherical Angle is either Right or Oblique.

9. A Spherical Right Angle is that which is conteined, by two Arches of the greateſt Circles in the Superficies of the Sphere cutting one another at Right Angles, that is, the one being right or perpendicular to the other: thus the Braſs Meridian cutteth the Horizon at right Angles; and thus the Meridians drawn upon the Globe, as well as the Braſs Meridian, do all of them cut the Equator at Right Angles.

10. An Oblique Spherical Angle, is that which is conteined by two Arches of the greateſt Circles in the Superficies of the Sphere, not being right or perpendicular to one another.

11. An Oblique Spherical Angle is Obtuſe, or Acute.

12. An Obtuſe Spherical Angle, is that which is greater than a Right Angle. An Acute is that which is leſs than a Right Angle.

13. If two of the greateſt Circles of the Sphere ſhall paſs through one anothers Poles, thoſe two great Circles ſhall cut one another at Right Angles. Thus the Brazen Meridian doth interſect the Equinoctial and Horizon.

14.

14. If two of the greatest Circles of the Sphere, shall intersect one another, and pass through each others Poles, they shall intersect one another at unequal or Oblique Angles, the Angle upon the one side of the intersection being Obtuse, or more than a Right, and the Angle upon the other side of the intersection being Acute or less than a Right. Thus in *Fig.* 1. The Arch *P M* doth intersect the Meridian and Horizon, but not in the Poles of either, therefore the Angle *H P M* upon one side of the intersection of that Arch with the Meridian, is more than a Right Angle; And the Angle *M P R* upon the other side of the intersection is less. And so likewise the Angle *P M H* upon the one side of the intersection of the Arch *P M* with the Horizon *H R*, is greater than a right Angle; and the Angle *R M P* upon the other side of the Intersection is less than a Right.

15. A Spherical Angle is measured by the Arch of a great Circle described from the Angular point between the sides of the Angle, those sides being continued unto Quadrants. Thus the Arch of the Equator *T Q* in *Fig.* 1. is the measure of the Angle *M P R*, or *T P Q*, the sides *P T* and *P Q* being Quadrants.

And the measure thereof in the Projection may thus be found: lay a Ruler from *P* to *T*, and it will cut the Primitive Circle in *V*; and the Arch *V Q* being taken in your Compasses and applyed to your Line of Chords, will give the Quantity of the Angle propounded.

16. The Complement of a Spherical Arch or Angle, is so much as it wanteth of a Quadrant, if the Arch or Angle given be less than a Quadrant; or so much as it wanteth of a Semi-

cle,

cle, if it be more than a Quadrant.

17. An Arch of a great Circle cutting the Arch of another great Circle, shall interfect one another at Right Angles, or make two Angles, which being taken together, shall be equal unto two Right. Thus in Fig. 1. The Axis *PAS* of Equinoctial Colure doth cut the Equator *EAQ* at Right Angles, but the Meridian *PMS* doth cut the Horizon *HMR* at Oblique Angles, making the Angle *PMR* less than a Right, and the Angle *SMR* more than a Right, and both together equal to a Semicircle.

18. From these general Definitions proper to Spherical Lines or Arches, the general Affections of these Arches may easily be discerned, I mean the various Positions of the Globe of the Earth, in respect of all and singular the Inhabitants thereof.

19. And the whole Body of the Sphere or Globe, in respect of the Horizon, is looked upon by the Earths Inhabitants, either in a Parallel, a Right, or an Oblique Sphere.

20. A Parallel Sphere is, when one of the Poles of the World is elevated above the Horizon to the Zenith, the other depressed as low as the Nadir, and the Equinoctial Line joyned with the Horizon. They which there inhabite (if any such be) see not the Sun or other Star rising or setting, or higher or lower in their diurnal revolution. And seeing that the Sun traveleth the whole Zodiack in a Year, and that half the Zodiack, is above the Horizon and half under it, it cometh to pass, that the Sun setteth not with them, for the space of six Months, nor giveth them any Light for the space of other six Months, and so

maketh

maketh but one Day and Night of the whole Year.

21. A Right Sphear is, when both the Poles of the World do lie in the Horizon, and the Equinoctial Circle is at his greatest distance from it, passing through the Zenith of the place. And in this position of the Sphere, all the Celestiall Bodies, Sun, Moon, and other Planets, and fixed Stars, by the daily turning about of the Heaven, do directly ascend above, and also directly descend below the Horizon, because the Motions which they make in their Daily motion do cut the Horizon Perpendicularly, and as it were at Right Angles. In this Position of the Sphere, all the Stars may be observed to rise and set in an equal space of time, and to continue as long above the Horizon, as they do under it, the Day and Night to those Inhabitants, being always of an equal length.

22. An Oblique Sphere is, when the Axis of the World (being neither Direct nor Parallel to the Horizon) is inclined obliquely towards both sides of the Horizon, as in *Fig.* II. Whence it cometh to pass, that so much as one of the Poles is elevated above the Horizon, upon the one side, so much is the other depressed under the Horizon, upon the other side.

And in this Position of Sphere, the Days are sometimes longer than the Nights, sometimes shorter, and sometimes of equal length. When the Sun is in either of the Equinoctial Points, the Days and Nights are equal, but when he declineth from the Equator towards the elevated Pole, the Days are observed to encrease, and when he declineth from the Equator towards the opposite Pole, of the Pole depressed,

depressed, the Days do decrease, as is manifest in *Fig.* 1. For when the Sun riseth at *M*, the Line *M S* above the Horizon is the Semidiurnal Arch of the longest day. When he riseth at *C*, the Arch *C V* above the Horizon, is the Semidiurnal Arch of the shortest Day. And when he riseth at *A*, the Days and Nights are of equal length, the Semidiurnal Arch of 180, being equal to the Seminocturnal Arch *AQ*.

CHAP. III.

Of the kind and parts of Spherical Triangles; and how to project the same upon the Plane of the Meridian.

HAving shewed what a Spherical Triangle is, and of what Circles it is composed, with the general Affections of such Lines: I will now shew how many several sorts of Triangles there are, of what Circular parts they do consist, and such Affections proper to them as will render the solution of them more clear and certain.

1. Spherical Triangles are either Right or Oblique.

2. A Right Angled Spherical Triangle, is that which hath one or more Right Angles.

3. A Spherical Triangle which hath three Right Angles, hath always his three sides Quadrants. As in *Fig.* 1. The Spherical Triangle *AZR*, the Angles *ZRA*, *RAZ* and *AZR* are right Angles, and the three sides *AZ*, *ZR* and *AR* are Quadrants also.

5. A

5. A Triangle that hath two right Angles, hath the sides opposite to those Angles Quadrants, and the third side is the measure of the third Angle. As in *Fig.* 1. The sides of the Spherical Triangle TPQ, namely TP and PQ are Quadrants, and the Angles opposite to these sides, to wit, PTQ and TQP are Quadrants also, and the third Angle TQ is the measure of the third Angle TPQ. But the Right Angled Triangle which hath one Right and two Acute Angles, is that which cometh most commonly to be resolved.

6. The Legs of a right Angled Spherical Triangle are of the same Affection with their opposite Angles; as in the Triangle ZAR *Fig.* 1. The side ZA is a Quadrant, and the Angle at A is right, because Z is the Pole of the Arch AR and ZA is perpendicular thereunto. And in the Triangle $RA\mathcal{E}$ the side $RZ\mathcal{E}$ being more then a Quadrant the Angle $RA\mathcal{E}$ is more then a Quadrant also, being more then the Right Angle RAZ. And in the right Angled Spherical Triangle APR the side PR being less then a Quadrant, the Angle PAR is less then a Quadrant also, being less then the right Angle RAZ.

7. An Oblique angled Spherical Triangle is either acute or obtuse.

8. An Acute angled Spherical Triangle hath all his Angles Acute, and each Side less then a Quadrant; As in the Triangles, ZFP. *Fig.* 2. The Angles at Z and P are acute, as appeareth by inspection; and the Angle at F is acute also because the Measure thereof $CD = EM$ is less then a Quadrant.

9. An Oblique Angled Spherical Triangle hath all his Angles either acute or obtuse: *viz.* Acute and mixt.

10. The Sides of a Spherical Triangle may be turned into Angles, and the Angles into Sides; The Complement of the greatest Side or greatest Angle to a Semicircle being taken in each conversion. For Example. If it were required to turn the Angles of the Oblique Angled Spherical Triangle *Z F P* into sides in *Fig.* 3. *E A* is the measure of the Angle at *P*, and *A D* in the Triangle *A D C* equal thereunto, *A C* is the Complement of *F Z P* to a Semicircle, and *K M* the the Measure of the Angle at *F* is equal to *D C*, and so the Sides of the Spherical Triangle *A D C* are equal to the Angles of the Spherical Triangle *F Z P*, making the side *A C* equal to the Complement of the Angle *Z* to a Semicircle.

11. In Right Angled Spherical Triangles the Sides intending the Right Angle we call the Legs; The Side subtending it the *Hypotenuse*.

12. In every Spherical Triangle besides the Area or space contained, there are six parts. *viz.* Three Sides and three Angles and of these six there must be always three given to find the rest, but in right Angled Spherical Triangles there are but five of the six parts parts which come into question, because one of the Angles being right is allways known, and so any two of the other five being given, the three remaining parts whether Sides or Angles, may be found. But before I come to the solution of these Triangles whether right or oblique, I will first shew how they may be represented upon the Globe, and projected upon the plane of of the Meridian.

13. A right Angled Spherical Triangle may be represented upon the Globe in this manner: Elevate one of the Poles of the Globe above the Horizon, to the quantity of one of the given Legs, so shall the distance between the Æquinox and the Zenith be equal thereunto, and at the Zenith fasten the Quadrant of altitude, so shall there be delineated upon the Globe the right Angled Spherical Triangle *A Z B* as may be seen in *Fig.* 1. In which the outward Circle *H Z R* doth represent the Brass Meridian, *A A Q* the Equator, and *Z C* the Quadrant of altitude.

14. An Oblique Angled Spherical Triangle may be represented upon the Globe in this manner. Number one of the given sides from one of the Poles to the Zenith; and there fasten the Quadrant of Altitude, upon which number another side, the third upon the great Meridian, from the Pole towards the Equinoctial, then turn the Globe till the Side numbred upon the Quadrant of Altitude, and the Side numbred upon the great Meridian shall intersect one another; so shall there be delineated upon the Globe the Oblique Angled Spherical Triangle *Z F P* in *Fig.* 3. In which *Z P* is numbred upon the Brass Meridian from the Pole of the World to *Z* the Zenith, *Z F* the Azimuth Circle represents the Quadrant of Altitude, and *P F* the great Meridian upon the Globe intersecting the Quadrant of Altitude at *F*.

15. A Right or Oblique Angled Spherical Triangle being thus delineated upon the Globe, there needs no further instructions, as to the measure of the sides, all that is wanting, is the laying down the Angles comprehended by those sides, and the finding out the measure of these Angles

being ſo laid down. And that this may be the better underſtood, I will firſt ſhew; how the ſeveral Circles upon the Globe before deſcribed, may be projected upon the Plane of the Meridian, and the ſeveral uſeful Triangles that are deſcribed by ſuch Projection with ſuch Aſtronomical Propoſitions as are conteined and reſolvable by theſe Triangles.

16. The Circles in the firſt Figure are the Meridian, Æquator, Horizon, Æquinoctial Colure, and the Tropicks. The Braſs Meridian without the Globe, is a perfect Circle deſcribed by taking 60 Degrees from your Line of Chords, as the Circle *HZRN* in *Fig.* 1. Within which all the other are projected. The Horizon, Æquator, Æquinoctial Colure, Eaſt and Weſt Azimuths are all ſtreight Lines. Thus the Diameter *HAR* repreſents the Horizon, *EAQ* the Equator, *PAS* the Equinoctial Colure and *ZAN* the Eaſt and Weſt Azimuths, in the drawing of theſe there is no difficulty, *PMS* is a Meridian, and *ZGN* an Azimuth Circle, for the drawing of which there are three points given and the Centers of the Meridians do always fall in the Equinoctial extended if need be, the Centers of the Azimuth Circles do fall in the Horizon extended if need be, and for the drawing of theſe Circles there needs no further direction, ſuppoſing the middle point given to be in the Æquator or Horizon, but yet the Centers of theſe Circles may be readily found, by the Lines of Tangents or Secants, for the Tangent of the Complement of *AT* ſet from *A* to *D*, or the Secant of the Complement ſet from *A* to *D* will give the Center of the Meridian *PTS*. The other two Circles in the 1. *Fig.*

are the Tropicks whose Centers are thus found, each Tropick is Deg. 23½ from the Equinoctial, which distance being set upon the Meridian from Æ to ♋ and Æ to ♑, if you draw a Line from A to ♋ and another perpendicular thereunto from ♋ it will cut the Axis *S AP* extended in the Center of that Tropick, by which extent of the compasses the other Tropick may be drawn also. Or thus the Co-tangent of Æ ♋ set from ♋ to the Axis extended will give the Center as before, and thus may all other Parallels be described.

17. In the second and third figures, the two extream points given in the Meridians are not equidistant from the third, for the drawing of which Circles, if the common way of bringing three points into a Circle be not liked; you may do thus, from the given point at *F* and the Center *A* draw the Diameter *T AS*, and cross the same at Right Angles with the Diameter *B AG*, a Ruler laid from *G* to *F* will cut the primitive Circle in *L*, make *EL* = *BL* a Ruler laid from *G* to *E* will cut the Diameter *S AT* in *V* the Center of the Circle *B DG*. Which Circle doth cut the Diameter *H AR* in the Pole of *Z F*, and the Diameter Æ *AQ* in *D* in the Pole of *PFX*, and a Ruler laid from *Z* to *C* will cut the Primitive Circle in ♈, and making *♈O* equal to ♈ a Ruler laid from *Z* to *O* will cut the Diameter *H AR*, extended in the Center of the Circle *Z F*.

18. Having drawn the Circle *Z FI*, in *Fig.* 13. The Circle *P EX*, or any other passing through the point *F*, may easily be described. Draw Æ *Q* at right Angles to *P X*, a Ruler laid from *G* unto (*e*) will cut the Primitive Circle in (*m*) make *mn* = *Bn*, a Ruler laid from *G* to *n*

shall cut the Diameter TFS in q make $Fq = Fq$ so shall FQ be the Radius, and the Center of the Circle PFX as was desired.

19. The preceeding directions are sufficient for the projecting of several Circles of the Globe before described upon the Plane of the Meridian, and the parts of those Circles so described may thus be measured. In *Fig.* 1. $HZ = CZ = AZ$ 90 Degrees. Whence it followeth, that the Quadrant CZ is divided into Degrees from its Pole M, by the Degrees of the Quadrant HZ, that is a Ruler laid from M to any part of the Quadrant HZ will cut as many Degrees in CZ as it doth in the Quadrant HZ, and thus the Arch $GF = HK$ the Arch $CB = HL$, and the Arch $\beta F = LK$.

20. That which is next to be considered is the projecting or laying down the Angles of a Triangle, and the measuring of them being projected, and the Angles of a Triangle are either such as are conteined between two right Lines as the Angle A in the Triangle PAR; or such as are conteined by a streight and a Circular Line, as the Angle PMR. *Fig.* 1. Or such as are conteined by two circular Lines, as the Angles FZP or ZFP in *Fig.* 3. The projecting or measuring the first sort of these Angles, needs no direction.

21. To project an Angle conteined by a streight and a circular line as the Angle $\text{Æ}BZ$ in *Fig.* 1. Do thus, lay a Ruler from N to C, and it will cut the Primitive Circle in K make $ZX = HK$, a Ruler laid from N to X will cut the Diameter HAR in the point M the Pole of the Circle ZCN, a Ruler laid from M to B the Angular point propounded, will cut the primitive Circle in I, make

NI

$NT = HL$ a Ruler laid from N to T will cut the Circle ZCN in W, a Ruler laid from B to W will cut the Primitive Circle in A, make AQ equal to the Angle propounded, and draw the Diameter BAQ, then is the Angle $\text{Æ}BZ$ or $NBQ = NQ$ as was required.

22. If the Angle had been projected and the measure required, a Ruler laid from M to B would give L and making $NT = HL$ a Ruler laid from M to T would give W, from B to W would give A, and AQ would be the measure of the Angle propounded.

23. To project an Angle conteined by two circular lines, one of them being an Arch of the Primitive Circle, as the Angle $\text{Æ}ZB$, *Fig.* 1. Do thus, set off the quantity of the Angle given from H to G, a Ruler laid from Z to G will cut the Diameter HAR in the point C, so may you draw the Circle ZCN and the Angle HZC will be equal to the Arch $HG = HC$ as was required.

24. If the Angle had been projected and the measure required, a Ruler laid from Z to C would cut the Primitive Circle in G and HG would be the measure of the Angle propounded.

25. To project an Angle conteined by two oblique Arches of a Circle, as the Angle ZFP in *Fig.* 3. You must first find the Pole of one of the two Circles conteining the Angle propounded, suppose ZBI, a Ruler laid from C the Pole thereof to F, the Angular point propounded, will cut the Primitive Circle in a make ab equal to the Angle propounded, a Ruler laid from F to b will cut the Diameter $\text{Æ}AQ$ in D the Pole of the Circle PEX, a Ruler laid from G to e will cut the Primitive Circle in m, make $mn = Bm$.

ler laid from *G* to *n* will cut the Diameter *TAS* in *p*, make *Aq* = *Ap* ſo ſhall *Fp* be the Radius and the Center of the Circle *PFX* and the Angle *ZFP* = *ab*, as was propounded.

26. If the Angle had been projected and the meaſure required; through the point *F* draw the Diameter *TFS* and the Diameter *BAG* at right Angles thereunto, a Ruler laid from *G* to *F* will cut the Primitive Circle in *K*, and making *KE* = *BK* a Line drawn from *G* to *E* will cut the Diameter *TAS* in the Center of the Circle *GDB* cutting the Diameter *HAR* in *C* the Pole of the Circle *ZBI*, and the Diameter *ÆAQ* in *D*, the Pole of the Circle *PEX* and a Ruler laid from *F* to *C* and *D* will cut the Primitive Circle in *a* and *b* the meaſure of the Angle required.

Or a Ruler laid from *F* to *K* and *M* will cut the Primitive Circle in Deg. the meaſure of the Angle propounded as before.

Or thus a Ruler laid from *C* and *D* to *F* will cut the Primitive Circle in *e* and *h* ſet 90 Degrees from *e* and *h* to *f* and *l* a Ruler laid from *C* to *f* will cut *ZBI* in *M* and a Ruler laid from *D* to *l* will cut *PEX* in *K*. This done a Ruler laid from *F* to *K* and *M* will cut the Primitive Circle in *g* and *d* the meaſure of the Angle as before.

And in *Fig.* 2. The quantity of the Angle *ZFP* may thus be found. A Ruler laid from *C* the Pole of the Circle *ZFI* to *F* the angular point will cut the Primitive Circle in *a*, ſet off a Quadrant from *a* to *b*, a Ruler laid from *C* to *b* will cut the Circle *ZFI* in the point *M*. In like manner a Ruler laid from *D* the Pole of the Circle *PEX*, will cut the Primitive Circle in *D*, ſet off a Quadrant from *A* to *b*, a Ruler laid from *D* to *P* will cut

cut the Circle *P F X* in *K*: Lastly a Ruler laid from *F* to *K*, and *M* will cut the Primitive Circle in *N S* the measure of the Angle *K F M* or *Z F P*, as was propounded.

27. Having shewed how a right or oblique Angled Spherical Triangle may be projected upon the Plane of the Meridian, as well as delineated upon the Globe, we will now consider the several Triangles usually represented upon the Globe, with the several Astronomical and Geographical Problems conteined in them, and resolved by them.

28. The Spherical Triangles usually represented upon the Globe are eight; whereof there are five Right angled Triangles, have their Denomination from their *Hypotenusas*.

The first is called the Ecliptical Triangle, whose *Hypotenusa* is an Arch of the Ecliptick, the Legs thereof are Arches of the Æquator and Meridian, this is represented upon the Globe, by the Triangle *A D F*, in *Fig.* 1. In which the five Circular parts, besides the Right Angle are;

1. The *Hypotenuse* or Arch of the Ecliptick *A F*.
2. The Leg or Arch of the Æquator, *A D*.
3. The Leg or Arch of the Meridian *D F*.
4. The Oblique Angle of the Æquator with the Ecliptick and the Suns greatest Declination *D A F*.
5. The Oblique Angle of the Ecliptick and Meridian, or the Angle of the Suns position *A F D*.

The two next I call Meridional, because the *Hypotenusas* in them both, are Arches of a Meridian. One of these is noted with the Letters

M P R

MP in *Fig.* 1, In which the five Circular parts are;

1. The *Hypotenuſa* or Arch of a Meridian *P M.*

2. The Leg or Arch of the Horizon *M R*, the Suns Azimuth North.

3. The Leg or Arch of the Braſs Meridian, repreſenting the height of the Pole *P R.*

4. The Oblique Angle of the Meridian upon the Globe; with the Braſs Meridian, or Angle of the Hour from Midnight. *P.*

5. The Oblique Angle of the Suns Meridian with the Horizon, or the Complement of the Suns Angle of Poſition *P M R.*

The other Right Angled Meridional Triangle is noted with the Letters *A E G* in *Fig.* 1. In which the 5 Circular parts are.

1. The *Hypotenuſa* or preſent Declination of the Sun, *A E.*

2. The Leg or Suns Amplitude at the hour of ſix, *A G.*

3. The other Leg or Suns height at the ſame time *E G.*

4. The Angle of the Meridian with the Horizon, or Angle of the Poles elevation, *E A G.*

5. The Angle of the Meridian with the Azimuth, or the Angle of the Suns poſition, *A E G.*

The fourth Right Angled Spherical Triangle, I call an Azimuth Triangle, becauſe the *Hypotenuſa* doth cut the Horizon in the Eaſt and Weſt Azimuths, as is repreſented by the Triangle *A D V.* in *Fig.* 1. In which the 5 Circular parts are,

1. The

1. The *Hypotenusa*, or Arch of the Suns or Stars Altitude *AV*.

2. The Leg or Declination of the Sun or Star, *DV*.

3. The other Leg, or Right Ascension of the Sun or Star, *AD*.

4. The Oblique Angle or Angle of the Poles elevation, *DAV*.

5. The other Oblique Angle or Angle of the Sun or Stars Position, *DVA*.

The fifth and last Right Angled Spherical Triangle, that I shall mention, I call an Horizontal Triangle, because the *Hypotenusa* thereof is an Arch of the Horizon, and is represented by the Triangle *AMT* in *Fig.* 1. In which the 5 Circular parts are;

1. The *Hypotenusa* and Arch of the Horizon, or Amplitude of the Sun at his rising or setting, *AM*.

2. The Leg conteining the Sun or Stars Declination *TM*.

3. The other Leg or Ascensional difference *AT*, that is, the difference between *DT* the Right Ascension and *DA* the Oblique Angle.

4. The Oblique Angle of the Horizon and Equator, or height of the Equator *TAM*.

5. The other Oblique Angle, or Angle of the Horizon and Meridian *AMT*.

The Oblique Angled Spherical Triangles usually represented upon the Globe are three. The first I call the Complemental Triangle, because the sides thereof are all Complements, and this is represented by the Triangle *FZP* in *Fig.* 1. Whose Circular parts are;

1. The Complement of the Poles elevation *ZP*.

2. The

2. The Complement of the Suns Declination, *FP*.

3. The Complement of the Suns Altitude or Almicantar *FZ*.

4. The Suns Azimuth or Diſtance from the North *FZP*.

5. The hour of the day or diſtance of the Sun from Noon *ZPF*.

6. The Angle of the Suns Poſition *ZFP*.

The ſecond Oblique Angled Spherical Triangle, I call a Geographical or Nautical Triangle, becauſe it ſerveth to reſolve thoſe Problems, which concern Geographie and Navigation, and this is alſo repreſented by the Triangle *FZP* in *Fig.* 1. Whoſe parts are.

1. The Complement of Latitude as before *ZP*.

2. The diſtance between the two places at *Z* and *F* or ſide *FZ*.

3. The Complement of the Latitude of the place at *F* or ſide *FP*.

4. The difference of Longitude between the two places at *Z* and *F* or the Angle *FBZ*.

5. The point of the compaſs leading from *Z* to *F* or Angle *FZP*.

6. The point of the Compaſs leading from *F* to *Z*, or Angle *ZFP*.

The third Oblique Angled Spherical Triangle is called a Polar Triangle, becauſe one ſide thereof is the diſtance between the Poles of the World, and the Poles of the Zodiack. This Triangle is repreſented upon the Cœleſtial Globe, by the Triangle *FSP* in *Fig.* 4. In which the Circular parts are;

1. The diſtance between the Pole of the World,

World, and the Pole of the Ecliptick, or the Arch *S P*.

2. The Complement of the Stars Declination, *F P*.

3. The Complement of the Stars North Latitude, from the Ecliptick or the Arch *F S*.

4. The Angle of the Stars Right Ascension *F P S*.

5. The Complement of the Stars Longitude *F S P*.

6. The Angle of the Stars Position *S F P*.

29. And thus at length I have performed, what was proposed in the 15 of this Chapter, that is, I have shewed how the several Circles of the Globe, may be projected upon the Plane of the Meridian, the several useful Triangles that are described by such projection, with such Astronomical Propositions as are contained and resolvable by those Triangles; And although the most accurate way of resolution is by the Doctrine of Trigonometry and the Canon of Lines and Tangents, yet it is not impertinent to do the same upon the Globe it self, which as to the sides is easie, but to measure or lay down the Angles is sometimes a little labourious.

In the Right Angled Spherical Triangle *Æ B Z* in *Fig.* 1. The measure of the Angle *Æ Z B* is reckoned in the Horizon from *H* to *C* but to lay down or measure the Angle *Æ B Z* the readiest way is to describe the Triangle again, making *Æ Z* = *Æ B* and *Æ B* = *Æ Z*, so will the Angle *Æ B Z* stand where the Angle *Æ Z B* is, and may be measured in the Horizon as the other was.

And

And so in the Oblique Angled Spherical Triangle *F Z P* in *Fig.* 1. The Angles at *Z* and *F* are easily measured or laid down upon the Globe, but to perform the same with the Angle *Z F P*, you may represent it at the Pole or Zenith and find the measure in the Equator or Horizon.

90. And now having, as I hope, sufficiently prepared the young Student for the first part of Astronomy, the Doctrine of the *Primum Mobile*, by shewing how the Heavens and the Earth are represented upon the Globe, or may be projected in Plane, I will now proceed to such Astronomical Propositions as are generally useful, and may be sufficient for an Introduction to this noble Science: to go through the several Triangles before propounded, will be very tedious, I will therefore shew the several Problems in one Right Angled and one Oblique Angled Spherical Triangle and the Canons by which they are to be resolved, and leave the rest for the Practice of my Reader. To this purpose I will next acquaint you with my Lord *Nepier's* Catholick Proposition for the solution of all Right and Oblique Angled Spherical Triangles.

CHAP.

CHAP. IV.

Of the Solution of Spherical Triangles.

IN Spherical Triangles there are 28 Varieties of Cases, 16 in Rectangular, and 12 in Oblique, whereof all the Rectangular and ten of the Oblique may be resolved by the two Axioms following.

1. *Axiom.* In all Right Angled Spherical Triangles having the same Acute Angle at the Base, the Sines of the *Hypotenusas* are proportional to the Sines of their Perpendicular.

2. *Axiom.* In all right Angled Spherical Triangles, the Sines of the Bases and the Tangents of the Perpendicular are proportional.

That all the Cases of a Right Angled Spherical Triangle may be resolved by these two Axioms, the several parts of the Spherical Triangle proposed, that so the Angles may be turned into sides, the *Hypotenusa*, into Bases and Perpendiculars and the contrary. By which means the proportions as to the parts of the Triangle given, are sometimes changed into Co-sines instead of Sines, and into Co-tangents instead of Tangents. Which the Lord *Nepier* observing; those parts of the Right Angled Spherical Triangle, which in conversion do change their proportion, he noteth by their Complements. *viz.* The *Hypotenuse* and the two Acute Angles: But the sides or Legs are not so noted,

ted, as in the Right Angled Spherical Triangle *MPR* in *Fig.* 1. And these five he calleth the Circular parts of the Triangle, amongst which the Right Angle is not reckoned.

2. Now if you reckon five Circulat parts in a Triangle, one of them must needs be in the middle, and of the other four, two are adjacent to that middle part, the other two are disjunct, and which soever of the five you call the middle part, for every one of them may by supposition be made so; those two Circular parts which are on each side of the middle are called extreams adjunct, and the other two remaining parts, are called extream disjunct, as in the Triangle *MPR* if you make the Leg *PR* the middle part, then the other Leg *MR* and the Angle Comp. *P*. Are the extreams conjunct, the Hyp. Comp. *MP* and Comp. *M*, are the extreams disjunct, and so of the rest, as in the following Table.

Mid.

Mid. part	Extr. conj.	Extr. disj.
Leg *PR*	Leg. *MR*	Comp. *M*
	Comp. *P*	Comp. *MP*
Leg *MR*	Leg. *PR*	Comp. *MP*
	Comp. *M*	Comp. *P*
Comp. *M*	Leg. *MR*	Comp. *P*
	Comp. *MP*	Leg. *PR*
Comp. *MP*	Comp. *M*	Leg. *PR*
	Comp. *P.*	Leg. *MR*
Comp. *P*	Comp. *MP*	Leg. *MR*
	Leg. *PR*	Comp. *M*

3. These things premised, the Ld. *Nepier* as a consectory from the two preceeding Axioms hath composed this Catholick and Universal Proposition.

The Rectangle made of the Sine of the middle part and Radius *is equal to the Rectangle made of the Tangents of the Extremes conjunct or the cosines of the Extremes disjunct.*

Therefore if the middle part be sought, the *Radius* must be in the first place; if either of the extremes, the other extreme must be in the first place.

Only note that if the middle part, or either of the extremes propounded be noted with its

Comp. in the circular parts of the Triangle, instead of the Sine or Tangent you must use the Cosine or Cotangent of such circular part or parts.

That these directions may be the better conceived, we have in the Table following set down the circular parts of a Triangle under their respective Titles, whether they be taken for the middle part, or for the extremes, conjunct or disjunct, and unto these parts, we have prefixed the Sine or Cosine, the Tangent or Cotangent, as it ought to be by the former Rule.

Mid. Par.	Extr. Conj.	Ext. Disj.
Sine *P R*	Tang. *M R*	Sine *M*
	Tang. *P.*	Sine *M P*
Sine *M R*	Tang. *P R*	Sine *M P*
	Cotang. *M.*	Sine *P*
Cosine *M*	Tang. *M R*	Sine *P*
	Cot. *M P*	Cos. *P R*
Cos. *M P*	Cotang. *M.*	Cos. *P R*
	Cotang. *P*	Cos. *M R*
Cosine *P*	Cot. *M P*	Cos. *M R*
	Tang. *P R*	Sine *M*

Now

Now then according to this Table and the former Rules.

1. Sine PR × Rad. = t MR × ct P.
2. Sine PR × Rad. = s M × s MP.
3. Sine MR × Rad. = t PR × ct M.
4. Sine MR × Rad. = s MP × s P.
5. Cos. M × Rad. = t MR × ct MP.
6. Cos. M × Rad. = s P × cs PR.
7. Cos. MP × Rad. = ct M × ct P.
8. Cos. MP × Rad. = cos. PR × cs MR.
9. Cos. P × Rad. = ct MP × t PR.
10. Cos. P × Rad. = cos. MR × s M.

By these 10 Rectangles may the 16 Cases of a Right angled Spherical Triangle be resolved, and some of them twice over; for although there are but 16 varieties in all Right angled Spherical Triangles, yet 30 Astronomical Problems may be resolved by one Triangle, as by the following Examples shall more clearly appear.

Of Right angled Spherical Triangles.

CASE 1.

The Legs given, to find the Angles.

IN the Right angled Spherical Triangle *MPR.* The given Legs are *MR* and *RP.* The Angles at *M* and *P* are required.

By the first of the 10 equal Rectangles s *PR* × Rad. = t *MR* × ct *P.* in which *P* is sought, therefore putting *MR* in the first place. The proportion is. t *MR.* Rad. : : s *PR.* ct *P.*

And by the third equal Rectangle. t *PR.* Rad. : : s *MR.* ct *M.*

CASE 2.

The Legs given, to find the Hypotenuse.

In the Right angled Spherical Triangle *MPR.* The given Legs are *MR* and *PR.* The Hypotenuse *MP* is required.

By the eighth of the 10 Rectangles cos. *MP* × Rad. = cos. *PR* × cos. *MR* in which *MP* the middle part is sought, therefore *Radius* must be put in the first place, and then the proportion is.

Rad. cos. *PR* : : cos. *MR.* cos. *MP.*

CASE 3.

CASE 3.

A Leg with an Angle opposite thereunto being given, to find the other Leg.

In the Right angled Spherical Triangle *MPR*, let there be given. The Leg *MR*. The Angle *P*. The Leg *PR* inquired.

By the first of the 10 Rectangles. Rad. t *MR* :: cot *P*. Sine *PR*. or The Leg *PR* and the Angle *M* given, to find *MR*.

By the 3 of the 10 Rectangles. Rad. t *PR* :: ct *M*. Sine *MR*.

CASE 4.

A Leg with an Angle conterminate therewith being given, to find the other Leg.

In the Right angled Spherical Triangle, *MPR*, The given Leg is *MR*, with the Angle *M*. The Leg *PR* is required.

By the 3 Rectangle. cot. *M*. Rad. :: Sine *MR*. t *PR*.

The given Leg *RP*, and Angle *P*. The Leg *MR* is required.

By the 1. Rectangle. ct *P*. Rad. :: sine *RP*. tang. *MR*.

CASE 5.

A Leg and an Angle conterminate therewith being given, to find the Hypotenuse.

In the Right angled Spherical Triangle *MPR*, let there be given,

The Leg { *MR* and the Angle *M* / *PR* and the Angle *P* } to find *MP*.

By the 5. Rectangle, t *MR*. Rad : : cos. *M*. ct *MP*,
By the 9. Rectangle. t *PR*. Rad. : : cos. *P*. ct *MP*.

CASE 6.

The Hypotenuse and a Leg given, to find the contained Angle.

In the Right angled Spherical Triangle *MPR*, let there be given,

The Hypotenuse *MP*, and Leg { *MR*. / *PR*. } To find { *M*. / *P*. }

By the 5. Rectangle, Rad. ct *MP* : : t *MR*. cos. *M*.
By the 9. Rectangle, Rad. ct *MR* : : t *PR*. cos. *P*.

CASE 7.

The Hypotenuse and one Angle given, to find the other Angle.

In the Right angled Spherical Triangle *MPR*, let there be given, The

The Hypotenuse MP } & Angle { M / P } To find the Angle { P. / M.

By the 7. Rectangle, cot. M. Rad :: cos. MP. cot. P.
By the 7. Rectangle cot. P. Rad :: cos. MP. cot. M.

CASE 8.

The Oblique Angles given, to find the Hypotenuse.

In the Right angled Spherical Triangle MPR, let there be given The Angles at P and M, To find the Hypotenuse PM.

By the 7. Rectangle. Rad. ct P :: cot. M. cos. MP.

CASE 9.

The Hypotenuse and an Angle given, to find the Leg conterminate with the given Angle.

In the Right angled Spherical Triangle MPR, let there be given,

The Hypotenuse PM } Angle { P. / M. } To find { PR. / MR.

By the 9. Rectangle, ct PM. Rad :: cos. P. t PR.
By the 5. Rectangle, ct PM. Rad :: cos. M. t MR.

CASE 10.

The Hypotenuse and an Angle given, to find the Leg opposite to the given Angle.

In the Right angled Spherical Triangle *MPR*, let there be given,

The Hypotenuse *PM* and the Angle { *M.* / *P.* } To find { *PR.* / *MR.* }

By the 2. Rectangle, Rad. s *MP* :: s *M.* Sine *PR.*
By the 4. Rectangle, Rad. s *MP* :: s *P.* Sine *MR.*

CASE 11.

A Leg and an Angle opposite thereunto being given, to find the Hypotenuse.

In the Right angled Spherical Triangle *MPR*, let there be given,

The Leg { *PR.* / *MR.* } and the Angle { *M* / *P* } to find the Hypotenuse *PM.*

By the 2. Rectangle, s *M.* Rad :: s *PR.* s *MP.*
By the 4. Rectangle, s *P.* Rad :: s *MR.* s *PM.*

CASE 12.

The Hypotenuse and a Leg given, to find the Angle opposite to the given Leg.

In the Right angled Spherical Triangle *PMR*, the

the Hypotenuse *MP* and the Leg *MR* are given, the Angle at *P* is required.

By the fourth Rectangle Sine *MP* to, Rad :: s *MR*. s *P*.

The Hypotenuse *MP* and Leg *PR* given, the Angle *M* is required.

By the second Rectangle. s *MP*. Rad :: s *PR*. s *M*.

CASE 13.

The Angle and Leg conterminate with it being given, to find the other Angle.

In the Right angled Spherical Triangle *PMR*, let there be given,

The Angle { *M* } and the Leg { *MR* } to find the Angle { *P* }
{ *P* } { *PR* } { *M* }

By the tenth Rectangle, Rad. cs *MR* :: s *M*. cs *P*.
By the sixth Rectangle, Rad. s *P* :: cs *PR*. cs *M*.

CASE 14.

An Angle and a Leg opposite thereunto being given, to find the other Angle.

In the Right angled Spherical Triangle *MPR*, let there be given,

The Angle { *P* } and the Leg { *MR* } to find the Angle { *M* }
{ *M* } { *PR* } { *P* }

By the 10. Rectangle, cs *MR*. Rad :: cs *P*. cs *M*.
By the 6. Rectangle, cs *PR*. Rad :: cs *M*. s *P*.

CASE 15.

CASE 15.

The Oblique Angles given, to find a Leg.

In the Right angled Spherical Triangle *MPR*, let there be given, the Angles at *M* and *P*, to find the Legs *MR* and *PR*.

By the 10. Rectangle, s *M*. Rad :: cs *P*. cs *MR*.
By the 6. Rectangle, s *P*. Rad :: cs *M*. cs *PR*.

CASE 16.

The Hypotenuse and one Leg given, to find the other Leg.

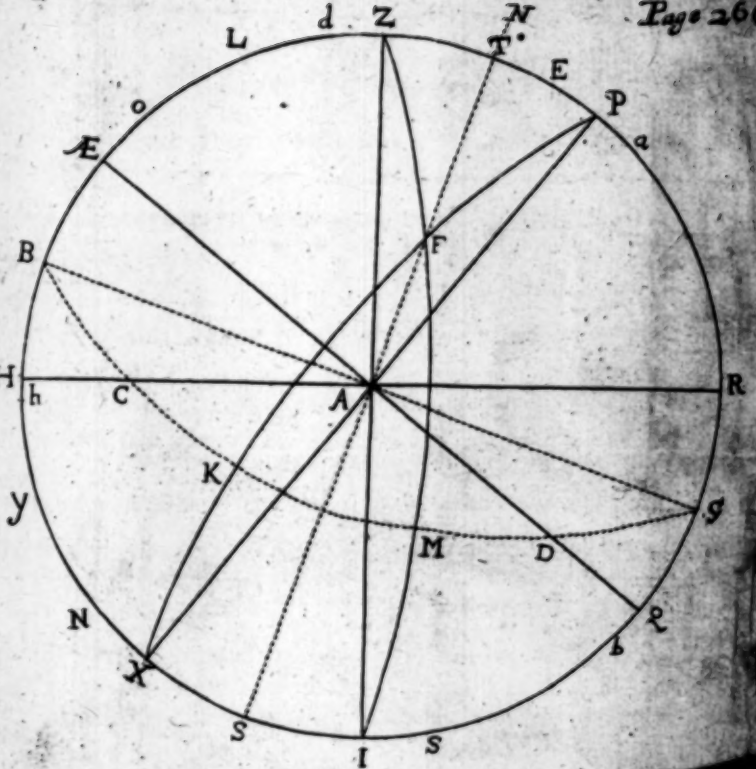
In the Right angled Spherical Triangle *MPR*, let there be given,

The Hypotenuse $\left\{\begin{matrix} PR \\ MP \end{matrix}\right.$ and the Leg $\left\{\begin{matrix} MR \\ PR \end{matrix}\right.$ to find the Leg $\left\{\begin{matrix} MR, \\ PR. \end{matrix}\right.$

By the 8. Rectangle, $\left\{\begin{matrix} \text{cs } PR. \text{ Rad} :: \text{cs } MP. \text{ cs } MR. \\ \text{cs } MR. \text{ Rad} :: \text{cs } MP. \text{ cs } PR. \end{matrix}\right.$

Thus I have given you the Proportions by which the 16 Cases of a Right angled Spherical Triangle may be resolved, In which there are contained 30 Astronomical Problems. Two in every Case except the Second and the Eighth. In both which Cases there are but two Problems. And thus I have done with Right angled Spherical Triangles.

4. In Oblique angled Spherical Triangles there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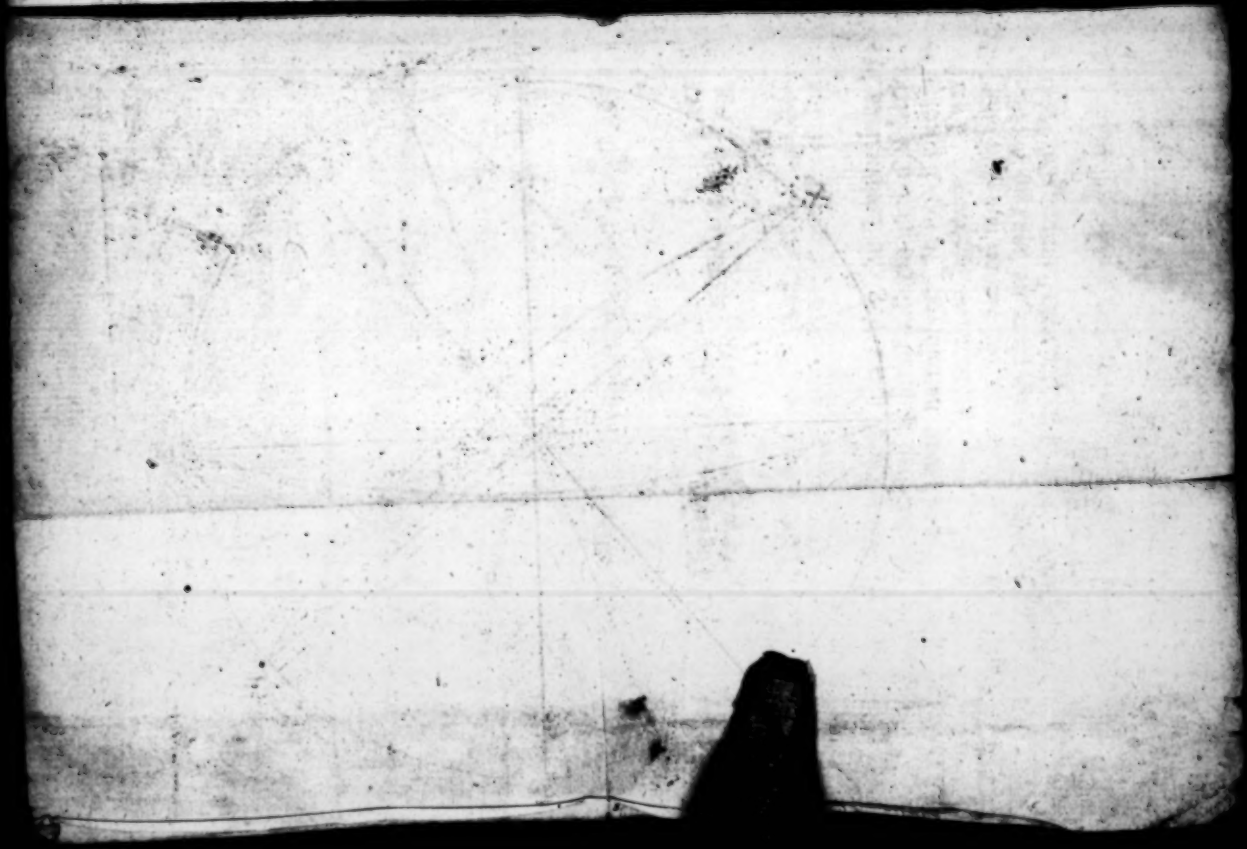

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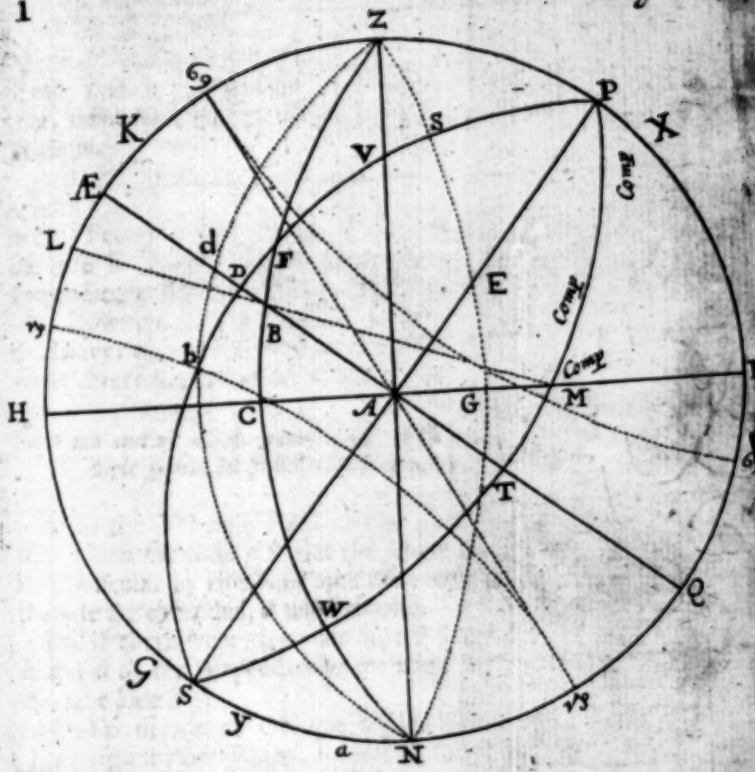


there are twelve Cases, ten whereof may be resolved by the Catholick Proposition; If the Spherical Triangle propounded be first converted into two right, by letting fall of a Perpendicular, sometimes within, sometimes without the Triangle.

5. If the Angles at the Base be both acute or both obtuse, the Perpendicular shall fall within the Triangle; but if one of the Angles of the Base be acute and the other obtuse, the Perpendicular shall fall without the Triangle.

6. However the Perpendicular falleth, it must be always opposite to a known Angle, for your better direction, take this General Rule.

From the end of a Side given, being adjacent to an Angle given, let fall the Perpendicular.

As in the Triangle *FPS* in *Fig.* 4. If there were given the Side *FS* and the Angle at *S*, the Perpendicular by this Rule must fall from *F* upon the Side *SP* extended, if need require.

But if there were given the Side *PS* and the Angle at *S*, the Perpendicular must fall from *P* upon the Side *FS*.

7. To divide an Oblique angled Spherical Triangle into two Right, by letting fall a Perpendicular upon the Globe it self, is not necessary, because all the Cases may be resolved without it, but in projection it is convenient to inform the fancy: and seeing the reason by which it is done in projection doth depend upon the nature of the Globe, I will here shew it both ways, first upon the Globe, and then by projection.

8. An

An Oblique angled Spherical Triangle may be divided into two Right, by letting fall a Perpendicular upon the Globe it ſelf, in this manner. In the Oblique angled Spherical Triangle *FPS* in *Fig.* 4. let it be required to let fall a Perpendicular from *P* upon the Side *FS*. Suppoſe the Point *P* to ſtand in the Zenith, where the Arch *FS* ſhall cut the Zodiack, which in this Figure is at *K*, make a mark, and from this Point of Interſection of the Circle upon which the Perpendicular is to fall with the Zodiack, reckon 90 Degrees, which ſuppoſe to be at *P*; a thin Plate of Braſs with a Nut at one end thereof, whereby to faſten it to the Meridian, as you do the Quadrant of Altitude, being graduated as that is, but of a larger extent (for that a Quadrant in this caſe will not ſuffice) being faſtned at *P* and turned about till it cut the Point *L* in the Zodiack, will deſcribe upon the Globe the Arch of a great Circle *PEL*, interſecting the Side *FS* at Right Angles in the Point *E*, becauſe the Point *L* in the Zodiack is the Pole of the Circle *SFK*, now all great Circles which paſſing through the Point *L*, ſhall interſect the Circle *SKG*, ſhall interſect it at Right Angles; by the 13. of the 2. Chapter.

9. And hence to divide an Oblique angled Spherical Triangle into two Right by projection is eaſie, as in the Triangle *FPS*, the Pole of the Circle *SFK* is *L*, therefore the Circle *BLP* ſhall cut the Arch *FS* at Right Angles in the Point *E*. And becauſe the Point *M* is the Pole of the Circle *BFP*, therefore the Circle *GMS* ſhall cut the Circle *BPP* at Right Angles in the Point *D*, the Side *FP* being extend-

ed;

ed. Come we now to the several Cases which after this preparation may be resolved, by the Catholick Proposition.

CASE I.

Two Sides with an Angle opposite to one of them being given, to find the Angle opposite to the other.

In the Oblique angled Spherical Triangle *FPS*, in *Fig.* 4. the Sides and Angles given and required will admit of six Varieties; all which may be resolved by the Catholick Proposition, at two operations, but those two may be reduced to one, as by the following Analogies to every Variety will plainly appear.

Given	Required	
FP		Rad. *s PS* :: *s PSF*. s PE
1. *PS*	*PFS*	*s PF*. Rad :: s PE. *s PFS*
PSF		*s PF*. *s PS* :: *s PSF*. *s PFS*
FP		Rad. *s FP* :: *s F*. s PE
2. *PS*	*PSF*	*s PS*. Rad :: s PE. *s PSF*
PFS		*s*. *PS*. *s FP* :: *s PFS*. *s PSF*
PS		Rad. *s SF* :: *s F*. s DS
3. *FS*	*FPS*	*s PS*. Rad :: s DS. *s SPD*
PFS		*s*. *PS*. *s SF* :: *s PFS*. *s PSF*
PS		Rad. *s PS* :: *s SPD*. s DS
4. *FS*	*PFS*	*s FS*. Rad :: s DS. *s SF*
FPS		*s*. *FS*. *s PS* :: *s SPF*. *s SF*
FS		Rad. *s FS* :: *s S*. s FC
5. *FP*	*FPS*	*s FP*. Rad :: s FC. *s FFC*
FSP		*s*. *FP*. *s FS* :: *s PSF*. *s FPS*

FS 6. FP FSP FPS	Rad. s FP :: s FPC. s FC s FS. Rad :: s FC. s S s. FS. s FP :: FPS. s PSF.

CASE 2.

Two Sides with an Angle oppoſite to one of them being given, to find the contained Angle.

In this Caſe there are ſix Varieties, all which may be reſolved by the Catholick Propoſition, according to the Table following.

Given Required	
FP 1. PS FPS PSF EPS + FPE = FPS	1. cot PSF. Rad :: cs PS. ct EPS 2. ct PS. Rad :: cs EPS. t EP 3. Rad. t EP :: ct FP. cs FPE ct PS. cs EPS :: ct FP. ct FPE
FP 2. PS FPS PFS EPS + EPF = FPS	1. cot PFS. Rad :: cs PF. ct EPF 2. ct PF. Rad :: cs EPF. t EP 3. Rad. t EP :: cot PS. cs EPS cot PF. cs EPF :: ct PS ct EPS
PS 3. FS PSF PFS FSD − PSD = PSF	1. cot PFS. Rad :: cs FS. ct FSD 2. ct FS. cs FSD :: Rad. t DS 3. Rad. t DS :: ct PS. cs PSD ct FS. cs FSD :: ct PS. cs PSD
PS 4. FS PSF FPS FSD − PSD = PSF	1. cot FPS. Rad :: cs PS. ct PSD 2. ct PS. cs PSD :: Rad. t DS 3. Rad. t DS :: ct FS. cs FSD ct PS. cs PSD :: ct FS. cs FSD
FS 5. FP PFS FSP SFC − PFC = PFS	1. cot FSP. Rad :: cs FS. ct SFC 2. ct FS. cs SFC :: Rad. t FC 3. Rad. t FC :: ct FP. cs PFC ct FS. cs SFC :: ct FP. cs PFC

FS

FS	1. *cot FPS. Rad :: cs FP. ct PFC*
6. *FF* *PFS*	2. *cot FP. cs PFC :: Rad. t FC*
FPS	3. Rad. t FC :: *ct FS. cs SFC*
SFC—PFC = PFS	*ct FP. cs PFC :: ct FS. cs SFC.*

CASE 3.

Two Sides and an Angle oppoſite to one of them being given, to find the third ſide.

The Varieties in this Caſe, with their reſolution by the Catholick Propoſition, are as followeth.

Given Required	
FP	1. *ct PS :: cs PSF. t ES*
1. *PS* *FS*	2. *cs ES. cs PS :: Rad. cs EP*
PSF	3. Rad. cs EP :: *cs FP. cs FE*
ES + FE = FS	*cs ES. cs PS :: cs FP. cs FE*
FP	1. *cot FP. Rad :: cos PFS. t FE*
2. *PS* *FS*	2. *cos FE. cos FP :: Rad. cos EP*
PFS	3. Rad. cos EP :: *cos PS. cos SE*
SE + FE = FS	*cos FE. cos FP :: cos PS. cos SE*
PS	1. *cot FS. Rad :: cos PFS. t FD*
3. *FS* *FP*	2. *cos FD. cos FS :: Rad. cs SD*
PFS	3. Rad. cos SD :: *cos PS. cs PD*
FD—PD = FP	*cos FD. cos FS :: cs PS. cs PD*
PS	1. *cot PS. Rad :: cos FPS. t PD*
4. *FS* *FP*	2. *cos PD. cos PS :: Rad. cos SD*
FPS	3. *Rad. cos SD :: cos FS. cs FD*
FD—PD = FP	*cos PD. cos PS :: cos FS. cs FD*

FS 5. *FP* *PS* *FSP* *SC—PC=PS*	1. *cot FS. Rad :: cos FSP. t SC* 2. *cos SC. cos FS* :: Rad. cos FC 3. Rad. cos FC :: *cos FP. cos PC* *cos SC. cos FS. cos FP. cos PC*
FS 6. *FP* *PS* *FPS* *SC—PC=PS*	1. *cot FP. Rad* :: *cos FPS. t PC* 2. *cos PC. cos FP* :: Rad. cos FC 3. Rad. cos FC :: *cos FS. cos SC* *cos PC. cos FP* :: *cos FS. cos SC*

CASE 4.

Two Angles with a Side oppoſite to one of them being given, to find the Side oppoſite unto the other.

The Varieties in this Caſe, with their Reſolution by the Catholick Propoſition, are as follow-eth.

Given	Required	
PFS 1. *FPS* *PS*	*FS*	Rad. *s PS* :: *s DPS*. s SD s. FP. Rad :: s SD. *s FS* *s. PFS. s PS* :: *s FPS. s FS*
PFS 2. *FPS* *FS*	*PS*	Rad. *s FS* :: *s PFS*. s SD *s. FPS*. Rad :: s SD. *s PS* *s. FPS. s FS* :: *s PFS. s PS*
FPS 3. *PSF* *FP*	*FS*	Rad. *s FP* :: *s FPS*. s FC *s. PSF*. Rad :: s FC. *s FS* *s. PSF. s FP* :: *s FPS. s FS*

FPS

	FPS	Rad. *s FS* :: *s PSF*. s FC
4.	*PSF* :: *FP*	*s FPS*. Rad :: s FC. *s FP*
	FS	*s FPS s FS* :: *s PSF*. *s FP*
	PSF	Rad. *s PS* :: *s PSF*. s PE
5.	*SFP* *FP*	*s SFP*. Rad :: s PE. *s FP*
	PS	*s SFP*. *s PS* :: *s PSF*. *s FP*
	PSF	Rad. *s FP* :: *s PFS*. s PE
6.	*SFP* *PS*	*s PSF*. Rad :: s PE. *s PS*
	FP	*s PSF*. *s FP* :: *s PFS*. *s PS*

CASE 5.

Two Angles and a side opposite to one of them being given, to find the Side between them.

The Varieties and Proportions, are as followeth.

Given Required	
PFS	1. *ct PS*. *Rad* :: *cs DPS*. *PD*
1. *FPS* *FP*	2. *ct DPS*. *s PD* :: Rad. t DS
PS	3. Rad. t DS :: *ct PFS*. *s FD*
FD—PD = FP	*ct DPS*. *s PD* :: *ct PFS*. *s FD*
PFS	1. *ct DFS*. *Rad* :: *cs PFS*. *t FD*
2. *FPS* *FP*	2. *cot PFS*. *s FD* :: *Rad*. *t DS*
FS	3. *Rad*. *t DS* :: *ct FPS*. *s PD*
FD—PD = FP	
FPS	1. *cot FP*. *Rad* :: *cs FPC*. *t PC*
3. *PSF* *PS*	2. *cot FPC*. *s PC* :: Rad. t FC
FP	3. Rad. t FC :: *ct PSF*. *s SC*
SC—PC = PS	*cot FPC*. *s PC* :: *ct PSF*. *CS*

FPS	1. *cot FS. Rad* :: *cs PSF. t SC*
4. *PSF* *PS*	2. *cot PSF. s SC* :: Rad. t FC
FS	3. Rad. t FC :: *cot FPS. s PC*
SC — *PC* = *PS*	*cot PSF. s SC* :: *cot FPS. s PC*
PSF	1. *cot PS. Rad* :: *cs PSF. s SE*
5. *SFP* *FS*	2. *cot PSF. s SE* :: Rad. t PE
PS	3. Rad. t PE :: *cot SFP. s FE*
FE + *SE* = *FS*	*cot PSF. s SE* :: *cot SFP. s FE*
PSF	1. *cot FP. Rad* :: *cs SFP. t FE*
6. *SFP* *FS*	2. *cot SFP. s FE* :: Rad. t PE
FP	3. Rad. t PE :: *cos PSF. s SE*
FE + *SE* = *FS*	*cot. SFP. s FE* :: *cs PSF. s SE*

CASE 6.

Two Angles and a Side oppoſite to one of them being given, to find the third Angle.

The Varieties and Proportions are as follow-eth.

Given	Required	
PFS		1. *ct DPS. Rad* :: *cs PS. ct PSD*
1. *FPS*	*PSF*	2. *s PSD. cs DPS* :: Rad. cs DS
PS		3. cs DS. Rad :: *cs DFS. s FSD*
FSD — *PSD* = *PSF*		*cs DPS. s PSD* :: *cs DFS. s FSD*
PFS		1. *ct PFS. Rad* :: *cs FS. ct FSD*
2. *FPS*	*PSF*	2. *s FSD. cs PFS* :: Rad. cs DS
FS		3. cs PDS. Rad :: *cs DPS. cs PSD*
FSD — *PSD* = *PSF*		*cs PFS. s FSD* :: *cs DPS. cs PSD*

FPS

FPS	1. ct FPC. Rad :: cs FP. ct PFC
3. PSF PFS	2. s PFC. cs FPC :: Rad. cs FC
FP	3. cs FC. Rad :: cs PSF. s FC
SFC—PFC = PFS	cs FPC. s PFC :: cs PSF. s SFC
FPS	1. cot PSF. Rad :: cos FS. ct SFC
4. PSF PFS	2. s SFC. cs PSF :: Rad. cs FC
FS	3. cs FC. Rad :: cs CPF. s PFC
SFC—PFC = PFS	cs PSF. s SFC :: cs CPF. s PFC
PSF	1. cot PSF. Rad :: cs PS. ct SPE
5. SFP FPS	2. s SPE. cs PSF :: Rad. cs PE
PS	3. cs PE. Rad :: cs SFP. s FPE
FPE + SPE = FPS	cs PSF. s SPE :: cs SFP. s FPE
PSF	1. cot SFP. Rad :: cs FP. ct FPE
6. SFP FPS	2. s FPE. cs SFP :: Rad. cs PE
FP	3. cos PE. Rad :: cs PSF. s SPE
FPE + SPE = FPS	cs SFP. s FPE :: cs PSF. s SPE

CASE 7.

Two Sides and their contained Angle being given, to find either of the other Angles.

The Varieties and Proportions are as follow-eth.

Given Required	
FS	1. ct FP. Rad :: cs PFS. t FE
1. FP FSP	2. ct PFS. s FE :: Rad. t PE
PFS	3. t PE. Rad :: s ES. ct PSF
FS—FE = ES	s EF. ct PFS :: s ES. ct PSF

FS 2. FP FPS PFS FD—FP = PD	1. cot FS. Rad :: cs PFS. t DF 2. cot PFS. s DF :: Rad. t DS 3. t DS. Rad :: s PD. ct SPD s DF. ct PFS :: s PD. ct SPD
FP 3. PS PSF FPS PS + PC = CS	1. cot FP. Rad :: cos FPC. t PC 2. cot FPC. s PC :: Rad. t FC 3. t FC. Rad :: s CS. cot FSP s PC. ct FPC :: s CS. ct FSP
FP 4. PS SFP FPS FP + PD = FD	1. cot PS. Rad :: cos SPD. s PD 2. cot SPD. s PD :: Rad. t DS 3. t DS Rad :: s FD. cot SFP s PD. ct SPD :: s FD. cot SFP
PS 5. FS SFP PSF FS—SE = FE	1. cot PS. Rad :: cs PSF. t SE 2. cot PSF. s SE :: Rad. t PE 3. t PE. Rad :: s FE. cot SFP s SE. ct PSF :: s FE. ct SFP
PS 6. FS FPS PSF SC—PS = PC	1. cot FS. Rad :: cs PSF. t SC 2. cot PSF. s SC :: Rad. t FC 3. t FC. Rad :: s PC. cot FPC s SC. cot PSF :: s PC. ct FPC

CASE 8.

Two Sides and their contained Angle being given, to find the third Side.

The Varieties and Proportions are as follow-eth.

Given

Given Required	
FS	1. *ct FP. Rad :: cs PFS. t FE*
1. *FP* *PS*	2. *ct FE. cs FP :: Rad. cos PE*
PFS	3. Rad. cs PE :: *cs ES. cs PS*
FS—FE = ES	*cs FE. cs FP :: cs ES. cs PS*
FP	1. *ct PS. Rad :: cs SPD. t PD*
2. *SP* *FS*	2. *ct PD. cs PS :: Rad. cos DS*
FPS	3. Rad. cos DS :: *cs FD. cs FS*
FP + PD = FD	*cs PD. cs PS :: cs FD. cs FS*
PS	1. *ct PS. Rad :: cs PSF. t. ES*
3. *PS* *FP*	2. *cs ES. cs PS :: Rad. cos PE*
PSF	3. Rad. cos PE :: *cos FE. cos FP*
FS—ES = FE	*cs ES. cs PS :: cos FE. cs FP*

CASE 9.

Two Angles and their contained Side being given, to find one of the other Sides.

Given Required	
PFS	1. *ct PFS. Rad :: cs FP ct FPE*
1. *FPS* *PS*	2. *ct FP. cs FPE :: Rad. t PE*
FP	3. t PE. Rad :: *cs EPS. ct PS*
FPS—FPE = EPS	*cs FPE. ct FP :: cs EPS. ct PS*
PFS	1. *cot FPC. Rad :: cs FP. t PFC*
2. *FPS* *FS*	2. *cot FP. cs PFC :: Rad. t FC*
FP	3. t FC. Rad :: *cs SFC. ct SF*
SFP + PFC = SFC	*ct FP. cs PFC :: cs SFC. ct SF*

FPS 3. PSF SF PS PSF+PSD=FSD	1. ct SPD. Rad :: cs PS. ct PSD 2. ct PS. cos PSD :: Rad. t DS 3. t DS. Rad :: cs FSD. ct SF cs PSD. ct PS :: cs FSD. ct SF
FPS 4. PSF FP PS FPS—EPS=FPE	1. ct PSF. Rad :: cs PS. ct SPE 2. ct PS. cs SPE :: Rad. t PE 3. t PE. Rad :: cs PPE. ct FP cs SPE. ct PS :: cs FPE. ct FP
PSF 5. SFP FP SF SFC—SFP=CFP	1. ct PSF. Rad :: cs SF. ct SFC 2. ct SF. cs SFC :: Rad. t FC 3. t FC. Rad :: cs CFP. ct FP cs SFC. ct SF :: cs CFP. ct FP
PSF 6. SFP PS SF FSD—FSP=PSD	1. ct SFP. Rad :: cs FS. ct FSD 2. ct FS. cs FSD :: Rad. t SD 3. t SD. Rad :: cos PSD. ct PS cs FSD. ct FS :: cs PSD. ct PS

CASE 10.

Two Angles and the Side between them being given, to find the third Angle.

The Varieties and Proportions are as followeth.

Given Required	
SFP 1. FPS PSF FP FPS—FPE=EPS	1. ct SFP. Rad :: cs FP. ct FPE 2. s FPE. cs F :: Rad. cs PE 3. Rad. cs PE :: s EPS. cs PSF s FPE. cs PFS :: s SPE. cs PSF

FPS

FPS	1. *ct SPD. Rad :: cs PS. ct PSD*
2. *PSF* *SFP*	2. *s PSD. cs SPD :: Rad. cs DS*
PS	3. *Rad. cs DS :: s FSD. cs SFP*
PSF + PSD = FSD	*s PSD. cs SPD :: s FSD. cs SFP*

PSF	1. *ct PSF. Rad :: cs SF. ct SFC*
3. *SFP.* *FPS*	2. *s SFC. cs PSF :: Rad. cs FC*
SF	3. *Rad. cs FC :: s PFC. ct FPS*
SFC — SFP = PFC	*s SFC. cs PSF :: s PFC. cs FPS*

CASE 11.

The three Sides being given, to find an Angle.

This Case may be resolved by the Catholick Proposition also, according to the direction of the Lord *Nepier*, as I have shewed at large in the Second Book of my *Trigonometria Britannica*, Chap. 2. but may as I conceive be more conveniently solved, by this Proposition following.

As the Rectangle of the Square of the Sides containing the Angle inquired;

Is to the Square of *Radius*: So is the Rectangle of the Square of the difference of each containing Side, and the half sum of the three Sides given.

To the Square of the Sine of half the Angle inquired.

In this Case there are three Varieties, as in the Triangle *F Z P. Fig.* 3.

Given	Required	
ZP 1. PF FZ	ZPF	s ZP × s PF. Rad. q. s ½ Z — ZP × s ½ Z — PF. Q s ½ ZPF
ZP 2. PF FZ	PFZ	s PF × s FZ. Rad. q. s ½ Z — PF × s ½ Z — FZ. Q s ½ PFZ
ZP 3. PF FZ	FZP	s ZP × s FZ Rad. q. s. ½ Z — ZP × ½ Z — ZF. Q s ½ FZP

CASE 12.

The three Angles given, to find a Side.

This is the Converſe of the laſt, and to be reſolved after the ſame manner, if ſo be we convert the Angles into Sides, by the tenth of the third Chapter: for ſo the Sides of the Triangle *ACD* will be equal to the Angles of the Triangle *FZP* in *Fig.* 3.

That is
- *AD* = *ÆE* the meaſure of the Angle *ZPF*.
- *DC* = *KM* the meaſure of the Angle *ZFP*.
- *AC* = *HB* the Complement of *FZP* to a Semicircle.

The

$$\text{The Angle}\begin{cases} DAC = QR = ZP. \\ ACD = rM = Hf = Z\,e = ZF. \\ ADM = sK = Æl = P\,b = PF. \end{cases}$$

And thus the Sides of the Triangle *ZPF* are equal to the Angles of the Triangle *ACD*. The Complement of the greateſt Side *PF* to a Semicircle being taken for the greateſt Angle *ADC*.

And in this Caſe therefore, as in the preceding, there are three Varieties which make up ſixty Problems in every Oblique angled Spherical Triangle; which actually to reſolve in ſo many Triangles, as have been mentioned, would be both tedious, and to little purpoſe; I will therefore ſelect ſome few, that are of moſt general uſe in the Doctrine of the Sphere, and leave the reſt to thine own practice.

CHAP. V.

CHAP. V.

Of ſuch Spherical Problems as are of moſt General Uſe in the Doctrine of the Primum Mobile *or Diurnal Motion of the Sun and Stars.*

PROBLEM I.

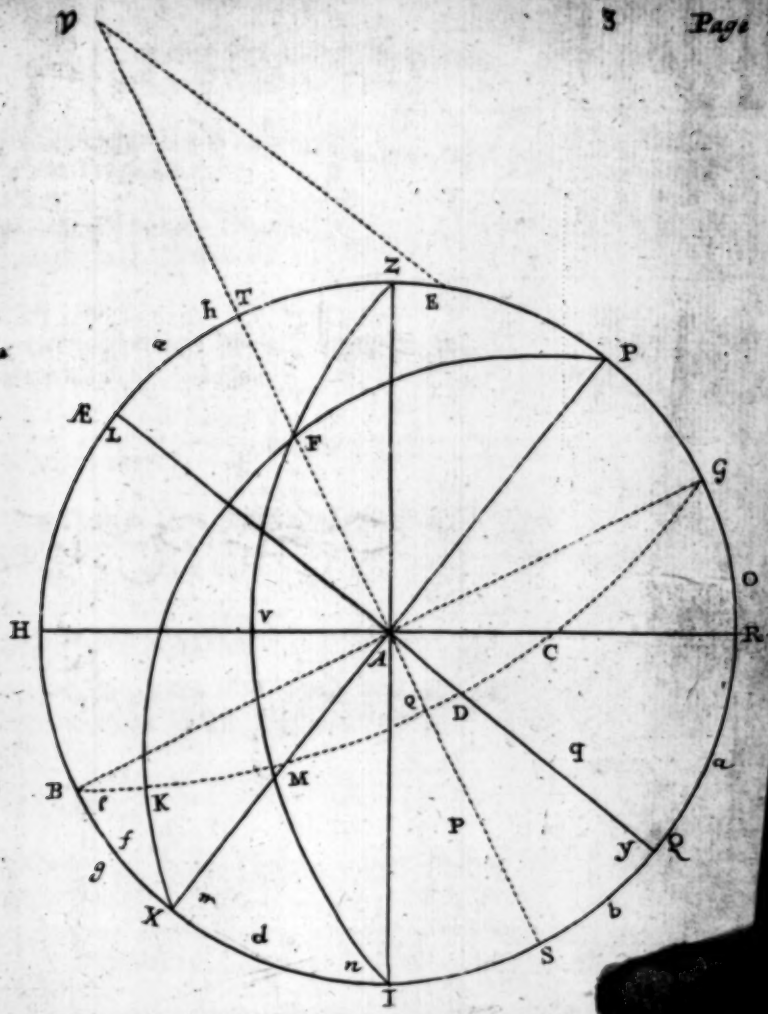
The greateſt Declination of the Sun being given, to find the Declination of any Point of the Ecliptick.

THE Declination of the Sun or other Star, is his or their diſtance from the *Equator*, and as they decline from thence either Northward or Southward; ſo is their Declination reckoned North or South.

2. The Sun's greateſt Declination, which in this and many other Problems is ſuppoſed to be given, with the Diſtance of the Tropicks, Elevation of the *Equator*, and Latitude of the Place, may thus be found.

Take with a Quadrant, the Sun's greateſt and leaſt Meridian Altitudes, on the longeſt and ſhorteſt days of the year, which ſuppoſe at *London* to be as followeth.

The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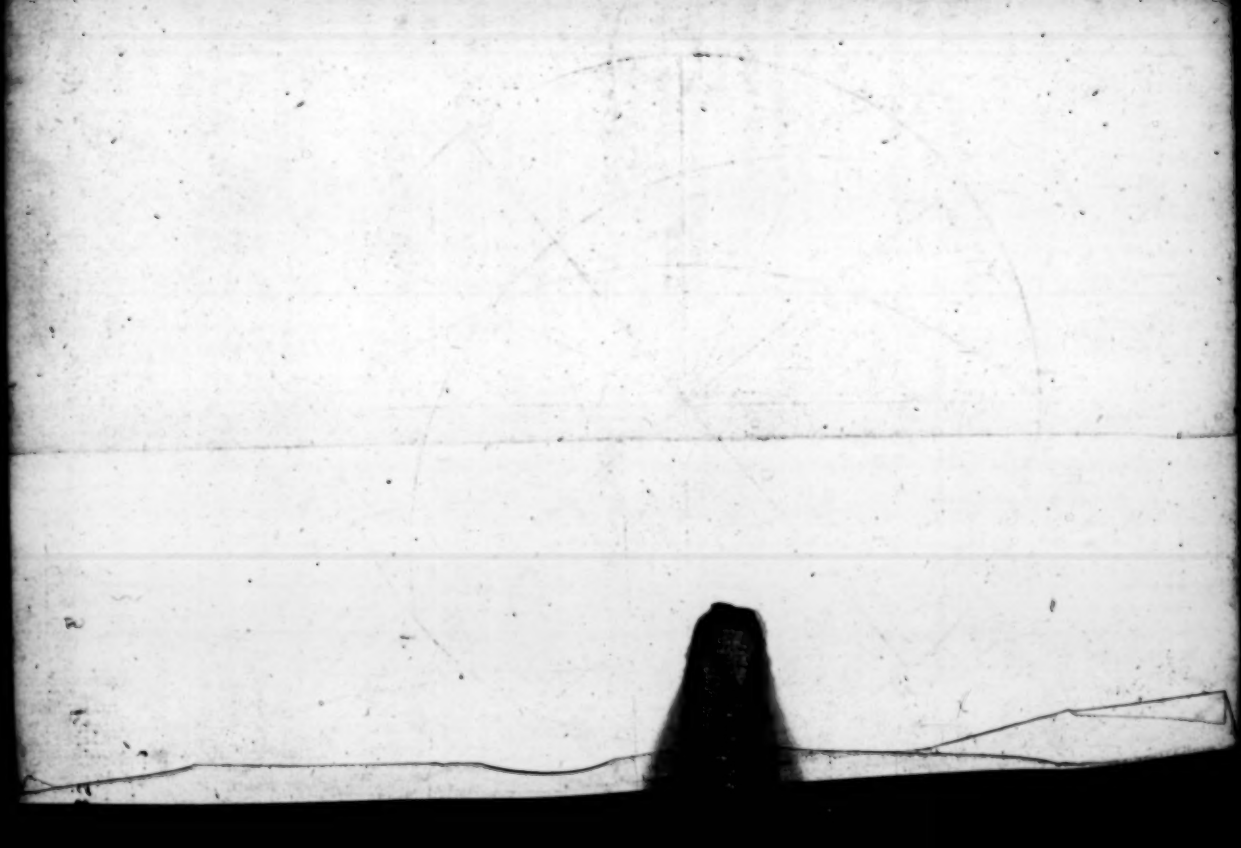


The Sun's { greatest / least } Meridian Altitude { Hs. 61.9916 / Hv. 14.9416 }

Their difference is the distance of the Tropicks { s. v. 47. 050 }

Half that Difference, is the Sun's greatest Declination { Æs. 23. 525 }

Which deduct from the Sun's greatest Altitude, the remainer is the height of the *Equator* { Hæ. 38.467 }

The Complement is the height of the Pole { ÆZ or PR. 51. 533 }

Now then in the Right angled Spherical Triangle *ADF* in *Fig.* 1. there being given.

1. The Angle of the Sun's greatest Declination *DAF*. 23. 525.

2. The Sun's supposed distance from ♈ to ♎ *AF*. 60 deg.

The Sun's present Declination *DF* may be found, by the 10 Case of Right angled Spherical Triangles.

As the *Radius*

Is to the Sine of *DAF*. 23. 525.	9. 60113517
So is the Sine of *AF* 60.	9. 93753063
To the Sine of *DF*. 20. 22.	9. 53866580

PROBLEM 2.

PROBLEM 2.

The Sun's greateſt Declination, with his Diſtance from the next Æquinoctial Point being given, to find his Right Aſcenſion.

In the Right angled Spherical Triangle *ADF* in *Fig.* 1. Having the Angle of the Sun's greateſt Declination *DAF.* 23. 525. And his ſuppoſed diſtance from ♈ or ♎, the *Hypotenuſa AF.* 60. The Right Aſcenſion of the Sun, or Arch of the *Æquator*, *AD* may be found, by the ninth Caſe of Right angled Spherical Triangles,

As the Cotang. of the Hypot. *AF.* 60.	9. 76143937
Is to the *Radius*	10. 00000000
So is the Coſine of *DAF.* 23. 525.	9 96231533
To the Tang. of *AD.* 57. 80.	10. 20087596

PROBLEM 3.

To find the Declination of a Planet or Fixed Star with Latitude.

In the Oblique angled Spherical Triangle *FPS* in *Fig.* 4. we have given, 1. *PS* = ♑♋ the greateſt Declination of the Ecliptick, 2. The Side *FS* the Complement of the Stars Latitude from the Ecliptick at *K.* 3. The Angle *PSF* the Complement of the Stars Longitude. To find *PF* the Complement of Declination. By the eighth Caſe of Oblique angled Spherical Triangles, the Proportions are.

As

As the Cot. of *P S.* 23. 525. 10. 3611802
Is to the *Radius.* 10. 0000000
So is the Cos. of *P S F.* 20 deg. 9. 9729858
To the Tang. of *S E.* 22. 25. 9. 6118056

F S. 86 deg. — *E S.* 22. 25. = *F E.* 63. 75.

As the Cos. of *E S.* 22. 25. *Comp. Arith.* 0. 0336046
To the Cosine of *P S.* 25. 525. 9. 9623154
So the Cos. *F E.* 63. 75. 9. 6457058
To the Cos. *P F.* 64. 01. 9. 6416258

Whose Complement, is *F T.* 25. 99. the Declination sought.

PROBLEM 4.

To find the Right Ascension of a Planet, or other Star with Latitude.

The Declination being found by the last Problem, we have in the Oblique angled Spherical Triangle *P F S* in *Fig.* 4. All the Sides with the Angle *F S P* 20 deg. or the Complement of the Stars Longitude. Hence to find *F P S* by the first Case of Oblique angled Spherical Triangles, I say

As the Sine of *P F.* 64. 01. *Comp. Arith.* 0. 0163059
Is to the Sine of *F S P.* 20. 9. 5340516
So is the Sine of *F S.* 86. 9. 9984407
To the Sine of *F P S.* 22. 28. 9. 5787982

Whose Complement 67. 72. is the Right Asc. of a Star ♊. 10. North Lat. 4.

PRO-

PROBLEM 5.

The Poles Elevation, Sun's greateſt Declination and Meridian Altitude being given, to find his true place in the Zodiack.

If the Meridian Altitude of the Sun be leſs than the height of the *Æquator*, deduct the Meridian Altitude from the height of the *Æquator*, the Remainer is the Sun's Declination towards the South Pole: but if the Meridian Altitude of the Sun be more than the height of the *Æquator*, deduct the height of the *Æquator* from the Meridian Altitude, what remaineth, is the Sun's Declination towards the North Pole, in theſe Northern Parts of the World: the contrary is to be obſerved in the Southern Parts.

Then in the Right angled Spherical Triangle *ADF* in *Fig.* 1. we have given the Angle *FAD* the Sun's greateſt Declination.

The Leg *DF* the Sun's preſent Declination, To find *AF* the Sun's diſtance from the next Equinoctial Point.

Therefore by the Caſe of Right angled Spherical Triangles.

As the Sine of *FAD*. 23. 525.	*Comp. Ar.* 0. 3988648
Is to the Sine of *DF*. 23. 5.	9. 5945468
So is the *Radius*.	10. 0009000
To the Sine of *AF*. 80. 04.	9. 9934116

PROBLEM 6.

PROBLEM 6.

The Poles Elevation and Sun's Declination being given, to find his Amplitude.

The Amplitude of the Sun's rising or setting is an Arch of the Horizon intercepted betwixt the *Æquator* and the place of the Sun's rising or setting; and it is either Northward or Southward, the Northward Amplitude is when he riseth or setteth on this Side of the *Æquator* towards the North Pole; and the Southern when he riseth or setteth on that Side of the *Æquator* which is towards the South Pole: That we may then find the Sun's Amplitude or Distance from the East or West Point, at the time of his rising or setting. In the Right angled Spherical Triangle *ATM*, in *Fig.* 2. let there be given the Angle *TAM.* 38. 47. the Complement of the Poles Elevation; and *TM.* 23. 15. the Sun's present Declination: To find *AM* the Sun's Amplitude.

By the eleventh Case of Right angled Spherical Triangles.

As the Sine of *MAT.* 38. 47. *Comp. Ar.*	0. 2061365
Is to the *Radim.*	10. 0000000
So is the Sine of *MT.* 23. 15.	9. 5945468
To the Sine of *AM.* 39. 19.	9. 8006833

PROBLEM 7.

PROBLEM 7.

To find the Aſcenſional Difference.

The Aſcenſional Difference is nothing elſe, but the Difference between the Aſcenſion of any Point of the Ecliptick in a Right Sphere, and the Aſcenſion of the ſame Point in an Oblique Sphere; As in *Fig.* 1. *AT* is the Aſcenſional difference between *DA* the Sun's Aſcenſion in a Right Sphere, and *DT* the Sun's Aſcenſion in an Oblique Sphere. Now then in the Right angled Spherical Triangle *AMT*, we have given. The Angle *MAT*. 38. 47. the Complement of the Poles Elevation. And *MT*. 23. 15. To find *AT* the Aſcenſional difference.

As *Rad.*	
To the Cot. of *MAT*. 38. 47. *Com. Ar.*	10.0999136
So is Tang. *MT*. 23. 55.	9.6310051
To the Sine of *AT*. 32. 56.	9.7309187

PROBLEM 8.

Having the Right Aſcenſion and Aſcenſional Difference, to find the Oblique Aſcenſion and Deſcenſion.

In *Fig.* 1. *DT* repreſents the Right Aſcenſion, *AT* the Aſcenſional Difference. *DA* the Oblique Aſcenſion which is found by deducting the Aſcenſional Difference *AT*. from the Right Aſcenſion *DT*. according to the Direction following.

If

If the Declination be

- N.
  - Subt.
    - North
      - The Ascentional Difference from the Right, and it giveth the Oblique Ascension.
      - The Ascensional Difference to the Right, and it giveth the Oblique Descension.
    - Add
- S.
  - Add
    - South
      - The Ascensional Difference to the Right, and it giveth the Oblique Ascension.
      - The Ascensional Difference from the Right, and it giveth the Oblique Descension.
  - Subt.

Right Ascension of ♊ 0 deg.	57.	80
Ascensional Difference	27.	62
Oblique Ascension ♊ 0 deg.	30.	18
Oblique Descension ♊ 0 deg.	85.	42

PROBLEM 9.

To find the time of the Sun's rising and setting, with the length of the Day and Night.

The Ascensional Difference of the Sun being added to the Semidiurnal Arch in a Right Sphere, that is, to 90 Degrees in the Northern Signs, or substracted from it in the Southern, their Sum or Difference will be the Semidiurnal Arch, which

doubled is the Right Arch, which biſected is the time of the Sun riſing, and the Day Arch biſected is the time of his ſetting.

As when the Sun is in 0 deg. ♊. his Aſcenſional Difference is 27. 62. which being added to 90 degrees, becauſe the Declination is North, the Sum will be 117. 62 the Semidiurnal Arch.

The double whereof is 235. 22 the Diurnal Arch, which being converted into time makes 15 hours 41 minutes: for the length of the Day, whoſe Complement to 24; is 8 hours 19 minutes the length of the Night; the half whereof is 4 hours 9 minutes 30 Seconds the time of the Sun's riſing.

PROBLEM 10.

The Poles Elevation and the Sun's Declination being given, to find his Altitude at any time aſſigned.

In this Problem there are three Varieties. 1. When the Sun is in the *Æquator*, that is, in the beginning of ♈ and ♎ in which caſe ſuppoſing the Sun to be at *B*, 60 degrees or four hours diſtant from the Meridian, then in the Right angled Spherical Triangle *B Z Æ*, in *Fig.* 1. we have given, *Æ Z*, 51. 53. the Poles Elevation, and *B Æ* 60 degrees, to find *B Z*.

Therefore by the 2 Caſe of Right angled Spherical Triangles.

As

As the *Radius*	
To the Cosine of *ÆZ.* 51. 53.	9. 7918635
So is the Cosine of *BÆ.* 60.	9. 6981700
To the Cosine of *BZ.* 71. 88.	9. 4918935

Whose Complement *BC.* 18. 12. is the ⊙ Altitude required.

The second Variety is when the Sun is in the Northern Signs, that is, in ♈. ♉. ♊. ♋. ♌. ♍. in which Case supposing the Sun to be at *F* in *Fig.* 1. Then in the Oblique angled Spherical Triangle *FZP*, we have given. 1. *PZ* 38. 47 the Complement of the Poles Elevation. 2. *FP.* 67. 97 the Complement of Declination. 3. *ZPF.* 45 the Distance of the ⊙ from the Meridian. To find *FZ.*

Therefore by the eighth Case of Oblique angled Spherical Triangles.

As the Cotang. of *ZP.* 38. 47.	10. 0997059
Is to the *Radius.*	10. 0000000
So is the Cosine of *ZPF.* 45.	9. 8494850
To the Tang. of *SP.* 29. 33.	9. 7497791

Then from *FP.* 67. 97
Deduct *SP.* 29. 33
There rests *FS.* 38. 64

As the Cosine of *S P.* 29.33. *Comp. Ar.* 0.0595768
To the Cosine of *P Z.* 38.47. 9.8937251
So is the Cosine of *F S.* 38.64. 9.8926982
To the Cosine of *F Z.* 45.45. 9.8460001

Whose Complement *F C.* 44.55 is the ☉ Altitude required.

The third Variety is when the Sun is in the Southern Signs as in ♎. ♏. ♐. ♑. ♒. ♓. And in this Case supposing the ☉ to be ♐ 10 degrees, and his Declination South *D b* 22.03. and his Distance from the Meridian 45 as before, then in the Oblique angled Spherical Triangle *Z b P* in *Fig.* 1. we have given *Z P.* 38.47. The Side *b P* 112.03. and the Angle *Z P b* 45. To find *Z b.*

Therefore by the 8 Case of Oblique angled Spherical Triangles.

As the Cotang. of *Z P.* 38.47. 10.0997059
Is to the *Radius.* 10.0000000
So is the Cosine of *Z P b.* 45. 9.8494850
To the Tang. of *S P.* 29.33. 9.7497791

Then from *b P.* 112.03
Deduct *S P.* 29.33
There rests *b S.* 82.70

As the Cosine of *P S.* 29.33. *Comp. Ar.* 0.0595768
To the Cosine of *Z P.* 38.47. 9.8937251
So the Cosine of *b S.* 82.70 9.1040246
To the Cosine of *Z b.* 83.45. 9.0573265

Whose Complement 6.55 is the ☉ Altitude required.

PROBLEM. 11.

PROBLEM 11.

Having the Altitude of the Sun, his Distance from the Meridian, and Declination, to find his Azimuth.

The *Azimuth* of the Sun is an Arch of the Horizon intercepted between the Meridian and the Vertical Line passing by the Sun, being understood by the Angle *HZC* in *Fig.* 1. or Arch *HC*. And in all the Varieties of the last Problem, may be found, by the first Case of Oblique angled Spherical Triangles.

Thus in the Triangle *ZBP.*

As the Sine of *BZ.* 71.88. *Comp. Ar.*	0.0220903
Is to the Sine of *BPZ.* 69.	9.9375306
So is the Sine of *BP.* 90.	10.0000000
To the Sine of *BZP.* 65. 67.	9.9596209

In the Triangle *ZFP.* I say.

s. ZF. s. ZPF :: s. FP, s. FZP.

In the Triangle *ZbP.* I say.

Sine *Zb.* Sine *ZPb* :: Sine *bP.* Sine *bZP.*

PROBLEM 12.

The Poles Elevation, with the Sun's Altitude and Declination given, to find his Azimuth.

In the Oblique angled Spherical Triangle *PZP* in *Fig.* 1. let there be given.

1. *PP.* 67. 97 the Complement of the ☉ Declination.

2. *ZP.* 38. 47 the Complement of the Poles Elevation.

3. *FZ.* 45. 46 the Complement of the ☉ Altitude.

And let the Angle *FZP* the ☉ *Azimuth* be required.

By the 11 Case of Oblique Angled Spherical Triangles.

As the Sine *ZP* × Sine *FZ*, Is to the Square of *Radius.*

So is the Sine $\frac{1}{2}$ *Z* of the Sides *ZP* × $\frac{1}{2}$ *Z* or — *ZF.*

To the Square of the Sine of half the Angle *FZP.*

The Sum of the three Sides is 151. 89

The half Sum is 75. 945 from which deduct *PZ* 38. 47. The difference is 37. 475

And the Difference between 75. 945 and *FZ* is 30. 495.

Sine

Sine of PZ. 38. 47. *Comp. Ar.*	0. 2081365
Sine of FZ. 45. 45. *Comp. Ar.*	0. 1471308
s. $\frac{1}{2}$ Z *ctr*— PZ. 37. 475.	9. 7842686
s. $\frac{1}{2}$ Z *ctr*— FZ. 30. 495.	9. 7054045
Square of the Sine of $\frac{1}{2}FZP$.	19. 8428618
Sine of 57. 94.	9. 9214309

The double whereof is 115. 88 the ☉ *Azimuth* from the North. And the Complement 64. 12, is the ☉ *Azimuth* from the South.

PROBLEM 13.

To find the Point of the Ecliptick Culminating, and its Altitude.

Before we can know what Sign and Degree of the Ecliptick is in the *Medium Cœli*; we must find the Right Ascension thereof, to do which, we must add the Sun's Right Ascension to the time afternoon, being reduced into Degrees and Minutes of the *Æquator*, the Sum is the Right Ascension of the *Medium Cœli*.

Example. Let the time given, be *March* the 20. 1674. at one of the Clock in the Afternoon.

At which time the Sun's place is in ♈. 10 deg. 23 Centesms.

To find the Right Ascension thereof, in the Right angled Spherical Triangle ADF in *Fig.* 1. we have given; The Angle of the Sun's greatest Declination DAF 23. 525 and the Sun's distance from the next Equinoctial Point AF 10. 23.

Therefore by the ninth Case of Right angled Spherical Triangles.

As the *ct. AF.* 10. 23.	10. 7435974
Is to *Radius.*	10. 0000000
So is *cs D AF* 22. 525.	19. 9623154
To *t AD* 9. 39.	9. 2187180

To which adding the Equinoctial Degrees answering to one hour, *viz.* 15. the Sum is 24. 39 the Right Ascension of the Mid Heaven. Hence to find the Point culminating, in the Right angled Spherical Triangle *ADF* in *Fig.* 1. we have given *AD* 24. 39 and *DAF* 23. 525 to find *AF.*

Therefore by the fifth Case of Right Angled Spherical Triangles,

As *t AD* 24. 39.	10. 6564908
Is to *Radius.*	10. 0000000
So is *cs D AF* 23. 525.	9. 9623154
To *ct AF* 26. 31.	10. 3058246

Therefore the Point culminating is ♈ 26. 31.

To find the Altitude thereof above the Horizon we have given in the same Triangle *DAF* 23. 525. and *AF* 26. 31. to find *DF.*

Therefore by the tenth Case of Right angled Spherical Triangles,

As *Radius.*	10. 0000000
Is to *s AF*—26, 31.	9. 6466268
So is *s DAF* 23. 525.	9. 6011352
To the *s DF* 10. 19.	9. 2477628

Which

Which is the North Declination of the Point of the Ecliptick culminating, and being added to the height of the *Æquator* at *London* 38. 47 the Sum is 48. 66 the Altitude of the Mid Heaven as was required.

PROBLEM 14.

Having the greatest obliquity of the Ecliptick together with the Distance of the Point given from the Equinoctial, to find the Meridian Angle, or Intersection of the Meridian with the Ecliptick.

Having drawn the Primitive Circle *HZRN* in *Fig.* 5. representing the Meridian, and the two Diameters *HAR*, and *ZAN*, set off the height of the Pole from *R* to *P*. 51. 53, and from *N* to *S*, and draw the Diameters *PAS* for the *Axis* of the World, and *ÆAQ* for the *Æquator*; this done, the Right Ascension of the Mid Heaven being given, as in the last Problem 24. 39 with the Point culminating. ♈. 26. 31, and the Declination thereof 10. 19, if you set 10 deg. 19 Centesmes from *Æ* to *F* and *e* to *X*, you may draw the Diameters *FAX* and *c Ad* at Right Angles thereunto, and because the *Imum Cœli* is directly opposite to the Point culminating, that is, in ♎ 26. 31, if you set 26. 31 from *X* to *b*, a Ruler laid from *c* to *b* will cut the Diameter *FX* in *G*, and then making *XbZXb* you have the three Points *bGb*, by which to draw that Circle, which will cut the *Æquator ÆAQ* in ♎, and so you have the three Points *X*♎*F* by which to describe the Arch of the Ecliptick ♈*F*♎*X*.

And

And in the Right angled Spherical Triangle $\gamma \mathcal{E} F$ we have given. The Angle $\mathcal{E}\gamma F$. 23. 525 the Sun's greatest Declination, and γF. 26. 31. the Point culminating, to find the Angle $\gamma F \mathcal{E}$.

Therefore by the seventh Case of Right angled Spherical Triangles.

As the *cs* $\mathcal{E}\gamma F$. 23. 525.	10. 3611802
Is to the *Radius*.	10. 0000000
So is the *cs* γF. 26. 31.	9. 9525062
To the *cot*. $\gamma F \mathcal{E}$. 68. 60.	9. 5913260

Which is the Angle of the Ecliptick with the Meridian.

PROBLEM 15.

To find the Angle Orient, or Altitude of the Nonagesime Degree of the Ecliptick.

In *Fig.* 5. the Pole of the Ecliptick $\gamma P \cong X$ is at *m*, and so you have the three Points $Z\,m\,N$ to draw the Vertical Circle $Z\,k\,N$ cutting the Ecliptick at Right Angles in the Point *4*: And then in the Right angled Spherical Triangle $F\,a\,Z$, we have given; FZ 41. 34 the Complement of FH the Altitude of the Mid Heaven; And the Angle $a\,P\,Z$ 68. 68 the Angle of the Ecliptick with the Meridian. To find $Z\,a$.

Therefore by the tenth Case of Right angled Spherical Triangles.

As

As the *Radius.*

To the Sine of *F Z.* 41. 34. 9. 8198890
So is the Sine of *Z F a.* 68. 68. 9. 9691128

To the Sine of *Z a.* 37. 97. 9. 7891027

Whose Complement is *a k* the Measure of the Angle *a g k* 52. 03 the Angle of the Ecliptick with the Horizon, or Altitude of the Nonagesime Degree.

PROBLEM 16.

To find the place of Nonagesime Degree of the Ecliptick.

In *Fig.* 5. *F* represents the Point of the Ecliptick in the Mid Heaven, which according to Problem 14 is ♈. 26. 31 which being known, in the Triangle *F Z a*, we have also given, *F Z* 41. 34 and the Angle *Z F a.* 68. 68. To find *F a.*

Therefore by the ninth Case of Right angled Spherical Triangles.

As the *cot.* of *F Z.* 41. 34. 10. 0556361
Is to the *Radius.* 10. 0000000
So is the *cos.* of *Z F a.* 68. 68. 9. 5605957

To the *tang.* of *F a.* 17. 73. 9. 5049596

Which being added to ♈ *F* 26. 31 the sum is ♉ *a.* 44. 04 the place of the Nonagesime Degree of the Ecliptick at *a.*

PROBLEM 17.

PROBLEM 17.

The Mid Heaven being given, to find the Points of the Ecliptick Ascending and Descending.

Having found by the last Problem, the place of Nonagesime Degree of the Ecliptick at *a* to be in ♉. 14. 04, if you add 90 Degrees or three Signs thereto, the Ascendant at *e* will be in ♌ 14. 04, and the Point descending by adding of six Signs will be in ♎ 14. 02. But these with the Cusps of the other Houses of Heaven may be otherwise found in this manner.

To the Right Ascension of the *Medium Cœli* or the tenth House, add 30, it giveth the Ascension of the eleventh House, to which adding 90 Degrees more, it giveth the Ascension of the twelfth House, *&c.* According to which direction, the Ascensions of the six Houses towards the Orient, are here set down in the following Table.

Now because the Circles of Position must according to these Directions cut the *Æquator* at 30 and 30 Degrees above the Horizon, if in *Fig.* 5. you set 30 Degrees from *E* to *n*, and *n* to *r*. A Ruler laid from *P* to *n* and *r*, shall cut the *Æquator* at *B* and *K*, and then you may describe the Circles of Position *HBR* and *HKR*, make *AT* = *AK* and *AV* = *AB*, and so you may describe the Circles *HTR* and *HVR*, and where these Circles do cut the	10.	24. 39
	11.	54. 39
	12.	84. 39
	1.	114. 39
	2.	144. 39
	3.	174. 39

Arch

Arch of the Ecliptick γF ♎ there are the Cusps of the Cœlestial Houses.

Thus a Ruler laid from *m*. the Pole of the Ecliptick to the Intersections *s. t. g. γ. v.* will cut the Primitive Circle in $\alpha. \gamma. \delta. \varepsilon. \eta.$ and the Arches $F\alpha = Fs. F\gamma = Ft. F\delta = Fg. F\varepsilon = F\gamma.$ and $F\eta = Fv$ being added to γB will give you the Cusps of the 11. 12. 1. 2 and 3 Houses, the other six are the same Degrees and Parts in the Opposite Signs.

Thus a Figure in Heaven may be erected by Projection, the Arithmetical Computation now followeth; In which the height of the Pole above each Circle of Position is required, the which in the Projection is easily found; as the Pole of the Circle of Position *HBR* is at the Point *D*. and so you have the three Points *S, D, P,* to describe that Circle by, which will cut the Circle *HBR* at Right Angles in the Point *C*. and the Arch *PC* is the height of the Pole above that Circle of Position, and may be measured by the Directions given in the nineteenth of the third Chapter.

In like manner the height of the Pole above the Circle of Position *HKR*, will be the Arch *PE*.

To compute the same Arithmetically in the Right angled Spherical Triangle *HÆB* in *Fig.* 5. we have given *ÆH*. 38. 47 the height of the *Equator*. *ÆB* 30. the difference of Ascension between the 10 and 11 Houses, to find *HB Æ* the Angle of that *Equator* with the Circle of Position.

Therefore by the first Case of Right angled Spherical Triangles.

As

As the Tang. of $H\mathcal{E}$, 38.47. 9.90000652
Is to the *Radius*. 10.00000000
So is the Sine of $\mathcal{E}B$. 30 9.69897000
To the Cotang. of $\mathcal{E}BH$. 57.81626. 9.79888348

Whose Measure in the Scheme is $\mathcal{E}C$, and the Complement thereof is CP. 32.18374 the height of the Pole required.

Therefore the height of the Pole above the Circle of Position HKR. In the Triangle $H\mathcal{E}K$, we have given, $H\mathcal{E}$ as before, and $\mathcal{E}K$. 60 to find $HK\mathcal{E}$. Therefore.

As the Tang. of $H\mathcal{E}$ 38.47. 9.90008652
Is to the *Radius*. 10.00000000
So is the Sine of $\mathcal{E}K$ 60. 9.93753063
To the Cotang. of $HK\mathcal{E}$ 42.53308. 10.03744411

Whose Measure in the Scheme is GL, and the Complement thereof is PL 47.46692. the height of the Pole required.

The height of the Pole above HDR is the same with HBR, and the height of the Pole above HTR is the same with HKR.

Having found the Ascensions of the several Houses together with the Elevation of the Pole above their Circles of Position, in the Oblique angled Spherical Triangle ΥBS, we have given,

1. The Angle ΥBS the Complement of $HB\mathcal{E}$.

2. The Angle $B\Upsilon S$. 23. The Sun's greatest Declination.

3. Their included Side ΥB. 54.39 the Ascension of the eleventh House. To find ΥS the

Point

Point of the Ecliptick, which is resolvable by the ninth Case of Right angled Spherical Triangles.

But in my *Trigonometria Britannica*, *Problem.* 54. for the resolving of Oblique angled Spherical Triangles, I have shewed how this Case as to our present purpose may be resolved, by these Proportions following.

1. $s\frac{1}{2}Z$ Ang. $s\frac{1}{2}X$ Ang :: $t\frac{1}{2}\Upsilon B$. $t\frac{1}{2}X$ Cru.
2. $cs\frac{1}{2}Z$ Ang. $cs\frac{1}{2}X$ Ang :: $t\frac{1}{2}\Upsilon B$. $t\frac{1}{2}Z$ Cru.

$\frac{1}{2}Z$ Cru $+\frac{1}{2}X$ Cru $=\Upsilon S$ the Arch of the Ecliptick desired.

For the Cusp of the Eleventh House.

TB Arch ΥB. 4439 the half whereof is 27. 195.
ΥBS. 122. 18374.
$B\Upsilon S$. 23. 525.

Z 145. 70874 — $\frac{1}{2}Z$ 72. 85437.
X. 198. 65874 — $\frac{1}{2}X$. 49. 32937.

$s\frac{1}{2}Z$. 72. 85437. *Comp. Arith.*	0. 01977589
$s\frac{1}{2}X$. 49. 32937.	9. 88000860
$t\frac{1}{2}\Upsilon B$. 27. 195.	9. 71081089
$t\frac{1}{2}X$ Cru. 22. 192.	9. 61059478

2. *Operation.*

$cs\frac{1}{2}Z$. 72. 85437. *Comp. Arith.*	0. 53012277
$ss\frac{1}{2}X$. 49. 32937.	9. 81395860
$t\frac{1}{2}\Upsilon B$. 27. 195.	9. 71081089
$t\frac{1}{2}Z$ Cru. 48. 611.	10. 05489226

1. Arch. 22. 192. Their Sum is 70. 803 the Point of the Ecliptick. 6z.

cs. ½ Z. 82. 51916. *Comp. Arith.* 0. 88517901
cs ½ X. 59. 00416. 9. 71164750
t. ½ ϒ A. 57. 195. 10. 19072348
t ½ Z Cru: 78. 397. 10. 68754999
1. Arch — 53. 296. Their Sum 121. 693 is the Point of the Ecliptick for the Ascendant.

For the Cusp of the Second House.

In the Oblique angled Spherical Triangle ϒ T γ. we have given,

1. ϒ T. 144. 39. The half whereof is 72. 195.
2. ϒ T γ. 122. 18374 } To find ϒ γ. The An-
3. T γ. 23. 525 } gles are the same with those of the Twelfth House. Therefore.

s. ½ Z. 80. 49596. *Comp. Arith.* 0. 00601663
s ½ X. 56. 97096. 9. 92351651

Their Sum 9. 92953314
t ½ ϒ T. 72. 195. 10. 49327695
t ½ X Cru. 69. 306. 10. 42281209

2. *Operation.*

cs ½ Z 80. 49596. *Comp. Arith.* 0. 78170174
cs ½ X 56. 97096. 9. 73628614

Their Sum 10. 51798788
t ½ ϒ T. 72. 195. 10. 49327695
t ½ Z Cru. 84. 34. 11. 01126483
1. Arch. 69. 306. Their Sum is 53. 740 is the Point of the Ecliptick for the Second House.

For

For the Cusp of the Third House.

In the Oblique angled Spherical Triangle $\gamma v \theta$, we have,

1. γv. 174. 39. The half whereof is 87. 19½. The Angls $\gamma v \theta$ and $v \gamma \theta$ are the same with those of the Eleventh House.

$s\frac{1}{2}$ Z. 72. 85437. *Comp. Arith.*	0. 01977589
$s\frac{1}{2}\gamma$ 49. 32937.	9. 88000800
Their Sum	9. 89978389
$t\frac{1}{2}$ Xv. 87. 195.	11. 30984054

For the Eleventh House.

For the Cusp of the Twelfth House.

In the Oblique angled Spherical Triangle γKF, we have given.

1. vK. 84. 39. The half whereof is. 42. 19½.
2. γKt. 137. 46692 } To find γf.
3. $K\gamma t$. 23. 525 }

Z. 160. 99192 $\frac{1}{2}$ Z. 80. 49596
X. 113. 94192 $\frac{1}{2}$ X. 56. 97096

$s\frac{1}{2}$ Z. 80. 49596. *Comp. Arith.*	0. 00601663
$s\frac{1}{2}$ X. 56. 97096.	9. 92351651
$t\frac{1}{2}\gamma K$. 42. 195.	9. 95740882
$t\frac{1}{2}$ X Cru. 37. 625.	9. 88694196

2. *Operation.*

ds. $\frac{1}{2}$ *Z.* 80.49596.	*Comp. Arith.*	0.78170174
cs $\frac{1}{2}$ *X.* 56.97096.		9.73628614
t $\frac{1}{2}$ ♈ *K.* 42.195.		9.95740882
t $\frac{1}{2}$ *Z* Cru. 71.496.		10.47539670

1. Arch. 37.625. Their Sum 113.6691 is the Point of the Ecliptick for the Twelfth House.

For the Cusp of the Ascendant.

In the Oblique angled Spherical Triangle ♈ *AG* we have,

1. ♈ *A.* 114.39. The half whereof is 57.195.
2. ♈ *AZ.* 141.5333. The Complement of *HAÆ* 38.46667.
3. *A v y.* 23.525.

Z.	165.05833	$\frac{1}{2}$ *Z.*	82.51916
X.	118.00833	$\frac{1}{2}$ *X.*	59.00416

s. $\frac{1}{2}$ *Z.* 82.51916.	*Comp. Arith.*	0.00371629
s. $\frac{1}{2}$ *X.* 59.00416.		9.93313477
t $\frac{1}{2}$ ♈ *A.* 57.195.		10.19072348
t $\frac{1}{2}$ *X.* 53.296.		10.12757454

2. *Operation.*

t $\frac{1}{2}$ *X* Cru. 86.468.	11.20962043

2. *Operation.*

2. *Operation.*

ts ½ Z. 72. 85437. *Comp. Arith.*	0. 53012277
cs ½ X. 49. 32937.	9. 81395860
Their Sum	10. 34408137
s ½ ♈ v. 87. 195.	11. 39984054
t ½ X Cru. 88. 729.	11. 65392191

1. Arch. 86. 468. Their Sum 175. 197 is the Point of the Ecliptick for the Third House.

And thus we have not only erected a Figure for the Time given, but composed a Table for the general erecting of a Figure in that Latitude; for by adding together the first and second Numbers in each Proportion for the first, second and third Houses there is composed two Numbers for each House, to each of which the Artificial Tangent of half the Ascension of each House being added, their Aggregates are the Tangents of two Arches, which being added together, do give the distance of the Cusp of the House from the first Point of *Aries*, as in the preceding Operations hath been shewed.

Only note, That if the Ascension of any House be more than a Semicircle, you must take the Tangent of half the Complement to a whole Circle. And to find the Cusp of the House, you must also take the Complement of the Sum of the Arches added together.

The Numbers according to the former Operations which do constitute a Table of Houses for the Latitude of *London*. 51. 32 are as followeth.

	11 and 3 Houses	Ascendant	12 and 2 Houses
1. Oper.	9. 89978389	9. 936851 ☌	9. 92953314
2. Oper.	10. 34408137	10. 5968263	10. 51798788

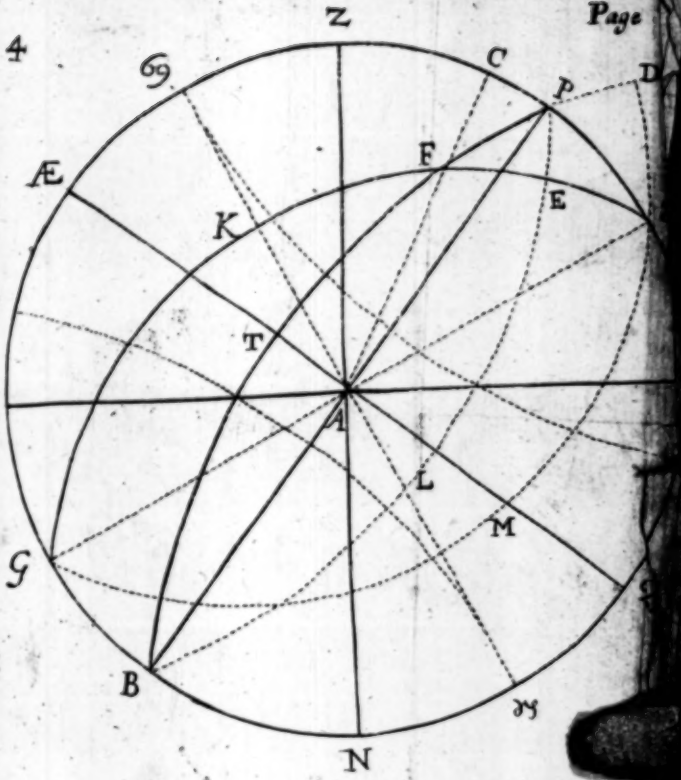
The Six Oriental Houses, by the preceding Operations.

10 House ♈ 26. 311
11 House ♊ 10. 803
12 House ♋ 23. 691
Ascendant ♌ 11. 693
2 House ♍ 3. 740
3 House ♍ 25. 197

The opposite Houses are in the opposite Signs and Degrees.

4 House ♎ 26. 311
5 House ♐ 10. 803
6 House ♑ 23. 691
7 House ♒ 11. 693
8 House ♓ 3. 740
9 House ♓ 25. 197

A Figure of the Twelve Cœlestial Houses.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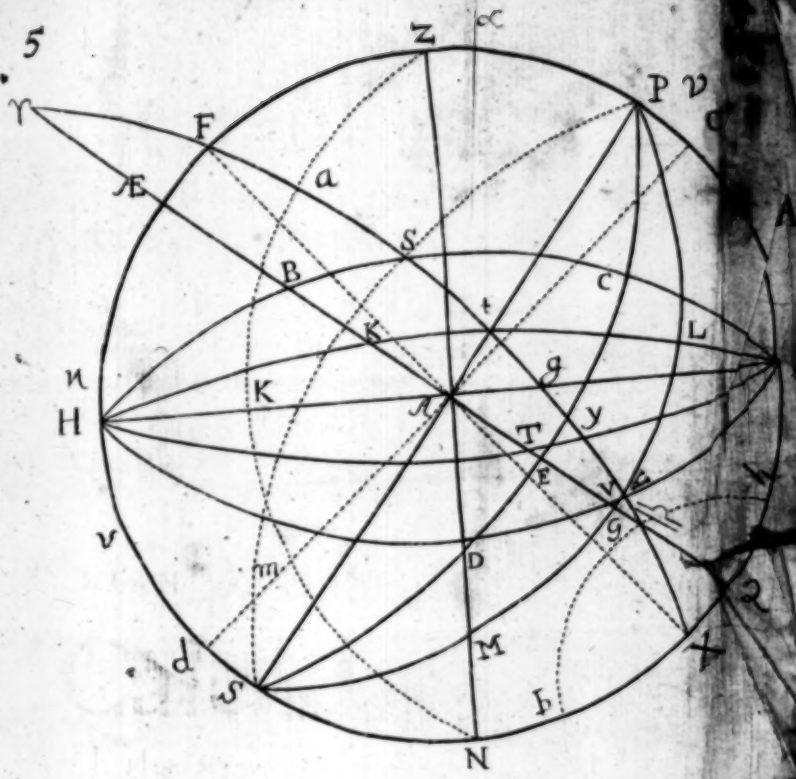


ASTRONOMY.

THE

Second Part:

OR,

AN ACCOUNT

OF THE

Civil Year,

With the Reason of the Difference

Between the

JULIAN & GREGORIAN

Calendars,

And the manner of Computing the Places of the *SUN* and *MOON*.

LONDON,

Printed for *Thomas Passinger*, at the *Three Bibles* on *London-Bridge*. 1679.

AN INTRODUCTION TO Astronomy.

The Second Book.

CHAP. I.

Of the Year Civil and Astronomical.

HAving shewed the Motion of the *Primum Mobile*, or Doctrine of the Sphere, which I call the Absolute Part of Astronomy; I come now unto the Comparative, that is, to shew the Motion of the Stars in reference to some certain Distinction of Time.

2. And the Distinction of Time is to be considered either according to Nature, or according to Institution.

3. The Distinction of Time according to Nature, is that space of Time, in which the Planets do finish their Periodical Revolutions from one certain Point in the Zodiack, to the same again, and this in reference to the Sun is called a Year, in reference to the Moon a Month.

4. The Sun doth pass through the Zodiack in 365 Days, 5 Hours, and 49 Minutes. And the Moon doth finish her course in the Zodiack, and return into Conjunction with the Sun, in 29 Days, 12 hours, 44 Minutes, and 4 Seconds. And from the Motion of these two Planets, the Civil Year in every Nation doth receive its Institution.

5. Twelve Moons or Moneths is the measure of the Common Year, in *Turkey* in every Moneth they have 29 or 30 Days, in the whole Year 354 Days, and in every third Year 355 Days.

6. The *Persians* and *Egyptians* do also account 12 Moneths to their Year; but their moneths are proportioned to the Time of the Suns continuance in every of the Twelve Signs; in their Year therefore which is Solar, there are always 365 Days, that is eleven Days more than the Lunar Year.

7. And the *Julian* Year which is the Account of all Christendom, doth differ from the other in this; that by reason of the Sun's Exces in Motion above 365 Days, which is 5 Hours, 49 Minutes, it hath a Day intercalated once in 4 Years, and by this intercalation, it is more agreeable to the Motion of the Sun, than the former, and yet there is a considerable difference between them, which hath occasioned the Church of *Rome* to make some further amendment of the Solar Year,

Year, but hath not brought it to that exactneſs, which might be wiſhed.

8. This intercalation of one Day once in 4 Years, doth occaſion the Sunday Letter ſtill to alter till 28 Years be gone about; The Days of the Week which uſe to be ſigned by the ſeven firſt Letters in the Alphabet, do not fall alike in every Common Year, but becauſe the Year conſiſteth of 52 Weeks and one Day, Sunday this Year will fall out upon the next Year's Monday, and ſo forward for ſeven years, but every fourth year conſiſting of 52 weeks and two days, doth occaſion the Sunday Letter to alter, till four times ſeven years, that is till 28 years be gone about. This Revolution is called the Cycle of the Sun, taking its name from the Sunday Letter, of which it ſheweth all the Changes that it can have by reaſon of the Biſſextile or Leap-year. To find which of the 28 the preſent is, add nine to the year of our Lord, (becauſe this Circle was ſo far gone about, at the time of Chriſts Birth) and divide the whole by 28, what remaineth is the preſent year, if nothing remain the Cycle is out, and that you muſt call the laſt year of the Cycle, or 28.

9. This Intercalation of one day in four years, doth occaſion the Letter *F* to be twice repeated in *February*, in which Moneth the day is added, that is, the Letter *F* is ſet to the 24 and 25 days of that Moneth, and in ſuch a year S. *Matthias* day is to be obſerved upon the 25 day, and the next Sunday doth change or alter his Letter, from which leaping or changing, ſuch a year is called Leap-year, aud the number of days in each Moneth is well expreſſed by theſe old Verſes.

rtThiy

Thirty days hath September, April, June *and* November.
February *hath* 28 *alone, All the reſt have thirty and one.*
But when of Leap-year cometh the Time,
Then days hath February *twenty and nine.*

That this year is ſomewhat too long, is acknowledged by the moſt skilful Aſtronomers, as for the number of days in a year the Emperours Mathematicians were in the right, for it is certain, that no year can conſiſt of more than 365 days, but for the odd hours it is as certain that they cannot be fewer than five, nor yet ſo many as ſix; ſo then the doubt is upon the minutes, 60 whereof do make an hour, a ſmall matter one would think, but how great in the conſequence we ſhall ſee. The Emperours year being more than 10 minutes greater than the Suns, will in 134 years riſe to one whole day, and by this means the Vernal or Spring Equinox, which in *Julius Cæſar*'s time was upon the 24 of *March*, is now in our time upon the 10 of *March*, 13 days backward, and ſomewhat more, and ſo if it be let alone will go back to the firſt of *March*, and firſt of *February*, and by degrees more and more backward ſtill.

10. To reform this difference, ſome of the late Roman Biſhops have earneſtly endeavoured. And the thing was brought to that perfection it now ſtandeth, by *Gregory* the Thirteenth, in the year 1582. His Mathematicians, whereof *Lilius* was the Chief, adviſed him thus: That conſidering there had been an Agitation in the Council

Council of *Nice* ſomewhat concerned in this matter upon the motion of that Queſtion, about the Celebration of Eaſter. And that the Fathers of the Aſſembly, after due deliberation with the Aſtronomers of that time, had fixed the Vernal Equinox at the 21 of *March*, and conſidering alſo that ſince that time a difference of ten whole days had paſt over in the Calendar, that is, that the Vernal Equinox, which began upon the 21 of *March*, had prevented ſo much, as to begin in *Gregorie*'s days at the 10 of the ſame, they adviſed, that 10 days ſhould be cut off from the Calendar, which was done, and the 10 days taken out of *October* in the year 1582. as being the moneth of that year in which that Pope was born; ſo that when they came to the fifth of the moneth they reckoned the 15, and ſo the Equinox was come up to its place again, and happened upon the 21 of *March*, as at the Council of *Nice*.

But that *Lilius* ſhould bring back the beginning of the year to the time of the *Nicene* Council and no further, is to be marvelled at, he ſhould have brought it back to the Emperours own time, where the miſtake was firſt entered, and inſtead of 10, cut off 13 days; however this is the reaſon why theſe two Calendars differ the ſpace of 10 days from one another. And thus I have given you an account of the year as it now ſtands with us in *England*, and with the reſt of the Chriſtian World in reſpect of the Sun, ſome other particulars there are between us and them which do depend upon the motion of the Moon, as well as of the Sun, and for the better underſtanding of them, I will alſo give you a brief account

account of her revolution. But first I will shew you, how the day of the moneth in any year propounded in one Country, may be reduced to its correspondent time in another.

11. Taking therefore the length of the year, to be in several Nations as hath been before declared, if we would find what day of the moneth in one Country is correspondent to the day of that moneth given in another, there must be some beginning to every one of these Accounts, and that beginning must be referred to some one, as to the common measure of the rest.

12. The most natural beginning of all Accounts, is the time of the Worlds Creation, but they who could not attain to the Worlds Beginning, have reckoned from their own, as the Romans from the building of *Rome*, the Greeks from their Olympicks, the Assyrians from *Nabonassar*, and all Christians from the Birth of Christ: the beginning of which and all other the most notable *Epochaes*, we have ascertained to their correspondent times in the Julian Period, which *Scaliger* contrived by the continual Multiplication of those Circles, all in former time of good use, and two of them do yet remain; the Circles yet in use are those of the Sun and Moon, the one, to wit, the Sun, is a Circle of 28 years, and the Circle of the Moon is 19, as shall be shewed hereafter. The third Circle which now serves for no other use than the constituting of the Julian Period, is the Roman Indiction, or a Circle of 15 years; if you multiply 28 the Circle of the Sun, by 19 the Circle of the Moon, the Product is 532, which being multiplied by 15, the Circle of the Roman Indiction, the Product is 7980, the

Number

Number of years in the Julian Period: whose admirable condition is to distinguish every year within the whole Circle by a several certain Character, the year of the Sun, Moon, and Indiction being never the same again until the revolution of 7980 years be gone about, the beginning of this Period was 764 Julian years before the most reputed time of the Worlds Creation; which being premised, we will now by Example shew you how to reduce the years of Forreigners to our Julian years, and the contrary.

1. *Example.*

I desire to know at what time in the Turkish Account, the fifth of *June* in the year of our Lord 1640. doth fall.

The Julian years complete are 1648, and are thus turned into days, by the Table of days in Julian years.

1000	Julian years give days	365250
600	Julian years give days	219150
40	Julian years give days	14610
8	Years give days	2922
	May complete	151
	Days	5
	The Sum is	602088

Now because the Turkish Account began *July* 16. *Anno Christi.* 622. you must convert these years into days also.

600

600	Julian years give days	219150
20	Years give days	7305
1	Year giveth days	365
	June complete	181
	Days	15
	The Sum is	227016

Which being ſubſtracted from		602088
There reſteth days		375072
900	Turkiſh years give days	318930
	There reſteth	56142
150	Turkiſh years give days	53155
	There reſteth	02987
8	Turkiſh years give days	2835
	There reſteth	152
	Giumadi. 4.	148
	There reſteth	4

Therefore the fifth of *June* 1649. in our Engliſh Account doth fall in the year 1058. of *Mahomet*, or the Turkiſh *Hegira*, the fourth day of the moneth *Giumadi*. 11.

2. *Example.*

I deſire to know upon what day of our Julian year the 17 day of the moneth in the 1069 year complete of the Perſian Account from *Jeſhegile* doth fall.

The

The beginning of this *Epocha* is from the *Epocha* of Christ in complete days 230639

1000	Persian years give	365000
60	Years give	21900
9	Years give	3285
	Chorial complete	90
	Days complete	16
	The Sum	620930
1000	Julian years Substracted	365250
	There rests	255680
700	Julian years	255675
	There rests	5

Therefore it falls out in the Julian year from Christ 1700. the fifth day of *January*.

He that understands this may by the like method convert the years of other *Epochas*, into our Julian years and the contrary.

The Anticipation of the Gregorian Calendar is more easily obtained, for if you enter the Table with the years of Christ complete, you have the days to be added to the time in the Julian Account, to make it answer to the Gregorian, which will be but ten days difference till the year 1700. and then the differenee will be a day more, until the year 1800. and so forward three days difference more in every 400 years to come, unless our year shall be reformed as well as theirs.

CHAP. II.

CHAP. II.

Of the Cycle of the Moon, what it is, how placed in the Calendar, and to what purpose.

THat the Civil Year in use with us and all Christians, doth consist of 365 days, and every fourth year of 366, hath been already shewed, with the return of the Sunday Letter in 28 years. In which time the Moon doth finish her course in the Zodiack no less than twelve times, which twelve Moons, or 354 days, do fall short of the Sun's year, eleven days in every common year, and twelve in the Bissextile or Leap-year.

And by Observation of *Meton* an *Athenian*, it was found out about 432 years before Christ, that the Moon in nineteen years did return to be in Conjunction with the Sun on the self same day, and this Circle of nineteen years is called the Cycle of the Moon, which being written in the Calendar against the day in every Moneth, in which the Moon did change, in Letters of Gold, was also called the Golden Number, or from the excellent use thereof, which was at first, only to find the New Moons in every Moneth for ever, but amongst Christians it serveth for another purpose also, even the finding of the time when the Feast of Easter is to be observed. The New Moons by this Number are thus found. In the first year of the Circle, or when the Golden Number is 1, where the Number 1 was set in the Calendar in any Moneth, that day is New Moon, in the

the second Year where you find the golden Number 2, in the third Year where you find the golden Number 3, and so forward till the whole Circle be expired; then you must begin with one again, and run through the whole Circle as before.

2. And the reason why the Calendar begins with the golden Number 3, not 1, is this. The Christians in *Alexandria* had used this Circle of the Moon two Years before the Nicene Council. And in the first of these Years the new Moon next to the Vernal Equinox was upon the 27th Day of the Egyptian month *Phamenoth* answering to the 23d of our *March*, against that Day therefore they placed the golden Number 1. And because there are 29 Days and a half from one new Moon to another, they made the distance between the new Moons to be interchangeably 29 and 30 Days, and so they placed the same golden Number against the 26 Day of *Phurmuthi* the Month following, and against the 26 Day of the Month *Pachon* and so forward, and upon this ground by the like progression was the golden Number set in the Roman Calendar; and so the golden Number 1 by their example was set against *March* 23. *April* 21. *June* 19. *July* 19. *August* 17. *September* 16. *October* 15. *November* 14. *December*. 13. But then because in the following Year the golden Number was 2. reckoning 30 Days from the 13th of *December*, the golden Number 2 was set to *January* 12. *February* 10. *March* 12. *April* 10. *May* 10. *June* 8. *July* 8. *August* 6. *September* 5. *October* 4. *November* 3. *December*. 2. From whence reckoning 13 Days as before, the golden Number 3

comes in course for the third Year to be set against the first of *January*.

But that you may know how the golden Number comes to be distributed in the Calendar according to the form in which it now is, you must consider that in 19 Solar Years there are not only 228 Lunar Months or 12 times 19 Lunar Months but 235 for the 11 Days which the common Solar Year doth exceed the Lunar, do in 19 Years arise to 209 Days, out of which there may be appointed 7 Months, 6 whereof will contain 30 Days apiece, and one Month 29 days; and these 7 Months are called Emboliſmical Months, because by a kind of injection or interposition they are reckoned in some of the 19 Years. And those Years in which they are reckoned are called Emboliſmical Years, to distinguish them from the common Years which always contain 354 Days, whereas 6 of these Emboliſmical Years do each of them contain 384 Days, and the seventh Emboliſmical Year in which the Month of 29 Days is reckoned, doth contain 383 Days.

3. The Emboliſmical Years in the Cycle of the Moon are properly these Seven. 3, 6, 9, 11, 14, 17, 19. because in the third Year 11 Days being thrice reckoned do amount to 33 Days, that is one Month of 30 Days and 3 Days over. Again in the sixth Year the 11 Days which the Solar exceed the Lunar, being thrice numbred, do amount to 33 Days. which with the 3 Days formerly reserved do make 36 Days, that is one Month of 30 Days and 6 Days over. Again in the Ninth Year there are also 33 Days, to which the 6 Days reserved being added, there will arise one Month more and 9 Days over. But in the

the Eleventh Year twice 11 Days being added to the 9 Days reserved, do make 31 Days, that is, one Month of 30 days and one day over, which being added to the supernumerary days in the fourteenth Year do make another Month of 30 Days and 4 Days over, and these being added to the supernumerary Days in the seventeenth Year do make another Month of 30 and 7 Days over, and these 7 Days being added to the 22 supernumerary Days in the Nineteenth Year of the Moons Cycle do make another Month of 29 Days.

4. But because there are 6939 Days and 18 Hours in 19 Solar Years, that is, 4 Days 18 Hours more then in the common and Emboliſmical Lunar Years, in which the excess between the Lunar and the Solar Year is supposed to be no more then 11 Days in each Year, whereas in every fourth Year the excess is one Day more, that is, 12 Days, that is, in 16 Years 4 Days, and in the remaining 3 Years three fourths of a day more. And that the new Moons after 19 Lunar Years or 235 Lunations do not return to the same days again, but want almost 5 days, it is evident that the civil Lunations do not agree with the Astronomical and that there must be yet some kind of intercalation used.

5. Now therefore in distributing the golden Number throughout the Calendar. If the new Moons should interchangeably consist of 30 and 29 days, and so but 228 Lunations in 19 Years, we might proceed in the same order in which we have begun, and by which as hath been shewed the third Year of the Golden Number falls upon the Calends of *January*. But for as much as there are first six Lunations of 30 days apiece and

one of 29 days to be interposed, therefore there must be 6 times 2 Lunations together consisting of 30 days and once three Lunations of 29 days. And that respect may be also had to the Bissextile days, although they are not expresst in the Calendar, that Lunation which doth contain the Bissertile day, if it should have been 29 days, it must be 30, if it should have consisted of 30 days it must consist of 31.

6. And because it was thought convenient, as hath been shewed, to begin with the third Year of the Cycle of the Moon, because the Golden Number 3 is set to the Calends of *January*, therefore in this Cycle the Embolismical Years are, 2, 5, 8, 11, 13, 16, 19. But yet that it may appear, that these Years are in effect the same, as if we had begun with the first Year of the Golden Number, save only that the eighth Year instead of the ninth is to be accounted Embolismical, I have added the Table follwing, in which it is apparent that the former Embolismical years do agree with these last mentioned.

7. But as I said before, it was thought more convenient to begin the account from the number 3 set to the Calends of *January*, because by so reckoning 30 and 29 days to each Lunation interchangeably, the same Number 3 falls upon *January* 31. *March* 1, and 31. *April* 29. *May* 29. *June* 27. *July* 27. *August* 25. *September* 14. *October* 23. *November* 22. *December* 21. As if the Lunar years were compleated upon the 20 of *December* there remain just 11 Days, which the Solar years doth exceed the Lunar.

8. And by ranking on and accounting 4 for the Golden Number of the next year, you will find

find it set on *January* 20, *February* 18, *March* 20, *April* 18, *May* 18, *June* 16, *July* 16, *August* 14, *September* 13, *Octob.* 12, *Novemb.* 11, *Decemb.* 10.

9. But in going on, and taking 5 for the Golden Number in the third year, we must remember that that is an Embolismical Year, and therefore that somewhere there must be 2 Months together of 30 days. And for this reason the Golden Number 5, is set to *January* 9, *February* 7, *March* 9, *April* 7, *May* 7, *June* 5, *July* 5, *August* 3, *September* 2, as also upon the second day of *October*, and not upon the first, that so there may be 2 Lunations together of 30, and the same Number 5 is also set to the thirty first of *October*, to make the Lunation to consist of 29 days, and to the thirtieth of *November* instead of the twenty ninth, that so a Lunation of

Cycle of the Moon.	Cycle of the Moon.	Embolismical Years.	Number of Days.
1	3		354
2	4		354
3	5	Embol.	384
4	6		354
5	7		354
6	8	Embol.	384
7	9		354
8	10		354
9	11	Embol.	384
10	12		354
11	13	Embol.	384
12	14		354
13	15		354
14	16	Embol.	384
15	17		354
16	18		354
17	19	Embol.	384
18	1		354
19	2	Embol.	384

so may again succeed as it ought.

10. In like manner in the sixth Year, having gone through the fourth and fifth as common years, you may see the Golden Number 8 set to the fifth of *April*, which should have been upon the fourth, and in the ninth Year the Golden Number 11 is set to the second of *February* which should have been upon the first.

And there is a particular reason, for which these numbers are otherwise placed from the eighth of *March* to the fifth of *April*, namely, that all the paschal Lunations may consist of 29 days: For thus from the eighth of *March* to the sixth of *April*, to both which days the Golden Number is 16, there are but 29 days. And from the ninth of *March* to the seventh of *April*, to both which days the Golden Number is 5, there are also 29 days, and so of the rest till you come to the fifth of *April*, which is the last Paschal Lunation, as the eighth of *March* is the first, but at any other time of the Year, the length of the Month in the Emboliſmical Year, may be fixed as you please.

12. And in this manner in the 17 years, in which the lunations of the whole Circle are finished, and in which the Golden Number is 19, the Month of *July* is taken at pleasure, to the thirtieth day whereof is set the Golden Number 19, which should have been upon the thirty first, and the same Number being notwithstanding placed upon the twenty eighth of *August*, that by the two Lunations of 29 days together, it might be understood, that the seventh Emboliſmical Month consisting of 29 days is there inserted, instead of a Month of 30 days. In which place the Emboliſmical or leaping Year of the Moon may

plainly

plainly be observed for that year is one day less than the rest, which the Moon doth as it were pass over. The which one day is again added to the 29 days of the last Month, that we may by that means come, as in other Years, to the Golden Number, which sheweth the New Moon in *January* following. And for this reason the Epact then doth not consist of 11 but of 12 days. And thus you see the reason, for which the Golden Numbers are thus set in the Calendar as here you see. In which we may also observe, that every following Number is made by adding 8 to the Number preceding, and every preceding Number is also made by adding 11 to the Number next following, and casting away 19 when the addition shall exceed it.

For Example, if you add 8 to the Golden Number 3 set against the first of *January*, it maketh 11, to which add 8 more and it maketh 19, to which adding 8 it maketh 27, from which substracting 19 the remainer is 8, to which again adding 8, the sum is 19, to which adding 8 the sum is 24, from which deducting 19 the remainer is 5, and so of the rest. In like manner receding backward, to the 5 add 11 they make 16, to the 16 add 11 they make 27, from which deducting 19 the remainer is 8, to which 11 being added the same is 19, to which 11 being added the sum is 30, from which deducting 19 the remainer is 11, to which 11 being added the sum is 22, from which deducting 19 the remainer is 3. And by this we may see that every following number will be in use 8 years after the preceding, and every preceding Number will be in use 11 years after the following, that is, the same will return to be in use after

8 Years and 11, and the other after 11 Years and 8, or once in 19 years.

CHAP. III.

Of the Use of the Golden Number in finding the Feast of Easter.

THe Cycle of the Moon or Golden Number is a circle of 19 years, as hath been said already, which being distributed in the Calendar as hath been shewn in the last Chapter, doth shew the day of the New Moon for ever; though not exactly: But the use for which it was chiefly intended, was to find the Paschal New Moons, that is, those new Moons on which the Feast of *Easter* and other moveable Feasts depend. To this purpose we must remember,

1. That the vernal Equinox is supposed to be fixed to the twenty first day of *March*.

2. That the fourteenth day of the Moon on which the Feast of *Easter* doth depend, can never happen before the Equinox; though it may fall upon it or upon the day following.

3. That the Feast of *Easter* is never observed upon the fourteenth day of the Moon, but upon the Sunday following; so that if the fourteenth day of the Moon be Sunday, the Sunday following is Easter day.

4. That the Feast of *Easter* may fall upon the fifteenth day of the Moon, or upon any other day unto the twenty first, inclusively.

5. That the Paschal Sunday is discovered by the

the proper and Dominical Letter for every Year The which may be found as hath been already declared, or by the proper Table for that purpose. Hence it followeth,

1. That the New Moon immediately preceding the Feast of *Easter*, cannot be before the eighth day of *March*, for if you suppose it to be upon *March* 6, the Moon will be 14 days old *March* 19, which is before the Equinox, contrary to the second Rule before given, and upon the seventh day of *March* there is no Golden Number fixed; and therefore the Golden Number 16, which standeth against *March* 8, is the first by which the Paschal New Moon may be discovered.

2. It followeth hence, That the last Paschal New Moon cannot happen beyond the fifth day of *April*, because all the 19 Golden Numbers are expressed from the eighth of *March* to that day. And if a New Moon should happen upon the sixth of *April*, there would be two Paschal New Moons that year, one upon the eighth of *March* and another upon the sixth of *April*, the same Golden Number 16 being proper to them both, but this is absurd because *Easter* cannot be observed twice in one year.

3. It followeth hence, That the Feast of *Easter* can never happen before the twenty second day of *March*, nor after the twenty fifth day of *April*: For if the first New Moon be upon the eighth of *March*, and that the Feast of *Easter* must be upon the Sunday following the fourteenth day of the Moon; it is plain that the fourteenth day of the Moon must be *March* 21 at the soonest: So that supposing the next day to be Sunday, *Easter* cannot

not be before *March* the twenty ſecond. And becauſe the fourteenth day of the laſt Moon falleth upon the eighteenth day of *April*, if that day be Saturday, and the Dominical Letter *D*, *Eaſter* ſhall be upon the nineteenth day, but if it be Sunday, *Eaſter* cannot be till the twenty fifth.

4. It followeth hence, That although there are but 19 days, on which the fourteenth day of the Moon can happen, as there are but 19 Golden Numbers, yet there are 35 days from the twenty ſecond of *March* to the twenty fifth of *April*, on which the Feaſt of *Eaſter* may happen, becauſe there is no day within thoſe Limits, but may be the Sunday following the fourteenth day of the Moon. And although the Feaſt of *Eaſter* can never happen upon *March* 22, but when the fourteenth day of the Moon is upon the twenty firſt, and the Sunday Letter *D*, nor upon the twenty fifth of *April*, but when the fourteenth day of the Moon is upon *April* 18, and the Dominical Letter *C*. Yet *Eaſter* may fall upon *March* 23, not only when the fourteenth day of the Moon is upon the twenty ſecond day which is Saturday, but alſo if it fall upon the twenty firſt which is Friday. In like manner *Eaſter* may fall upon *April* 24, not only when the fourteenth day of the Moon is upon the eighteenth day which is Monday, but alſo if it happen upon the ſeventeenth being Sunday. And for the ſame reaſon it may fall oftner upon other days that are further diſtant from the ſaid twenty ſecond of *March* and twenty fifth of *April*.

5. It followeth hence, That the Feaſt of *Eaſter* may be eaſily found in any Year propounded: For the Golden Number in any Year being given, if you

you look the ſame between the eighth of March and fifth of *April* both inclusively, and reckon 14 days from that day, which anſwereth to the Golden Number given, where your account doth end is the fourteenth day of the Moon: Then conſider which is the Dominical Letter for that Year, and that which followeth next after the fourteenth day of the Moon is Eaſter day. Example, In the year 1674 the Golden Number is 3, and the Sunday Letter *D*, which being ſought in the Calendar between the aforeſaid limits, the fourteenth day of the Moon is upon *April* the thirteenth, and the *D* next following is *April* 19. And therefore *Eaſter* day that Year is *April* 19. Otherwiſe thus.

In March *after the firſt* C,
Look the Prime wherever it be,
The third Sunday after Eaſter *day ſhall be.*
And if the Prime on Sunday be,
Reckon that for one of the Three.

6. Thus the Feaſt of *Eaſter* may be found in the Calendar, and from thence a brief Table ſhewing the ſame, may be extracted in this manner. Write in one Column the ſeveral Golden Numbers in the Calendar from the eighth of *March* to the fifth of *April*, in the ſame order obſerving the ſame diſtance. In the ſecond Column ſet the Dominical Letters in number 35 ſo diſpoſed, as that no Dominical Letter may ſtand againſt the Golden Number 16, but ſetting the Letter *D* againſt the Golden Number 5, write the reſt in this

this order. *E, F, G, A, B, &c.* and when you come to the Golden Number 8, ſet the Letter *C*, and there continue the Letters till you come to *C* again, becauſe when the Golden Number is 16, which in the Calendar is ſet to the eighth day of *March*, is new Moon, and the fourteenth day of that Moon doth fall upon the twenty firſt, to which the Dominical Letter is *C*, upon which the Feaſt of *Eaſter* cannot happen; and therefore in the third Column containing the day in which the Feaſt of *Eaſter* is to be obſerved, is alſo void. But in the next place immediately following, to wit, againſt the letter *D* is ſet *March* 22, becauſe if the fourteenth day of the Moon ſhall fall upon the twenty firſt of *March* being Saturday, the next day being Sunday, ſhall be the Feaſt of *Eaſter*.

To the Letters following, *E, F, G, A, B, &c.* are ſet 23, 24, 25, and ſo orderly to the laſt of *March*, and ſo forward till you come to the twenty fifth of *April*, by which Table thus made, the Feaſt of *Eaſter* may be found until the Calendar ſhall be reformed.

For having found the Golden Number in the firſt Column, the Dominical Letter for the Year next after it, doth ſhew the Feaſt of *Eaſter*, as in the former Example, the Golden Number is 3 and the Dominical Letter *D*, therefore *Eaſter* day is upon *April* 19. The other moveable Feaſts are thus found.

Advent Sunday is always the neareſt Sunday to St. *Andrews*, whether before or after.

Septua-

Septuagesima Sunday is Nine Weeks before *Easter*.

Sexagesima Sunday is Eight Weeks before *Easter*.

Quinquagesima Sunday is Seven Weeks before *Easter*.

Quadragesima Sunday is Six Weeks before *Easter*.

Rogation Sunday is five Weeks after *Easter*.

Ascension day is Forty Days after *Easter*.

Whitsunday is Seven Weeks after *Easter*.

Trinity Sunday is Eight Weeks after *Easter*.

G. N.	D. L.	Easter.
XVI		
V	D	22 March
	E	23
XIII	F	24
II	G	25
	A	26
X	B	27
	C	28
XVIII	D	29
VII	E	30
	F	31
XV	G	1 April
IV	A	2
	B	3
XII	C	4
I	D	5
	E	6
IX	F	7
	G	8
XVII	A	9
VI	B	10
	C	11
XIV	D	12
III	E	13
	F	14
XI	G	15
	A	16
XIX	B	17
VIII	C	18
	D	19
	E	20
	F	21
	G	22
	A	23
	B	24
	C	25

CHAP.

CHAP. IV.

Of the Reformation of the Calendar by Pope Gregory *the Thirteenth; and ſubſtituting a Cycle of Epacts in the room of the Golden Number.*

HItherto we have ſpoken of the Calendar which is in uſe with us, we will now ſhew you for what reaſons it is alter'd in the Church of *Rome*, and how the Feaſt of *Eaſter* is by them obſerved.

The Year by the appointment of *Julius Cæſar* conſiſting of 365 days 6 hours, whereas the Sun doth finiſh his courſe in the Zodiack, in 365 days 5 hours 49 minutes or thereabouts, it cometh to paſs that in 134 Years or leſs, there is a whole day in the Calendar more than there ought; in 268 years 2 days more; in 4002 years 3 days: and ſo ſince *Julius Cæſar*'s time the vernal Equinox hath gone backward 13 or 14 days, namely from the 24 of *March* to the tenth. Now becauſe the Equinox was at the time of the *Nicene* Council upon the twenty firſt of *March*, when the time for the obſerving of *Eaſter* was firſt univerſally eſtabliſhed, they thought it ſufficient to bring the Equinox back to that time, by cutting off 10 days in the Calendar as hath been declared, and to prevent any anticipation for the time to come, have appointed, that the Leap-year ſhall be thrice omitted in every 400 Years to come, and for memory ſake, appointed the firſt omiſſion to be accounted from the Year 1600, not from 1582, in which the reformation was made, becauſe it was not only near the time, in which the emendation was begun, but alſo becauſe the Equinox has not fully made an anticipation of 10 days from the

place

place thereof, at the time of the *Nicene* Council, which was *March* 21.

The Years then 1700, 1800, 1900, which ſhould have been Biſſextile Years, are to be accounted common years, but the Year 2000 muſt be a Biſſextile: In like manner the Years 2100, 2200, 2300, ſhall be common years, and the Year 2400 Biſſextile, and ſo forward.

2. Again, becauſe it was ſuppoſed that the Cycle of the Moon, or Golden Number was ſo fixed, that the new and full Moons would in every 19 years return to the ſame days again; whereas their not returning the ſame hours, but making an anticipation of one hour 27 minutes or thereabouts, it muſt needs be that in 17 Cycles or little more than 300 Years, there would be an anticipation of a whole day. And hence it is evident that in 1300 Years ſince the *Nicene* Council, the New and Full Moons do happen more than 4 days ſooner than the Cycle of the Moon or Golden Number doth demonſtrate: Whence alſo it comes to paſs, that the fourteenth day of the Moon by the Cycle is in truth the eighteenth day, and ſo the Feaſt of *Eaſter* ſhould be obſerved not from the fifteenth day of the Moon to the twenty firſt, but from the nineteenth to the twenty fifth.

3. That the Moon therefore being once brought into order, might not make any anticipation for the time to come, it is appointed that a Cycle of 30 Epacts ſhould be placed in the Calendar inſtead of the Golden Number, anſwering to every day in the Year; to ſhew the New Moons in theſe days, not only for 300 Years or thereabouts, but that there might be new Epacts without

out altering the Calendar, to perform the ſame thing upon other days as need ſhall require.

4. For the better underſtanding whereof, to the Calendar in uſe with us, we have annexed the Gregorian Calendar alſo: In the firſt Column whereof you have 30 numbers from 1 to 30, ſave only that in the place of 30 you have this Aſterisk *, But they begin with the Calends of *January*, and we continued and repeated after a Retrograde order in this manner, *29, 28, 27, &c. and that for this cauſe eſpecially, that the number being given which ſheweth the New Moons in every Month for one Year, you might by numbring 11 upwards excluſively find the number which will ſhew the New Moons the Year following, to wit, the Number which falleth in the eleventh place.

5. And theſe Numbers are called Epacts, becauſe they do in order ſhew thoſe 11 days, which are yearly to be added to the Lunar Year conſiſting of 354 days, that it may be in conformity with the Solar Year conſiſting of 365 days. To this purpoſe, as hath been ſaid concerning the Golden Number, theſe Epacts being repeated 12 times, and ending upon the twentieth day of *December*, the ſame Numbers muſt be added to the 11 remaining days, which were added to the firſt 11 days in the Month of *January*.

6. And becauſe 12 times 30 do make 360, whereas from the firſt of *January* to the twentieth of *December* incluſively, there are but 354 days, you muſt know that to gain the other ſix days, the numbers 25 and 24 are in every other Month both placed againſt one day, namely, to *February* 5, *April* 5, *June* 3, *Auguſt* 1, *September* 29, and *Novem-*

November 27. But why these two Numbers are chosen rather then any other, and why in these 6 Months the number 25 is sometimes writ to XVI, sometimes to XXV in a common character, and why the number 19 is set to the last day of *December* in a common Character, shall be declared hereafter.

7. Here only note that this Asterisk * is set instead of the Epact 30, because the Epact shewing the Number of days which do remain after the Lunation in the Month of *December*, it may sometimes fall out that 2 Lunations may so end, that the one may require 30 for the Epact, and the other 0, which would, if both were written, cause some inconveniences, and therefore this * Asterisk is there set, that it might indifferently serve to both. And the Epact 29 is therefore set to the second day of *January*, because after the compleat Lunation in the second of *December* there are 29 days, and for the like reason the Epact 28 is set against the third of *January*, because after the compleat Lunation in the third of *December* there are then 28 days over, and so the rest in order till you come to the thirtieth of *January*, where you find the Epact 1. because after the compleat Lunation on the thirtieth day there is only one day over.

8. And besides the shewing of the New Moons in every Month, which is and may be done by the Golden Number, the Epacts have this advantage, that they may be perpetual and keep the same place in the Calendar in all future ages, which can hardly be effected with the Golden Number, for in little more then 700 years, the New Moons do make an anticipation of one day, and then it

will be neceſſary to ſet the Golden Number one degree backward, and ſo the Golden Number which at the time of the Nicene Council was ſet to the firſt of *January*, ſhould in 300 years be ſet to the laſt of *December*, and ſo of the reſt, but the Epacts being once fixed ſhall not need any ſuch retraction or commutation. For as often as the New Moons do change their day either by Anticipation or by Suppreſſion of the Biſſextile year, you ſhall not need to do any more than to take another rank of 19 Epacts, inſteed of thoſe which were before in uſe. For inſtance, the Epacts which are and have been in uſe in the Church of *Rome* ſince the year of reformation 1582, and will continue till the year 1700, are theſe 10 following 1. 12. 23. 4. 15. 26. 7. 18. 29. 10. 21. 2. 13. 24. 5. 16. 27. 8. 19. And from the year 1700 the Epacts which will be in uſe are theſe. * 11. 22. 3. 14. 25. 6. 17. 28. 9. 20. 1. 12. 23. 4. 15. 26. 7. 18. and ſhall continue not only to the year 1800, but from thence until the year 1900 alſo; and although in the year 1800 the Biſſextile is to be ſuppreſſed, yet is there a compenſation for that Suppreſſion, by the Moons Anticipation. To make this a little more plain, the motion of the Moon, which doth occaſion the change of the Epact, muſt be more fully conſidered.

CHAP. V.

CHAP. V.

Of the Moons mean Motion, and how the Anticipation of the New Moons may be discovered by the Epacts.

THe Moon according to her middle motion doth finish her course in the Zodiack in 29 days, 12 hours 44 minutes, three seconds or thereabout, and therefore a common Lunar year doth consist of 354 days, 8 hours, 48 minutes, 38 seconds and some few thirds, but an Embolismical year doth consist of 383 days, 21 hours, 32 minutes, 41 seconds and somewhat more; and therefore in 19 years it doth exceed the motion of the Sun 1 hour, 27 minutes, 33 seconds *ferè*.

2. Hence it cometh to pass, that although the New Moons do after 19 years return to the same days; yet is there an Anticipation of 1 hour, 27 minutes, 33 seconds. And in twice 19 years, that is, in 38 years, there is an Anticipation of 2 hours, 55 minutes, 6 seconds, and after 312 years and a half, there is an Anticipation of one whole day and some few Minutes. And therefore after 312 years no new Moon can happen upon the same day it did 19 years before, but a day sooner. Hence it comes to pass that in the Julian Calendar, in which no regard is had to this Anticipation, the New Moons found out by the Golden Number must needs be erroneous, and from the time of the Nicene Council 4 days after the New Moons by a regular Computation.

3. And hence it follows also, that if the Golden Number, after 312 were upon due consideration

ration removed a day forwarder or nearer the beginning of the Months, they would ſhew the New Moons for 312 years to come. And being again removed after thoſe years, a day more would by the like reaſon do the ſame again. But it was thought more convenient ſo to diſpoſe 30 Epacts, that they keeping their conſtant places, 19 of them ſhould perform the work of the Golden Number, until by this means there ſhould be an Anticipation of one day. And when ſuch an Anticipation ſhould happen, thoſe 19 Epacts being let alone, other 19 ſhould be uſed, which do belong to the preceding day, without making any alteration in the Calendar.

4. And if this Anticipation would do the whole work, nothing were more plain, then to make that commutation of the 19 Epact once in 312 years: but becauſe the detraction of the Biſſextile days doth variouſly interpoſe and cauſe the 19 Epacts ſometimes to be changed into theſe that do precede, ſometimes into theſe that follow, ſometimes into neither, but to continue ſtill the ſame; therefore ſome Tables are to be made, by which we may know, when the commutation was to be made and into what Epacts.

4. Firſt therefore there was made a Table called *Tabula Epactarum Expanſa*, in this manner.

Firſt on the top were placed the 19 Golden Numbers in order, beginning with the Number 3, which in the old Calendar is placed againſt the Calends of *January*, and under every one of theſe Golden Numbers there are placed 30 Epacts all conſtituted from the loweſt number in the firſt rank in which the Epact is 1, and in that firſt rank the Golden Number is 3, the reſt from thence

thence towards the right Hand are made by the conſtant addition of it, and the caſting away of 30, as often as they ſhall exceed that number, only when you come to the 27, the Epact under the Golden Number 19, there muſt be added 12 inſtead of 11, that ſo the Epact following may be 9 not 8, for the Reaſons already given in this Diſcourſe concerning the Golden Number and Emboliſmical years. And this rank being thus made, the other Epacts are diſpoſed in their natural order aſcending upwards, and the number once again reſumed after the Epact 30 or rather this Aſterisk * ſet in the place thereof: only obſerve that under the Golden Number 12. 13. 14. 15. 16. 17. 18. 19. in the place of XX there is yet 25 in the common Character. And to the Epacts under the Golden Number 19, 12 muſt ſtill be added to make that Epact under the Golden Number 1. As was ſaid before concerning the loweſt Rank.

5. And on the left hand of theſe Epacts before thoſe under the Golden Number 3. are ſet 30 Letters of the Alphabet, 19 in a ſmall Character, and 11 in a great, in which ſome are paſſed by, for no other reaſon ſave only this, that their ſimilitude with ſome of the ſmall Letters, ſhould not occaſion any miſtake in their uſe, which ſhall be ſhewed in its place.

6. Beſides this Table there was another Table made which is called *Tabula Æquationis Epactarum*, in which there is a *ſeries* of years, in which the Moon, by reaſon of her mentioned anticipation doth need Æquation, and in which the number of Epacts ſigned with the letters of the Alphabet, are to be changed; being otherwiſe Æ-

quated where it needeth, by the ſuppreſſion of the Biſſextile days.

7. But it ſuppoſeth, that it was convenient to ſuppreſs the Biſſextiles once only in 100 years; and the Moon to be æquated, or as far as concerns her ſelf, the rank of Epacts to be changed, once only in 300 years, and the 12 years and a half more, to be referred till after the years 2400, they do amount unto 100 years, and then an æquation to be made: but then it muſt be made by reaſon of the interpoſing this hundred not in the three hundredth but the hundredth year. Moreover this æquation is to be made as in refereece to the Moon only, becauſe as the ſuppreſſion of the Biſſextiles intervene, the order of changing the ranks of Epacts is varied, as ſhall be ſhewed hereafter.

8. Again this Table ſuppoſeth, that ſeeing the New Moon at the time of the Nicene Council was upon the Calends of *January*, the golden Number 3 being there placed, that it would have been the ſame if the Epact * had been ſet to the ſame Calends, that is if the Epacts had been then in uſe. And therefore at that time the higheſt or laſt rank of Epacts was to be uſed, whoſe Index is *P*, and then after 300 years, the loweſt or firſt rank ſhould ſucceed, whoſe *Index* is *a*, (for the letters return in a Circle) and after 300 years more, the following rank whoſe *Index* is *b* and ſo forward; but that it is conceived, that the New Moon in the Calends of *January*, is more agreeable to the year of Chriſt 500, than the time of the Nicene Councel; and therefore as if the rank of Epacts under the letter *l* were ſutable to the year 500, it ſeemed

good

good to make uſe of that rank under the letter *a* in the year of Chriſt 800, and thoſe under the letter *b*, in the year 1100, and thoſe under the letter *c* in the year 1400.

9. Which being granted, becauſe in the year 1582, ten days were cut off from the Calendar, we muſt run backward, or in an inverted order count 10 *ſeries*, deſigned, ſuppoſe, by the letters *b. a. P. N. M. H. G. F. E. D.* ſo that from the year 1582 the *ſeries* of Epacts whoſe literal *Index* is *D*, is to be uſed, and this is that rank of Epacts which is now uſed in the Church of *Rome*.

10. And therefore as if this Table had its beginning from that year; the firſt number in the ſecond column is 1582, and then in order under it. 1600. 1700. 1800. 1900. 2000. *&c.* And in the third Column every fourth hundred year is marked for a Biſſextile, that is, 1600. 2400. 2800, *&c.* and in the fourth Column to every three hundreth Year is ſet this Character *C*, to ſhew in what year the Moon by her Anticipation of one day, doth need æquation; but in the year 1800 the double character is ſet *CC*, to ſignify that then another hundred years are gotten by the 12 years and a half reſerved, beſides and above the other 300 years; and this character is alſo ſet to the years 4300. 6800, and for the ſame reaſon.

But in the firſt Column, or on the left hand of theſe years are placed the Letters or *Indices* of thoſe ranks of Epacts in the former Table, which are to be uſed in thoſe years and when the Letters are charged. Thus againſt the year 1600 the Letter *D* is continued, to ſhew that from that

year, to the year 1700 the rank of Epacts is ſtill to be uſed, which do belong to that Letter. And for as much as the Letter *C* is ſet to the year 1700, it ſheweth that that rank of Epacts is then to be uſed, which do belong thereto, and ſo of the reſt.

11. The reaſon why theſe Letters in the firſt Column are ſometimes changed in 100 years, ſometimes in 200, ſometimes not in leſs then 300 Years, and that they are ſometimes taken forward, ſometimes backward, according to the order of the Alphabet, is becauſe the ſuppreſſion of the Biſſextiles do intervene with the lunar æquation: for if the Biſſextile were only to be ſuppreſſed, in theſe 300 or ſometimes 400 years, in which the Moon needeth æquation, the rank of Epacts in that caſe would need no commutation, but would continue the ſame for ever; and the golden Number would have been ſufficient, if the ſuppreſſion of the Biſſextile, and anticipation of the Moon, did by a perpetual compenſation cauſe the new Moons ſtill to return to the ſame days: but becauſe the Biſſextile is ofttimes ſuppreſſed, when the Moon hath no æquation, the Moon hath ſometimes an æquation when the Biſſextile is not ſuppreſſed, ſometimes alſo both are to be done and ſometimes neither; all which varieties may yet be reduced to theſe three Rules.

1. As often as the Biſſextile is ſuppreſſed without any æquation of the Moon, then the letter which ſerved to that time ſhall be changed to the next below it contrary to the order of the Alphabet. And the new Moons ſhall be removed one

one day towards the end of the Year.

2. As often as the Moon needeth æquation, without ſuppreſſion of the Biſſextile, then the Letter which was in uſe to that time ſhall be changed to the next above it according to the order of the Alphabet, that the New Moons may again return one day towards the beginning of the year.

3. As often as there is a Suppreſſion and an æquation both, or when there is neither, the Letter is not changed at all but that which ſerved for the former Centenary, ſhall alſo continue in the ſucceeding; becauſe the compenſation ſo made, the New Moons do neither go forward nor backward, but happen in the compaſs of the ſame days.

1. And this is enough to ſhew for what reaſon the letters are ſo placed in the Table, as there you ſee them: for in the year 1600 the Biſſextile being neither ſuppreſſed, nor the Moon æquated, the letter *D* uſed in the former Centenary or in the latter part thereof from the year 1582, is ſtill the ſame.

In the year 1700, becauſe there is a ſuppreſſion, but no æquation, the commutation is made to the Letter *C* deſcending.

In the Year 1800, becauſe there is both a ſuppreſſion and an æquation, the ſame letter *C* doth ſtill continue.

In the Year 2400, becauſe there is an æquation and no ſuppreſſion, there is an aſcenſion to the Letter *A*.

And thus you ſee not only the conſtruction of this Table, but how it may be continued to any other Year, as long as the World ſhall laſt.

12. And

12. And by these two Tables we may easily know which rank of the 30 Epacts doth belong to, or is proper for any particular age: for as in our age, that is, from the Year 1600 to the Year 1700 exclusively, that *Series* is proper whose *Index* is *D*, Namely, 23, 4, 15, 26, *&c.* so in the two Ages following, that is, from the Year 1700 to the Year 1900 exclusively, that *Series* is proper whose *Index* is *C*, namely these, 22, 3, 14, 25. and in the three ages following thence, that is from the Year 1900 to the Year 2100 exclusively, that *Series* is proper whose *Index* is *B*, namely these, 21, 2, 13, 24, *&c.* And so for any other.

Hence also it may be known, which of the 19 doth belong to any particular Year, for which no more is necessary, than only to know the Golden Number for the year given, which being sought in the head of the Table, and the *Index* of that Age in the side, the common Angle, or meeting of these two, will shew you the Epact desired: As in the year 1674 the Golden Number is 3 and the *Index D*; therefore in the common Angle I find 23 for the Epact that year, and sheweth the New Moons in every Month thereof.

And here it will not be unseasonable to give the reason, for which the Epact 25 not XXV is written under the Golden Numbers 12, 13, 14, 15, 16, 17, 18, 19. namely, because the ranks of Epacts, which under these greater Numbers hath this Epact 25, hath also XXIV, it would follow that in these Ages in which any of these Ranks were in use, the New Moon in 19 years will happen twice upon the same days; in those six Months in which the Epacts XXV and XXIV are set to the same day: Whereas the New Moons do not happen

pen on the same day till 19 years be gone about. To avoid this inconvenience, the Epact 25 not XXV is set under these great numbers, and the Epact 25 is in the Calendar, in these Months set with the Epact XXVI, but in the other Months with the Epact XXV.

14. Hence it cometh to pass, 1. That in these Years the Epacts 25 and XXIV do never meet on the same day. 2. That there is no danger that the Epacts 25 and XXVI should in these 6 Months cause the same inconvenience, seeing that the Epacts 25 and XXVI are never both found in the same Rank. 3. That the Epact 25 may in other Months without inconvenience be set to the same day with the Epact XXVI, because in these there is no danger of their meeting with the Epact XXIV on the same days. 4. That there is no fear that the Epacts XXV and XXIV being set on the same days, should in future Ages cause the same inconvenience, because the Epacts XXV and XXIV are not found together in any of the other Ranks. But that either one or both of them are wanting. Besides, when one of these Epacts is in use, the other is not, and that only which is in use is proper to the day. As in this our Age until the Year 1700 the Epacts in use are those in the rank whose *Index* is *D*. In which these two XXIV and XXV are not both found. And in the two following Ages, because the rank of Epacts in use is that whose *Index* is *C*, in which there is the Epact XXV, not XXIV, the New Moons are shewed by the Epact XXV not by XXIV. But because in three following Ages, the rank of Epacts in use is that whose *Index* is *B*, in which 25 and XXIV are both found, the New Moons

Moons are ſhewed by the Epact XXIV when the golden Number is 6. And by the Epact 25 when the golden Number is 17, and not by the Epact XXV.

15. And if it be asked why the Epact 19 in the common Character is ſet with the Epact XX againſt the laſt day of *December*; know that for the reaſons before declared, the laſt Emboliſmical Month within the ſpace of 19 years, ought to be but 29 days and not 30, as the reſt are; and therefore when the Epact 19 doth concur with the golden Number 19, the laſt Month or laſt Lunation beginning the ſecond of *December*, ſhall end upon the 30 and not upon the 31 of that Month, and the New Moon ſhould be ſuppoſed to happen upon the 31 under the ſame Epact 19, that 12 being added to 19 and not 11, you may have one for the Epact of the year following, which may be found upon the 30 of *January*, as if the Lunation of 30 days had been accompliſhed the Day before.

CHAP. VI.

How to find the Dominical Letter and Feaſt of Eaſter according to the Gregorian account.

HAving ſhewed for what reaſon, and in what manner the Epacts are ſubſtituted in the place of the golden Number, and how the New Moons may be by them found in the Calendar for ever; I ſhall now ſhew you how to find the Feaſt of Eaſter and the other moveable Feaſts

Feaſts according to the Gregorian or new account; and to this purpoſe I muſt firſt ſhew you how to find the Dominical Letter, for that the Cycle of 28 years will not ſerve the turn, becauſe of the ſuppreſſion of the Biſſextile once in a hundred years, but doth require 7 Cycles of 28 years apeice. The firſt whereof begins with *CB*, and endeth in *D*. The ſecond begins with *DC*, and endeth in *E*. The third begins with *ED*, and endeth in *F &c.* The firſt of theſe Cycles began to be in uſe 1582, in which year the dominical Letter according to the Julian account was *G*, but upon the fifteenth day of *October*, that Year was changed to *C*: for the fifth of *October* being Friday and then called the fifteenth, the Letter *A* became Friday, *B* Saturday, and *C* Sunday, the remaining part of the year, in which the Cycle of the Sun was 23, and the ſecond after the Biſſextile or leap Year, and ſo making *C*, which anſwereth to the fifteenth year of that Circle, to be 23, the Circle will end at *D*, and conſequently *CB*, which in the old account doth belong to the 21 year of the Circle, hath ever ſince been called the firſt, and ſo ſhall continue until the year 1700, in which the Biſſextile being ſuppreſſed, the next Cycle will begin with *DC* as hath been ſaid already. Under the firſt rank or order of Dominical Letters are written the years 1582 and 1600, under the ſecond 1700, under the third 1800, under the fourth 1900 and 2000, under the fifth 2100, under the ſixth 2200 and under the ſeventh 2300 and 2400. And again under the firſt Order, 2500, under the ſecond 2600, under the third 2700 and 2800, and ſo forward as far as you

pleaſe,

pleaſe, always obſerving the ſame order, that the 100 Biſſextile years may ſtill be joyned with the not Biſſextile immediately preceding.

1. And hence it appears, that the ſeven orders of Dominical Letters, are ſo many Tables, ſucceſſively ſerving all future Generations. For as the firſt Order ſerveth from the year 1582 and 1600 to the year 1700 excluſively, and the ſecond Order from thence to the year 1800 excluſively, ſo ſhall all the reſt in like manner which here are ſet down, and to be ſet down at pleaſure. And hence the Dominical Letter or Letters may be found for any year propounded, as if it were required to find the dominical Letter for the year 1674, becauſe the year given is contained in the centenary 1600. I find the Cycle of the Sun by the Rule already given to be 3. In the firſt order againſt the number 3, I find *G* for the Sunday Letter of that year, in like manner becauſe the year 1750 is contained under the Centenary 1700, the Cycle of the Sun being 27, I find in the ſecond rank the Letter *D* anſwering to that Number, and that is the Dominical Letter for that year, and ſo of the reſt.

3. Again for as much as the fifth Order is the ſame with that Table, which ſerves for the old account, therefore that order will ſerve the turn for ever where that Calendar is in uſe, and ſo this laſt will be of perpetual uſe to both the Calendars.

4. Now then to find the time in which the Feaſt of *Eaſter* is to be obſerved, there is but little to be added to that which hath been already ſaid concerning the Julian Calendar. For the Paſchal

chal Limits are the same in both, the difference is only in the Epacts, which here are used instead of the golden Number.

5. For the terms of the Paschal New Moons are always the eighth of *March* and the fifth of *April*: but whereas there are 11 days within these Limits to which no golden Number is affixed, there is now one day to which an Epact is not appointed, because there is no day within those Limits, on which in process of time a New Moon may not happen. And the reason for which the two Epacts XXV and XXIV are both set to the fifth of *April*, is first general, which was shewed before, namly that by doing the same in 5 other Months, the 12 time 30 Epacts might be contracted to the Limits of the lunar Year which consists of 354 days: but there is a particular reason also for it, that the Antients having appointed that all the Paschal lunations should consist of 29 days, it was necessary that some two of the Epacts should be set to one of these days in which the Paschal lunation might happen, the Epacts being 30 in number. And it was thought convenient to choose the last day, to which the Epact XXV belonging, the Epact XXIV should also be set; and hence by imitation it comes to pass, that these and not other Epacts are set to that day in other Months, in which two Epacts are to be set to the same days.

6. The use of these Epacts in finding the Feast of *Easter*, is the same with that which hath been shewed concerning the golden Numbers. For the Epact and the Sunday Letter for that year propounded being given, the Feast of *Easter* may be found in the Calendar after the same manner.

manner. Thus in the year 1674, the Epact is 23 and the Sunday Letter *G*, and therefore reckoning fourteen days from the eighth of *March* to which the Epact is set, the Sunday following is *March* 25, which is the day on which the Feast of *Easter* is observed.

7. And hence as hath been shewed in the third Chapter concerning the Julian Calendar, a brief table may be made to shew the feast of *Easter* and the other moveable Feasts for ever, in which there is no other difference, save only that the Epacts as they are in this new Calendar, are to be used as the golden Numbers are, which stand in the old Calendar. And a Table having the golden Numbers of the old Calendar set in one Column, and the Epacts as they are in the new Calendar set in another, will indifferently shew the movable Feasts in both accounts, as in the Year 1674, the golden Number is 3 and the Sunday Letter according to the Julian account is *D*, according to the Gregorian *G*, and the Epact 23, and therefore according to this Table our *Easter* is *April* 19, and the other, to wit, the Gregorian, is *March* 25. The like may be done for any other year past or to come.

CHAP. VII.

CHAP. VII.

How to reduce Sexagenary numbers into Decimal, and the Contrary.

EVery Circle hath antiently, and is yet generally supposed to be divided into 360 degrees, each degree into 60 Minutes, each Minute into 60 Seconds, and so forward as far as need shall require. But this partition is somewhat troublesom in Addition and Subtraction, much more in Multiplication and Division; and the Tables hitherto contrived to ease that manner of computation, do scarce sufficiently perform the work, for which they are intended. And although the Canon published by the learned *H. Gellibrand*, in which the Division of the Circle into 360 degrees is retained, but every degree is divided into 100 parts, is much better than the old Sexagenary Canon, yet some are of opinion, that if the Antients had divided the whole circle into 100 or 1000 parts, it would have proved much better then either; only they think Custome such a Tyrant, that the alteration of it now will not be perhaps so advantagious; leaving them therefore to injoy their own opinions, they will not I hope be offended if others be of another mind: for their sakes therefore, that do rather like the Decimal way of calculation, Having made a Canon of artificial Signs and Tangents for the degrees and parts of a Circle divided into 100 parts, I shall here also shew you, how to reduce sexagenary Numbers into Decimal, and the contra-

ry, as well in time as motion.

2. The parts of a Circle conſiſting of 360 degrees, may be reduced into the parts of a circle divided into 100 degrees or parts, by the rule of Three in this manner.

As 360 is to 100, ſo is any other Number of degrees, in the one, to the correſpondent degrees and parts in the other.

But if the ſexagenary degrees have Minutes and Seconds joyned with them, you muſt reduce the whole Circle as well as the parts propounded into the leaſt Denomination, and ſo proceed according to the rule given.

Example. Let it be required to convert 125 degrees of the Sexagenary Circle, into their correſpondent parts in the Decimal. I ſay, as 360 is to 100, ſo is 125 to 34, 722222, &c. that is, 34 degrees and 722222 Parts.

2. Example. Let the Decimal of 238 degrees 47 Minutes be required. In a whole Circle there are 21600 Minutes, and in 238 degrees, there are 14280 Minutes, to which 47 being added the ſum is 14327. Now then I ſay if 21600 give 100, what ſhall 14327. The Anſw. is 66,3287 &c. In like manner if it were required to convert the Hours and Minutes of a Day into decimal Parts, ſay thus, if 24 Hours give 100, what ſhall any other number of Hours give. Thus if the Decimal of 18 hours were required, the anſwer would be 75, and the Decimal anſwering to 16 Hours 30 Minutes is 68, 75.

But if it be required to convert the Decimal Parts of a Circle into its correſpondent Parts in Sexagenary. The proportion is; as 100 is

to

to the Decimal given, ſo is 360 to the Sexagenary degrees and parts required.

Example. Let the Decimal given be 345722222, if you multiply this Number given by 360, the Product will be 1249999992, that is cutting off 7 Figures, 124 degrees and 9999992 parts of a degree. If Minutes be required, multiply the Decimal parts by 60, and from the product cut off as many Figures, as were in the Decimal parts given, the reſt ſhall be the Minutes deſired.

But to avoid this trouble, I have here exhibited two Tables, the one for converting ſexagenary degrees and Minutes into Decimals, and the contrary. The other for converting Hours and Minutes into Decimals, and the contrary. The uſe of which Tables I will explain by example.

Let it be required to convert 258 degrees 34′. 47″, into the parts of a Circle decimally divided.

The Table for this purpoſe doth conſiſt of two Leaves, the firſt Leaf is divided into 21 Columns, of which the 1. 3. 5. 7. 9. 11. 13. 15. 17. 19 doth contain the degrees in a ſexagenary Circle, the 2. 4. 6. 8. 10. 12. 14. 16. 18 and 20 doth contain the degrees of a Circle Decimally divided, anſwering to the former, and the laſt Column doth contain the Decimal parts, to be annexed to the Decimal degrees. Thus the Decimal degrees anſwering to 26 Sexagenary are 7, and the parts in the laſt Column are 22222222 and therefore the degrees and parts anſwering to 26 Sexagenary degrees are 7. 22222222.

In like manner the Decimal of 62 degrees, 17. 22222222. And the Decimal of 258 degrees, 34′. 47″; is thus found.

The Decimal of 258 degrees is	71.66666666
The Decimal of 34 Minutes is	.15747040
The Decimal of 47 ſeconds is	.00362652
Their Sum	71.82776358

is the Decimal of 258 degrees, 34′. 47″ as was required.

In like mauner the Decimal of any Hours and Minutes may be found by the Table for that purpoſe.

Example. Let the Decimal of 7 Hours 28′ be required.

The Decimal anſwering to 7 h. is	29.16666667
The Decimal of 28 Minutes is	1.94444444
The Sum	31.11111111

is the Decimal Sought.

To find the degrees and Minutes in a ſexagenary Circle, anſwering to the degrees and parts of a Circle Decimally divided, is but the contrary work.

As if it were required to find the Degrees and minutes anſwering to this decimal 71.02776359, the Degrees or Integers being ſought in the 2. 4. 6 or 8 Columns *&c.* of the firſt Leaf of that Table, right againſt 71. I find 256 and in the laſt Column theſe parts 11111111, which being leſs than the Decimal given, I proceed till

till I come to 666667, which being the neareſt to my number given, I find againſt theſe parts under 71. Degrees 258, ſo then 258 are the degrees anſwering to the Decimal given and,

To find the Minutes and Seconds from I Subſtract the number in the Table	71.82776359 71.66666667
The remainer is	16109692
which being Sought in the next Leaf under the title Minutes, the next leaf is	11747640
And the Minutes 34, and this number being Subtracted the remainer is	00362652

Which is the Decimal of 47 ſeconds, and ſo the degrees and Minutes anſwering to the Decimal given are 258 degrees 34′ and 47″, the like may be done for any other.

CHAP. VIII.

Of the difference of Meridians.

HAving in the firſt part ſhewed how the places of the Planets in the Zodiack may be found by obſervation, and how to reduce the time of an obſervation made in one Country, to the correſpondent time in another, as to the day of the Month, by conſidering the ſeveral

measures of the year in several Nations, there is yet one thing wanting, which is, by an observation made of a Planets place in one Country to find when the Planet is in that place in reference to another; as suppose the ☉ by observation was found at *Uraniburg* to be in ♈. 3ᵈ. 13′. 14″. *March* the fourteenth 1583 at what time was the Sun in the same place at *London*? To resolve this and the like questions, the Longitude of places from some certain Meridian must be known; to which purpose I have here exhibited a Table shewing the difference of Meridians in Hours and Minutes, of most of the eminent places in *England* from the City of *London*, and of some places beyond the Seas also. The use whereof is either to reduce the time given under the Meridian of *London* to some other Meridian, or the time given in some other Meridian to the Meridian of *London*.

1. If it be required to reduce the time given under the Meridian of *London* to some other Meridian, seek the place desired in the Catalogue, and the difference of time there found, either add to or subtract from the times given at *London*, according as the Titles of Addition or Subtraction shew, so will the time be reduced to the Meridian of the other place as was required. Example. The same place at *London* was in the first Point of ♉, 6 Hours *P.M.* and it is required to reduce the same to the Meridian of *Uraniburg* I therefore seek in *Uraniburg* in the Catalogue of places, against which I find 50′ with the Letter *A* annexed, therefore I conclude, that the Sun was that day at *Uraniburg* in the first point of ♉, 6 Hours 50′. *P. M.*

2. If

2. If the time given be under ſome other Meridian, and it be required to reduce the ſame to the Meridian of *London*, you muſt ſeek the place given in the Catalogue, and the difference of time there found, contrary to the Title is to be added or ſubtracted from the time there given.

Example. Suppoſe the place of the Sun had been at *Uraniburg*, at 6 Hours 50'. *P. M.* and I would reduce the ſame to the Meridian of *London*; againſt *Uraniburg* as before I find 50' *A.* therefore contrary to the Title I Subtract 50' and the remainder 6 Hours is the time of the Suns place in the Meridian of *London*.

CHAP. IX.

Of the Theory of the Sun's or Earth's Motion.

IN the firſt part of this Treatiſe we have ſpoken of the primary Motion of the Planets and Stars, as they are wheeled about in their diurnal motion from Eaſt to Weſt, but here we are to ſhew their own proper motions in their ſeveral Orbs from Weſt to Eaſt, which we call their ſecond motions.

1. And theſe Orbs are ſuppoſed to be Elliptical, as the ingenious *Repler*, by the help of *Tycho's* accurate obſervations, hath demonſtrated in the Motions of *Mars* and *Mercury*, and may therefore be conceived to be the Figure in which the reſt do move.

2. Here then we are to conſider what an *Ellipſis* is, how it may be drawn, and by what Me-

thod the motions of the Planets according to that Figure may be computed.

3. What an *Ellipſis* is *Apollonius Pergæus in Conicis*, *Claudius Mydorgius* and others have well defined and explained, but here I think it ſufficient to tell the Reader, that it is a long Circle, or a circular Line drawn within or without a long Square; or a circular Line drawn between two Circles of different Diameters.

4. The uſual and Mechanical way of drawing this *Ellipſis* is thus; firſt draw a line to that length which you would have the greateſt Diameter to be, as the Line *AP* in Figure 8, and from the middle of this Line at *X*, ſet off with your compaſſes the Equal diſtance *XM* and *XH*.

5. Then take a piece of thred of the ſame length with the Diameter *AP* and faſten one end thereof in the point *M* and the other in the point *H*, and with your Pen extend the thred thus faſtened to the point *A*, and from thence towards *P* keeping the thread ſtiff upon your Pen, draw a line from *A* by *B* to *P*, the line ſo drawn ſhall be half an *Ellipſis*, and in like manner you may draw the other half from *P* by *D* to *A*. In which becauſe the whole thred is equal to the Diameter *AP*. therefore the two Lines made by thred in drawing of the *Ellipſis*, muſt in every point of the ſaid *Ellipſis* be alſo equal to the ſame Diameter *AP*. They that deſire a demonſtration thereof geometrically, may conſult *Apollonius Pergæus*, *Claudius Mydorgius* or others, in their treatiſes of Conical Sections, this is ſufficient for our preſent purpoſe, and from the equality of theſe two Lines with the Diameter, a brief Method of calculation of the

Planets

Planets place in an *Ellipsis*, is thus Demonstrated by Dr. *Ward* now Bishop of *Salisbury*.

6. In this *Ellipsis H* denotes the place of the Suns Center, to which the true motion of the Planet is referred, *M* the other *Focus* whereunto the equal or middle motion is numbred, *A* the *Aphelion* where the Planet is fartheſt diſtant from the Sun and ſloweſt in motion, *P* the *Perihelion* where the Planet is neareſt the Sun and ſloweſt in motion. In the points *A* and *P* the Line of the mean and true motion do convene, and therefore in either of theſe places the Planet is from *P* in æquality, but in all other points the mean and true motion differ, and in *D* and *C* is the greateſt elliptick Æquation.

8. Now ſuppoſe the Planet in *B*, the line of the middle motion according to this Figure is *MB*, the line of the true motion *HB*. The mean Anomaly *AMB*. The Eliptick æquation or *Proſthaphæreſis MBH*, which in this Example ſubtracted from *AMB*, the remainer *AHB* is the true Anomaly. And here note that in the right lined Triangle *MBH*, the ſide *MH* is always the ſame, being the diſtance of the *Foci*, the other two ſides *MB* and *HB* are together equal to *AP*. Now then if you continue the ſide *MB* till *BE* be equal to *BH* and draw the line *HE*, in the right lined Triangle *MEH*, we have given *ME=AD* and *MH* with the Angle *EMH*, to find the Angles *MEH* and *MHE* which in this caſe are equal, becauſe *EB=BH* by Contraction, and therefore the double of *BEH* or *BHE=MBH*, which is the Angle required.

And that which yet remaineth to be done, is the

the finding the place of the *Aphelion*, the true Excentricity or distance of the umbilique points, and the stating of the Planets middle motion.

CHAP. X.

Of the finding of the Suns Apogeon, *quantity of Excentricity and middle motion.*

THe place of the Suns *Apogeon* and quantity of Excentricity may from the obſervations of our countrey man Mr. *Edward Wright* be obtained in this manner, in the years 1596, and 1497, the Suns entrance into ♈ and ♎ and into the midſt of ♉. ♌. ♏ and ♒ were as in the Table following expreſſed.

	1596 D. H. M.	1597 D. H. M.	
January.	25.00.07	24.05.54	♒ 15
March.	9.18.43	10.00.37	♈ 0
April.	24.21.47	25.03.54	♉ 15
July.	28.01.43	28.09.56	♌ 15
September.	12.13.48	12.19.15	♎ 0
October.	27.15.23	27.21.50	♏ 15

And hence the Suns continuance in the Northern Semicircle from ♈ to ♎ in the year 1596 being Leap year, was thus found.

From

	d.	h.	′
From the 1. of *January* to ☉ Entrance ♎.	256.	13.	48.
From the 1. of *Jun* to ☉ Entrance ♈	69.	18.	43
Their difference.	186.	19.	05

In the year 1597 from the 1 of *January* to the time of the ☉ Entrance into ♎. 255.19.15
To the ☉ entrance into ♈. 69.09.37
Their difference is 186.18.38

And the difference of the Suns continuance in these Arks in the year 1596 and 1597 is 27′. and therefore the mean time of his continuance in those Arks is days 186. hours 18. minutes 51. seconds 30. And by consequence his continuance in the Southern Semicircle that is from ♎ to ♈ is 178 days. 11 hours, 8 minutes and 30 seconds.

In like manner in the year 1596 between his entrance into ♉ 15. and ♏ 15, there are days 185.17.36
And in the year 1597 there are days 185.17.56

And to find the middle motion answering to days 186. hours 18. Minutes 51. seconds 30 I say.

As 365 days, 6 hours, the length of the Julian, year is to 360, the degrees in a Circle.

So is 186 days, 18 hours, 51′. 30″ to 184 degrees. 03′. 56″.

In like manner the mean motion answering to

to 185 days, 17$^{h.}$ 46′ is 183 degrees, 02′. 09.″

Apparent motion from ♈ to ♎	180. 00. 00
Middle motion	184. 03. 56
Their Sum	364. 03. 56
Half Sum is the Arch. *SME*	182, 01, 58

In 1596 from 15 ♎ to 15 ♑ there are days 185, hours 01, minutes 36. In 1597. days 135. hours 4. 02′.

And the mean motion anſwering thereunto is. 182$^{d.}$ 30′. 36″.

Apparent motion from 15 ♉ to 15 ♏.	180.
Middle motion 185. 17. 56.	181. 04. 53
Half Sum is	183. 32. 26

From 15 ♎ to 15 ♑. Days. 185. 04$^{h.}$ 02′

Apparent motion	180.
Middle motion	182. 30. 36
Half Sum	181. 15. 18

Now then in *Fig.* from *PGC.* 181. 32. 26 deduct *NKD* 180, the Remainer is *DC* + *NP.* 1. 32. 26. Therefore *DC* or *NP.* 46. 13, whoſe Sine is *HA.*

And from *XPG.* 181. 15. 18 deduct *TNK* 180, the Remainer is *KG* + *TX* 1. 15. 18. Therefore *KG* or *TX* 37. 39, whoſe Sine is *HR.*

Now

Now then to find the Apogæon.

As *HA* 46'. 13"		5. 12851105
To *Rad.* So *HR* 37'. 39"		15. 03948202
To *Tang. HAR.* 39d. 10'. 04"		9. 91097097
GAM.	45	
Apogæon	95. 49. 56.	

Hence to find the excentricity AR.

As the Sine *HAR.*	39. 10. 04	9. 80043756
To *Rad.* So *HR.*	37. 39	15. 03948202
To *RA.*	1733. 99	5. 23904446

Or thus,

In the Triangle *ARγ* we have given *Aγ.* and *Rγ.*

As *Aγ.*	37. 39	5. 03948202
To *Rad.* So *Rγ.*	46. 13	15. 12851105
To *Tang. RAγ.*	50. 49. 56	10. 08902903
PAS.	45.	

Apogæon 95 *deg.* 49'. 56". as before.

Then for the Excentricity RA.

As the Sine of *RAγ.*	50. 49. 56	9. 88945938
Is to *Rγ.*	46'. 13"	5. 12851105
So is *Radius.* To *RA.*	1734. 01	5. 23905167

And this agreeth with the excentricity, used by

by Mr. *Street* in his *Astron. Carolina*, Pag. 23. But Mr. *Wing* as well by observation in former ages, as our own, in his *Astron. Instaur.* Pag. 39. doth find it to be 1788 or 1791. The work by both observations as followeth.

2. And first in the time of *Ptolemy*, *Annō Christi* 139 by comparing many observations together, he sets down for the measure neareſt truth, the interval between the vernal Equinox and the Tropick of Cancer to be days 93. hours 23. and minutes 03. And from the Vernal to the Autumnal Equinox, days 186. hours 13. and minutes 5.

	D.
The apparent motion from ♈ to ♋	90.36.00
Middle motion for 93$^{d.}$ 23$^{h.}$ 3′. is	92.36.42
The half Sum is *GP*	91.18.21
Apparent motion from ♈ to ♎	180.00.00
Middle motion for 186$^{d.}$ 13$^{h.}$ 5′. is	183.52.03
The half Sum is *GEK*	181.56.01
The half of *GEK* is *GE*.	90.58.01
And *GP* leſs *GE* is	00.20.20
Whoſe Sum is *AC* 59146.	

Again from *GEK* 181. 56. 02. deduct the Semicircle *FED* 180. the remainer is the ſumm *DK* and *FG*. 1. 56. 2. and therefore *DK*=*FG*. 58′. 01″. whoſe ſign is *BC*. 168753. *L* is the place of the *Aphelion*, and *AB* the Excentricity.

Now then in the Triangle *ABC*. in the Fig. 6 we have given the two ſides *AC* and *BC*. To find the Angle *BAC* and the Hypotenuſe *AB*.

For

For which the proportions are.

As the side *AC.* 59146 — 4.771921538
Is to the Radius. — 10.0000000
So is the side *BC.* 168755 — 5.227256653
To Tang. *BAC.* 70. 41. 10. — 10.45533127

Secondly for AB.

As the Sine of *BAC.* 70. 41. 10. — 9.974843527
Is to the side *AB.* 168755. — 5.227256653
So is the Radius. — 10.0000000
To the Hypot. *AB.* 1788. 10. — 5.252413133

Therefore the *Aphelion* at that time was in ♊ 12. 41. 10. And the excentricity. 1788.

3. Again *Anno Christi* 1652 the Suns place by observation was found to be as followeth.

April.	24. hours.	10.	}	♉.	15
October.	27. hours.	7. 10′	}	♍.	15
January.	24. hours.	11.20′	}	♎.	15
July.	27. hours.	16.30,	}	♌.	15

Hence it appeareth that the Sun is running through one Semicircle of the Ecliptick, that is from ♉ 15 to ♍ 15. 185 days 21 hours and 10′. And through the other Semicircle from ♎ 15 to ♌ 15, days 184. hours 5. therefore the Suns mean motion, according to the practice in the last example, from ♉ 15 to ♍ 15 is 181. 30.26. and from ♎ 15 to ♌ 15. 181. 16. 30.

Now then in Fig. 7. if we subtract the semicircle of the Orb *KMH.* 180. from *WPV* 181. 36. 26. the remainer is the sum of *KW* and *HV*

1-36

1.36.26. the Sine of half thereof 48'. 13" is equal to *AC*. 140252.

Again the mean motion of the Sun in his Orb from ♎ 15 to ♉ 15 is the Arch *SKP*. 181. 16. 30. whose excess above the Semicircle being bisected is 38. 15. whose Sine *CB*. 111345. now then in the Triangle *ABC* to find the Angle *BAC*, the proportion is.

As the side *AC*. 140252	5.14690906
Is to the *Radius*.	10.00000000
So is the Side *CB* 111345	5.04667072
To Tang. *BAC*. 38. 36. 21,	9.89966166

Which being deducted out of the Angle. ☊ *AS*. 45 it leaveth the Angle ☊ *AL* 6. 33. 39. the place of the ☉ *Aphelion* sought, and this is the quantity which we retain.

And for the excentricity BC.

As the Sum of *BAC*. 38.26.21	9.79356702
Is to the *Radius*.	10.00000000
So is the side *BC* 111345	5.04667072
To the Hypot. *AB*. 179103	5.25310370

So then *Anno Christi*. 1652. *Aphel.*	96.33.39
Anno Christi. 139. the *Aphelion*	70.41.10
Their difference is	25.52.29

And the difference of time is 1513 Julian years.

Hence to find the motion of the *Aphelion* for 2. years, say I, if 1513 years give 25.52.29, what shall one year give, and the answer is 00d. 01' 01",

$01^{h}.33'''.56^{iv}.44^{v}$. that is in Decimal numbers. 0.00475. 04447. 0555.

And the motion for. 1651 years. 7. 84298. 42c8862, which being deducted from the place of the *Aphelion Anno Christi.* 1652——26.82245. 3703703. The remainer, *viz.* 18. 97946. 9494841 is the place thereof in the beginning of the Christian *Æra*, which being reduced is, 68 deg. 19. min. 33. sec. 56. thirds.

4. The Earths middle motion, *Aphelion* and Excentricity being thus found, we will now shew how the same may be stated to any particular time desired, and this must be done by help of the Sun or Earths place taken by observation. In the 178 year then from the death of *Alexander*, *Mechir* the 27 at 11 hours *P. M. Hipparcus* found in the Meridian of *Alexand.* that the Sun entered ♈ 0. the which Vernal Equinox happened in the Meridian of London according to Mr. *Wings* computation at 9 hours 14', and the Suns *Aphelion* then may thus be found.

The motion of the *Aphelion* for one year, was before found to be. 0.00475.04447.0555. therefore the motion thereof for one day is 0.00001. 501491722. The Christian *Æra* began in the 4713 year compleat of the Julian Period, in which there are days 1721423. The *Æra Alexandri* began *November* the twelfth, in the year 4390 of the Julian Period, in which there are 1603397 days. And from the death of *Alexander* to the 27 of *Mechir* 178, there are days 64781, therefore from the beginning of the Julian Period, to the 178 year of the *Æra Alexandri*, there are days 1668178 which being deducted from the days in the Christian *Æra*.

1721423, the remainer is 53245, the number of days between the 178 year after the death of *Alexander*, *Mechir* 27, and the beginning of the Christian *Æra*.

Or thus. From the *Æra Alexandri* to the *Æra Christi* there are 323 Julian years, and 51 days, that is 118026 days. And from the *Æra Alexandri* to the time of the observation, there are 64781 days, which being deducted from the former, the remainer is 53245 as before. Now then if you multiply the motion of the *Aphelion* for one day, *viz.* 0.00001. 3014917 by 53245, the product is 0.69297.925665, which being deducted from the place of the *Aphelion* in the beginning of the Christian *Æra*, before found. 18.97046. 9494841. the remainer 18. 28649. 0239176 is the place of the *Aphelion* at the time of the observation, that is in Sexagenary numbers. deg. 65. 49. 53".

5. The place of the *Aphelion* at the time of the observation being thus found to be deg. 65. 49'. 53". The Suns mean Longitude at that time, may be thus computed.

In Fig. 8. In the Triangle *EMH* we have given the side *ME* 200000, the side *MH* 3576, the double excentricity before found, and the Angle *EMH* 114. 10'. 07". the complement of the *Aphelion* to a Semicircle, to find the Angle *MEH*, for which the proportion is,

As the Summ of the sides, is to the difference of the sides, so is the Tangent of the half Summ of the opposite Angles, to the Tangent of half their difference.

The side *ME*. 200000.

The side *MH* 3576.

Z. Of the sides. 203576. *Co. ar.*	4.69127343
X. Of the sides. 196424.	5.29321855
Tang. ½ *Z* Angles. 32′.54′.58.	9.91111512
Tang. ½ *X* Angles. 31.59.21.	
Angle *MEH.* 0.55.35.	9.79560710

The double whereof is the Angle *MBH* 1.51.10. which being Subtracted from 360 the remainer 358. 08. 50. is the estimate middle motion of the Sun, from which subtracting the *Aphelion* before found, 65. 49. 53. the remainer 292. 18. 57. is the mean Anomaly by which the absolute Æquation may be found according to the former operation.

Z.ME+*MH.* 203576. *Co. ar.*	4.69127343
X.ME−*MH.* 196424	5.29321855
Tang. ½ Anom. 56. 09. 28.	10.17359517
Tang. ½ *X.* 55. 12. 18.	10.15808715
Differ. 00. 57. 10.	

Doubled 1. 54. 20, which added to the middle motion before found gives the ☉ true place ♈. 00. 3′. 10″, which exceeds the observation 3′. 10″. therefore I deduct the same from the middle motion before found, and the remainer 358. 05. 50. is the middle motion at the time of the observation of *Hipparchus*, to which if you add the middle motion of the Sun for 53245 days, or for 323 Ægyptian years 131 days, 280. 46. 08′ the Summ, rejecting the whole Circles, is 278. 51. 48 the Suns mean Longitude in the beginning of the Christian *Æra*.

6. But one observation is not sufficient, where-

by to ſtate the middle motion for any deſired *Epocha*, we will therefore examine the ſame by another obſervation made by *Albategnius* at *Aracta* in the year of Chriſt 882, *March*: 15. hours 22. 21. but in the Meridian of *London* at 18 hours. 58′.

The motion of the *Aphelion* for 881 years, 74 days is 3. 80606853737, which being added to the place thereof in the beginning of the Chriſtian *Æra*, the place at the time of the obſervation will be found to be 22.78553148578, that is reduced, Deg. 82. 01′. 40″. And hence the Æquation according to the former operations is Deg. 2. 01′. 16″ which being deducted from a whole Circle, the remainer 357d. 58′. 44″ is the eſtimate middle motion at that time, from which deducting the *Aphelion* deg. 82. 01. 40. the remainer 275. 57. 04 is the mean anomaly, and the Æquation anſwering thereto is deg. 2. 02′. 18″ which being added to the middle motion before found, gives the ☉ place ♈. 00. 01′. 02″ which exceeds the obſervation 01′. 02″. therefore deduct the ſame from the middle motion before found, the remainer 357. 57′. 22″ is the middle motion of the ☉ at the time of the obſervation, from which deducting the middle motion for 881 years, 74 days, 18 hours, 58 minutes, *viz.* 80d. 06′. 10″. the remainer 277 deg. 51′. 12″. is the ☉ mean Longitude in the beginning of the Chriſtian *Æra*.

By the firſt obſervation it is deg. 278.51′. 48″
By the ſecond 277.51.12
Their difference is 1.00.36

He

He that deſires the ſame to this or any other *Epocha*, to more exactneſs, muſt take the pains to compare the Collection thereof from ſundry Obſervations, with one another, this is ſufficient to ſhew how it is to be found. Here therefore I will only add the meaſures ſet down by ſome of our own Nation, and leave it to the Readers choice to make uſe of that which pleaſeth him beſt.

The ☉ mean Longitude in the beginning of the Chriſtian Æra according to.

Vincent Wing is	9.	8d.	00″.	31″
Tho. Street is	9.	7.	55.	56
John Flamſted is	9.	7.	54.	39
By our firſt Computation	9.	8.	51.	48
By our ſecond	9.	7.	51.	12

In the Enſuing Tables of the ☉ mean Longitude, we have made uſe of that meaſure given by Mr. *Flamſted*, a little pains will fit the Tables to any other meaſure.

CHAP. XI.

Of the quantity of the Tropical and Sydereal Year.

THe year Natural or Tropical (ſo called from the Greek word Τρέπω, (which ſignifies to turn) becauſe the year doth ſtill turn or return into it ſelf) is that part of time in which the ☉ doth finiſh his courſe in the Zodiack

by coming to the ſame point from whence it began.

2. That we may determine the true quantity thereof, we muſt firſt find the time of the ☉ Ingreſs into the Æquinoctial Points, about which there is no ſmall difference amongſt Aſtronomers, and therefore an abſolute exactneſs is not to be expected, it is well that we are arrived ſo near the Truth as we are. Leaving it therefore to the ſcrutiny of after Ages, to make and compare ſundry Obſervations of the ☉ entrance into the Æquinoctial Points, it ſhall ſuffice to ſhew here how the quantity of the Tropical year may be determined, from theſe following obſervations.

3. *Albategnius*, *Anno Chriſti* 882 obſerved the ☉ entrance into the Autumnal Æquinox at *Aracta* in *Syria* to be *Sept.* 19. 1 hour 15′ in the Morning. But according to Mr. *Wings* correction in his *Aſtron. Inſtaur.* Page 44, it was at 1 hour 43′ in the Morning, and therefore according to the ☉ middle motion, the mean time of this Autumnal Æquinox was *Sept.* 16. $12^{h.}$ 14′. 25″. that is at *London* at $8^{h.}$ 54′. 25″.

4. Again by ſundry obſervations made in the year 1650. the ſecond from Biſſextile as that of *Albategnius* was, the true time of the ☉ ingreſs into ♎ was found to be *Sept.* 12. $14^{h.}$ 40′. and therefore his ingreſs according to his middle motion was *Sept.* 10. $13^{h.}$ 02.

5. Now the interval of theſe two obſervations is the time of 768 years, in which ſpace by ſubtracting the leſſer from the greater, I find an anticipation of 5 days, 9 hours, 52′. 25″. which divided by 768 giveth in the quotient 19′.

10′. 55″. 39‴ which being subtracted for 365 days, 6 hours, the quantity of the Julian year, the true quantity of the Tropical year will be 365 days, 5 hours, 49′. 04″. 21‴. Others from other observations have found it somewhat less, our worthy countryman Mr. *Edward Wright* takes it to be 365d. 5 hours. 48′.

Mr. *John Flamsted*, 5h. 29′. Mr. *Tho. Street* 5h. 49′. 01″. taking therefore the Tropical year to consist of 365 days, 5 hours, 49 Minutes, the Suns mean motion for one day is 0 deg. 59′, 8″. 19‴. 43iv. 47v. 21vi. 29vii. 23viii. or in decimal Numbers, the whole Circle being divided into 100 degrees, the ☉ daily motion is 0. 27379. 08048. 11873.

6. The Sydereal or Starry year is found from the Solar by adding the Annual Motion of the eighth Orb or præcession of the Æquinoctial Points thereunto, that præcession being first converted into time.

7. Now the motion of the fixed Stars is found to be about 50″. in a years time, as Mr. *Wing* hath collected from the several observations of *Timocharis*, *Hipparchus*, *Tycho* and others; and to shew the manner of this Collection, I will mention onely two, one in the time of *Timocharis*, and another in the time of *Tycho*.

8. *Timocharis* then as *Ptolemy* hath it in his *Almagist*, sets down the Virgins Spike more northwardly than the Æquinoctial, 1 deg. 24′. the time of this observation is supposed to be about 291 years before Christ, the Latitude 1 deg. 59′ South, and therefore the place of the Star was in ♍. 21d. 59′. And by the observation of *Tycho* 1601 current, it was in ♎

18. 16′. and therefore the motion in one year 50″, which being divided by 365 days, 6 hours, the quotient is the motion thereof in a days time. 00′. 8‴. 12^{iv}. 48^{v}. 47^{vi}. 18^{vii}. 30^{viii}. 13^{ix}. and in decimal Numbers, the motion for a year is 00385. 80246. 91358. The motion for a day. 00001. 05626. 95938.

9. Now the time in which the Sun moveth 50″, is 20′. 17″. 28‴, therefore the length of the ſydereal year is 365 days, 6 hours, 9′. 17″. 28‴. And the Suns mean motion for a day 59′. 8‴. 19‴. 43^{iv}. 47^{v}. 21^{vi}. 29^{vii}. 23^{viii}. converted into time is 00. 03′. 56″. 33‴. 18^{iv}. 55^{v}. 9^{vi}. 23^{vii}. 57^{viii}. which being added to the Æquinoctial day, 24 hours, giveth the mean ſolar day, 24 hours. 3.′ 56″. 33‴. 18^{v}. 55. 9. 23. 57.

10. And the daily motion of the fixed Stars, being converted into time is 32^{iv}. 51^{v}. 15^{vi}. 9^{vii}. 14^{viii}. 24^{ix}. and therefore the Æquinoctial day being 24 hours, the ſydereal day is 24 hours, 00′. 00″. 00‴. 32^{iv}. 51. 15. 9. 14 24.

11. Hence to find the præceſſion of the Æquinoctial Points, or Longitude of any fixed Star, you muſt add or ſubtract the motion thereof, from the time of the obſervation, to the time given, to or from the place given by obſervation, and you have your deſire.

Example. The place of the firſt Star in Aries found by *Tycho* in the year 1601 current, was in ♈. 27^{d}. 37′. 00. and I would know the place thereof in the beginning of the Chriſtian *Æra*.

The

The motion of the fixed Stars for 1600 years,	$22^{d}\ 13'.20''$
Which being deducted from the place found by observ.	27. 37. 00
The remainer.	5.231.40

is the place thereof in the beginning of the Christian *Æra*.

12. Having thus found the ☉ middle motion, the motion of the *Aphelion* and fixed Stars, with their places, in the beginning of the Christian *Æra*; we will now set down the numbers here exhibited *Æra Christi*. Mr. *Wing* from the like observations, takes the ☉ motion to be as followeth.

The ☉ mean Longitude	9.8.00.31
Place of *Aphelion*	2.8.20.03
The Anomaly	c6.29.40.28

The which in decimal Numbers are

The ☉ mean Longitude	77.22460.86419
Place of the *Aphelion*	18.98171.29629
The Anomaly	58.24289.56790

The mean motions for one year.

The ☉ mean Longitude	99.93364.37563.34
The *Aphelion*	00.00475.04447.05
The ☉ mean Anomaly	99.92889.33116.29

The ☉ mean motions for one day.

The

The ☉ mean Longitude	00.27379.08048.11
The *Aphelion*	00.00001.30149.17
The mean Anomaly	00.17377.77898.94

And according to theſe meaſures are the Tables made ſhewing the ☉ mean Longitude and Anomaly, for Years, Months, Days and Hours.

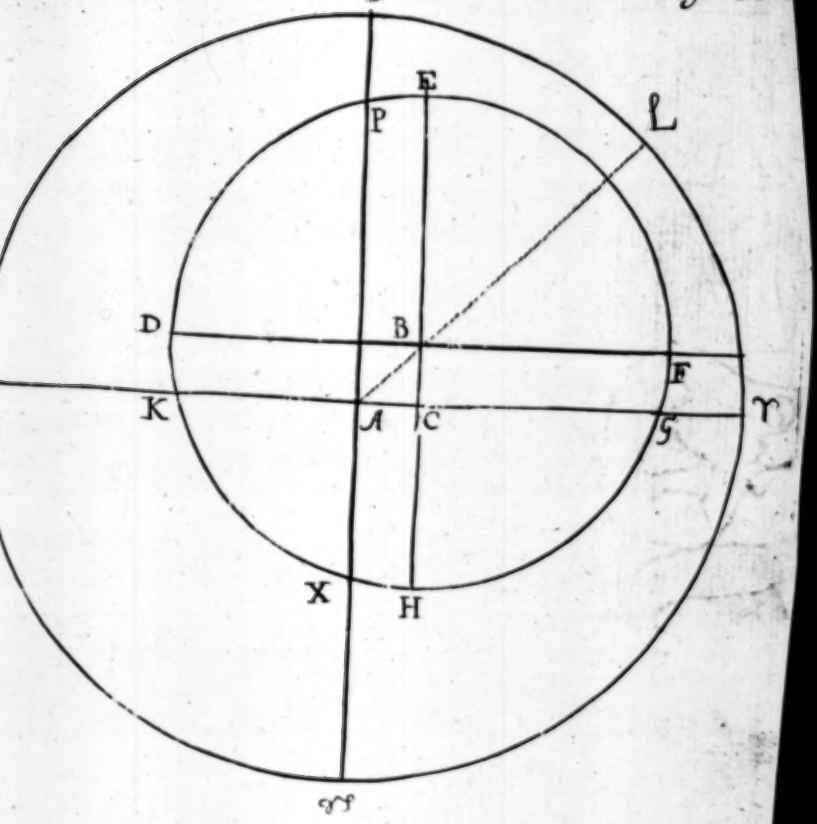
CHAP. XII.

The Suns mean motions otherwiſe ſtated.

SOme there are in our preſent age, that will not allow the *Aphelion* to have any motion, or alteration, but what proceeds from the motion of the fixed Stars, the which as hath been ſhewed, do move 50 ſeconds in a year, and hence the place of the firſt Star in Aries, in the beginning of the Chriſtian *Æra* was found to be ♈. 5.23^{d}.40.

Now then, if from the place of the *Aphelion Anno Chriſti*. 1652 as was ſhewed in the tenth Chapter, deg. 96. 33′. 39. we deduct the motion of the fixed Stars for that time. 28. 19. 12. the remainer 68. 14. 27 is the conſtant place of the *Aphelion*; but Mr. *Street* in his *Aſtronomia Carolina* Page 23, makes the conſtant place of the *Aphelion* to be 68^{d}. 20.00, and the ☉ excentricity 1732.

And from the obſervation of *Tycho* 1590 *March* the eleventh. in the Meridian of *Uraniburg*, but reduced to the Meridian of *London*. *March* the tenth, hour 23. 2′. He determines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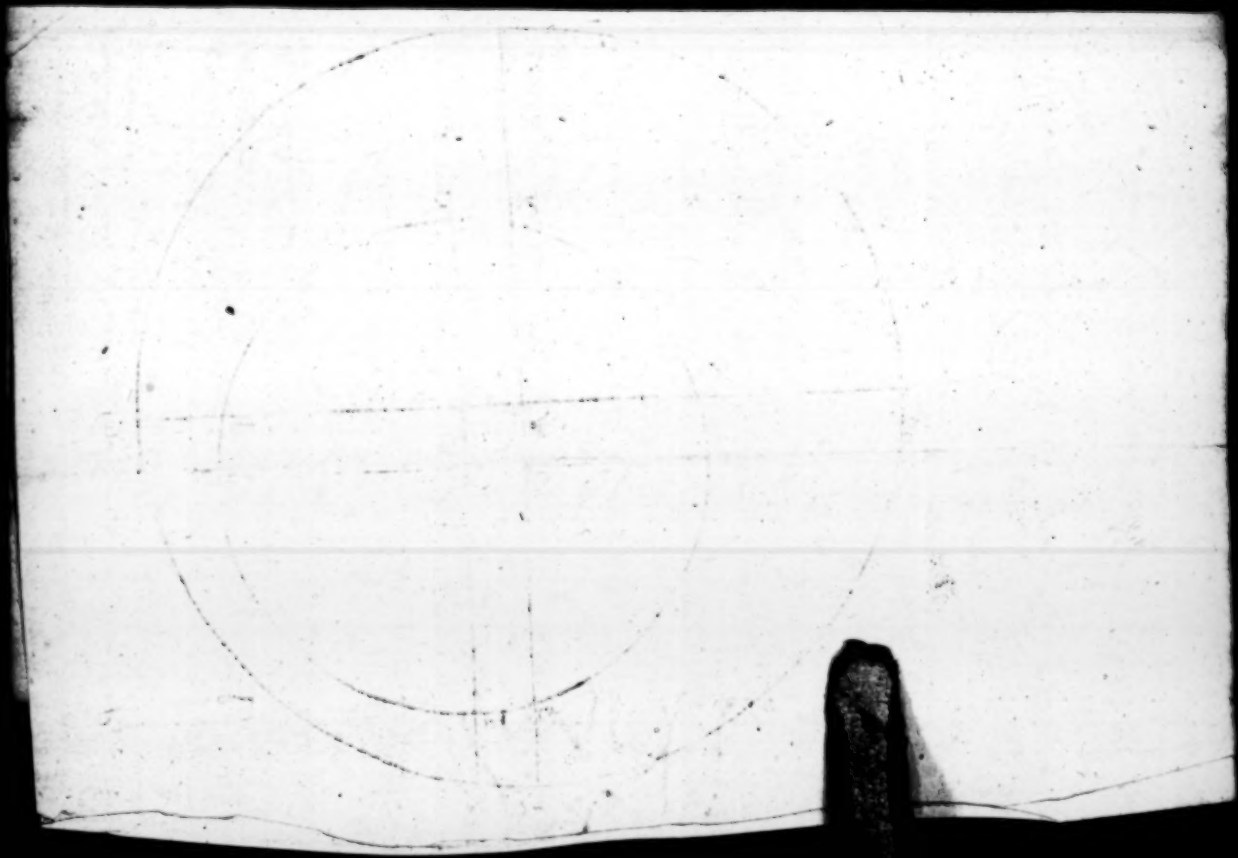

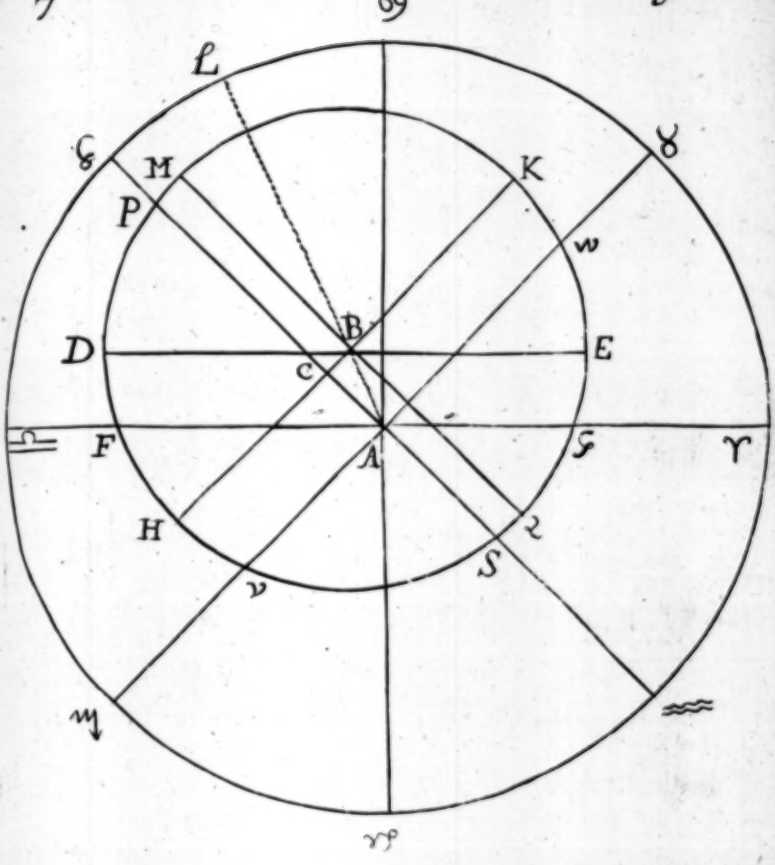
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determines the Earths mean Anomaly thus.

The place of the Sun observed	♈. 0. 33. 19
The præcession of Æquinox	0.27.27.22
The Earths Sydereal Longitude	5.03.05.57
The place of the *Aphelion* Subtract	8.08.20.00
The Earths true Anomaly	8.24.45.57
Æquation Subtract	1.58.47
The remainer is the Estimate M. Anom.	8.22.47.10
Æquation answering thereto add.	1.58.27
The Earths true Anomaly	8.24.45.37
The place of the *Aphelion*	8.08.20.00
Præcession of the Æquinox	0.27.27.22
Place of the Sun	♈.00.32.59
But the place by observation	♈.00.33.19
The difference is	001.001.20
Which being added to the mean Anom.	8.22.47.10
The mean Anomaly is	8.22.47.30
The absolute Æquation	1.58.27
The true Anomaly	8.24.45.57

Agreeing with observation.

And so the mean Anomaly *Æra Christi* is 6. 23. 19. 56. But Mr. *Flamsted* according to whose measure the ensuing Tables are composed, takes the mean Anomaly *Æra Christi.* to be 6. 24. 07. 091. The place of the *Aphelion* to be 8, 08. 23. 50. And so the Præcession of the Æquinox and *Aphelion* in the beginning of the

the Christian *Æra*. 8, 13. 47. 30. in decimal Numbers.

Æra Christi,

The Suns mean Anomaly	56.69976.85185
The Suns *Apogeon* and Præc. Æq.	20.49768.51851

The ⊙ mean motions for one Year.

The ⊙ mean Longitude	99.93364.37563.34
The Præcession of Æquin.	.00385.80246.91
The ⊙ mean Anomal.	59.92978.57316.43

The ⊙ mean Motions for one Day,

The ⊙ mean Longitude	00.27379.08048.11
The Præcession of Æqui.	00.00001.05699.30
The ⊙ mean Anom.	00.27378.02348.81

CHAP. XIII.

How to Calculate the Suns true place by either of the Tables of middle motion.

WRite out the *Epocha* next before the given time, and severally under that set the motions belonging to the years, months and days compleat, to the hours, scruples, current every one under his like (only remember that in the Bissextile years after the end of *February* the days must be increased by an unite) then adding all together, the sum shall be the ⊙ mean motion for the time given.

Example

Example.

Let the given time be *Anno Christi* 1672. *February* 23. hours 11. 34′. 54″. by the Tables of the ☉ mean Longitude and Anomaly, the numbers are as followeth.

		M. Longitud.	M. Anomal.
The *Epocha*	1660	80.67440.	53.79815
Years	11	99.81766.	99.76526
January		08.48751.	08.48711
Day.	23	06.29718.	06.29688
Hours	11	00.12548.	00.12548
	34	00.00646.	00.00646
	54	00.00017.	00.00017
		95.40886.	68.47951

By the Tables of the Suns mean Anomaly and præcession of the Æquinox, the numbers are these.

		Anomaly.	Præce. Æqui.
The *Epocha*	1660	53.76721.	26.90200
Years	11	99.77520.	00.04243
January		8.48718.	00032
Days	23	6.29694.	00024
Hours	11	0.12548.	26.94499
	34′	.00546.	68.45882
	54″	.00035.	95.40381

☉ mean Anomaly 68.45882

There

There is no great difference between the ☉ mean Longitude and Anomaly found by the Tables of mean Longitude and Anomaly, and that found by the Tables of mean Anomaly and Preceſſion of the Æquinox. The method of finding the Elliptical Æquation is the ſame in both, we will inſtance in the latter only, in which the ☉ mean Anomaly is Degrees 68. 45882. And the preceſſion of Æquin. deg. 26. 94499.

But becauſe there is no Canon of Sines and Tangents as yet publiſhed, ſuitable to this diviſion of the Circle into an 100 deg. or parts: We muſt firſt convert the ☉ mean Anomaly, and prec. of of the Æquin. given, into the degrees and parts of the common Circle: And this may be done either into degrees and decimal parts of a degree, or into deg. and minutes: if it were required to be done into degrees and minutes, the Table here exhibited for that purpoſe will ſerve the turn, but if it be required to be done into degrees and decimal parts, I judge the following method to be more convenient.

Multiply the degrees and parts given by 36, the Product, if you cut off one figure more towards the right hand than there are parts in the number given, ſhall be the degrees and parts of the common Circle.

Anomaly.	68.45882	Præc. Æquinox.	26.94499
	36		36
	41075292		16166994
	20537646		8083497
Anom.	246.451752	Præ. Æq.	97.001964

And

And if you multiply the parts of these Products, you will convert them into minutes.

Otherwise thus. Multiply the degrees and parts given by 6 continually, the second Product, if you cut off one figure more towards the right hand than are parts in the number given, shall be the degrees and parts of the common Circle. The third Product of the parts only shall give minutes, the fourth seconds, and so forward as far as you please. Example.

☉Mean Anom. 68.45882	Prec. Æq. 26.94499
6	6
41075292	16166994
246.451752	97.001964
6	6
27.10512	0.11784
6	6
6.3072	7.0704

And thus the mean Anom. is deg. 246.451742 or 27'.06. The Prec. Æq. 97.001964. or 00'.07".

Hence to find the Elliptical Æquation in degrees and decimal parts: In Fig. 8. we have given in the right lined plain Triangle *EMH*, the sides *ME*, and *MH*, and the Angle *EMH*, 66.451742. the excess of the mean Anomaly above a Semicircle, to find the Angle *MEH*.

The

The side *ME* 200000
The side *MH* 3468

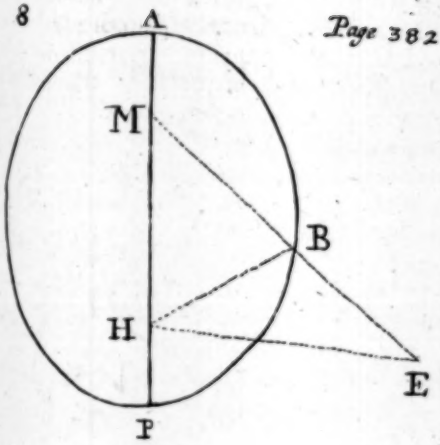
Z cru.	203468	*Co. ar.*	4.69150389
X cru.	196532		5.29343327
$t\frac{1}{2}Z$ angle.	56.774129		10.18374097
$t\frac{1}{2}X$ angle.	55.857087		10.16867813

MEH. 0.917042 the double whereof is the Angle *MBH.* 1.834084 or Elliptick Æquation sought, which being added to the mean Anomaly and præcession of the Æquinox, because the Anomaly is more than a Semicircle, the same is the Suns true place.

The ☉ mean Anomaly	246.451742
The Præcession of the Æquinox	97.001964
Elliptick Æquation	1.834084
The Suns true place.	345.287790

But because the Elliptick Æquation thus found doth not so exactly agree to observation as is desired, *Bullialdus* in Chap. 3. of his Book entituled *Astronomiæ Philolaicæ fundamenta clarius explicata*, Printed at *Paris*, 1657. shews how to correct the same by an Angle applied to the *Focus* of middle motion, subtended by the part of the ordinate line, intercepted between the *Ellipsis* and the Circle circumscribing it. This Mr. *Street* maketh use of in his *Astronomia Carolina*, and this I thought not amiss to add here.

In Fig. 9. let *ABCPDF* be supposed an *Ellipsis*, and the Circle *AGPK* described upon the extremes of the transverse Diameter, and the Ordinates *KN* and *OB* extended to *G* and *M* in the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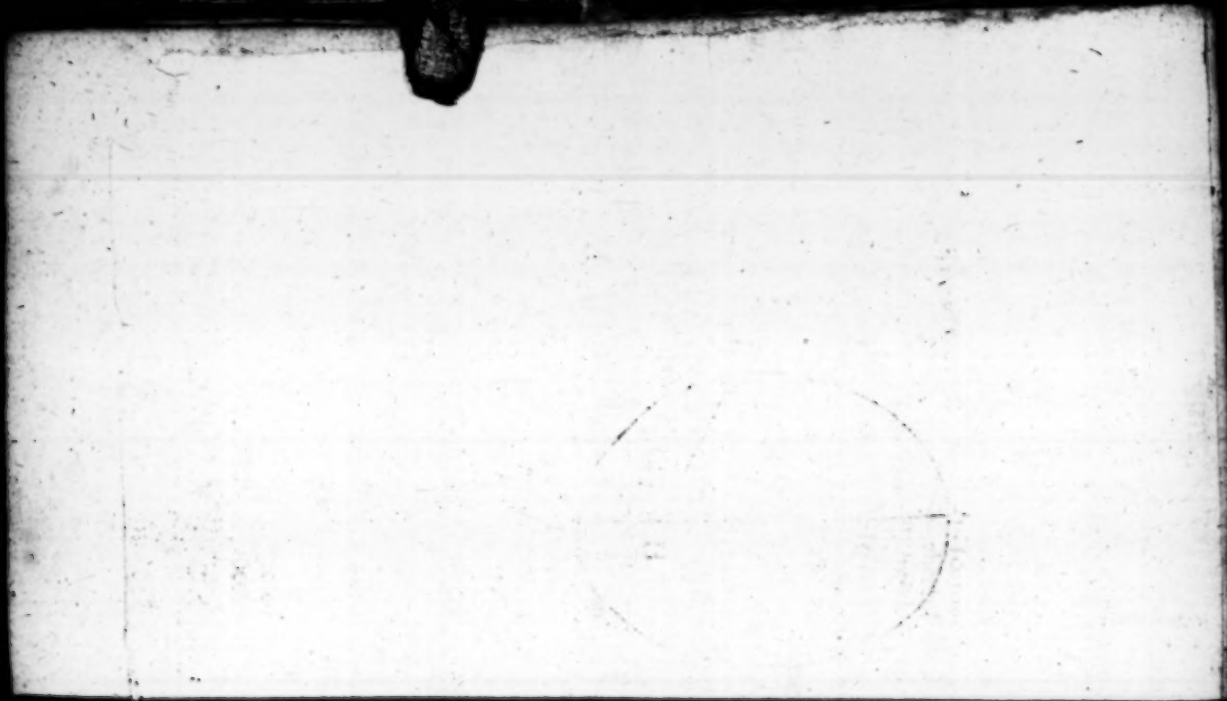


the Periphery of the Circle: then by the 21 of the first of *Apollonius*.

XN.GX::*OB tang. OEB.OM tang. OEM.*

And the Angle *OEM*—*OEB*=*BEM*=*ETT*, the variation to be deducted from the Elliptick Æquation *ETH*, the Remainer is the absolute Æquation *TTS* in the first Quadrant.

In the second and third Quadrants, the variation or difference between the mean and corrected Anomaly, must be added to the Elliptick Æquation, to find the true and absolute Æquation.

For *XN.XG.QV. tang. QEV.* the *comp. m. Anom. QR. t. QER.* and the Angle *VER*=*ECO* is the variation, and *ECO*+*ECH*=*OCH* is the absolute Æquation sought in the second Quadrant.

Again, *XN.XG*::*a D, tang. aED. a b, tang. aEB.* And *aEB*—*aED*=*DEf* the variation=*EFO* and *Ef0*+*EfH*=*0fH* the absolute Æquation sought in the third Quadrant.

Lastly, in the fourth Quadrant of mean Anomaly it is.

XN.XG::*ch. tang. cEH. eg. tang. cFg.* and *hEg* is the variation: And *EFH*—*EFγ*=*YFγ* the absolute Æquation sought in the fourth Quadrant.

And to find *XN* the conjugate Semi-diameter, in the right angled Triangle *ENX*, we have given, *EN*=*AX* and *EX* the semi-distance of the umbilick points. And Mr. *Briggs* in Chap. 19. of his Arithm. Logar. hath shewed, that the half Sum of the Logarith. of the sum and difference of the Hypotenuse, and the given leg. shall be the Logarith. of the other leg.

Now then $EN = AX$. 100000
The Leg EX. 1734

Their Sum	101734	5.00745001
Their difference	98266	4.99240328
The Z of the Logarithms,		9.99985329
½ Z. Logarith. XN. 99983		4.99992664

Now then in the former Example the mean Anomaly is 246 deg. 451741. and the excess above a semicircle is the ang. aED. 66.451742. Therefore.

As XN. 99983		4.99992664
Is to XG. 100000		5.00000000
So is the tang. aED 66.451742		10.36069857
To the tang. aEB 66.455296		10.36077193

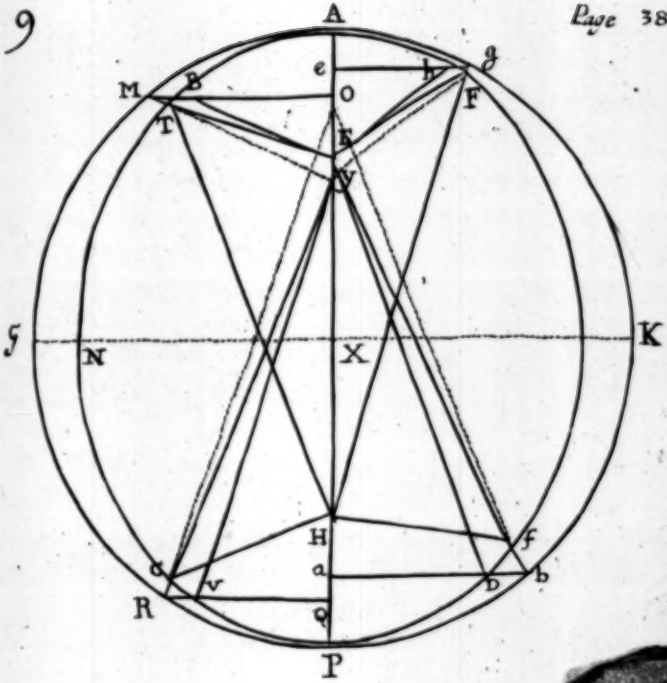
$aEB - aED = DEf$ .003544 the variation, which being added to the Elliptick Æquation before found, the absolute Æquation is 1.837628. and therefore the ☉ true place 345.291334. that is X. 15. 17. 28.

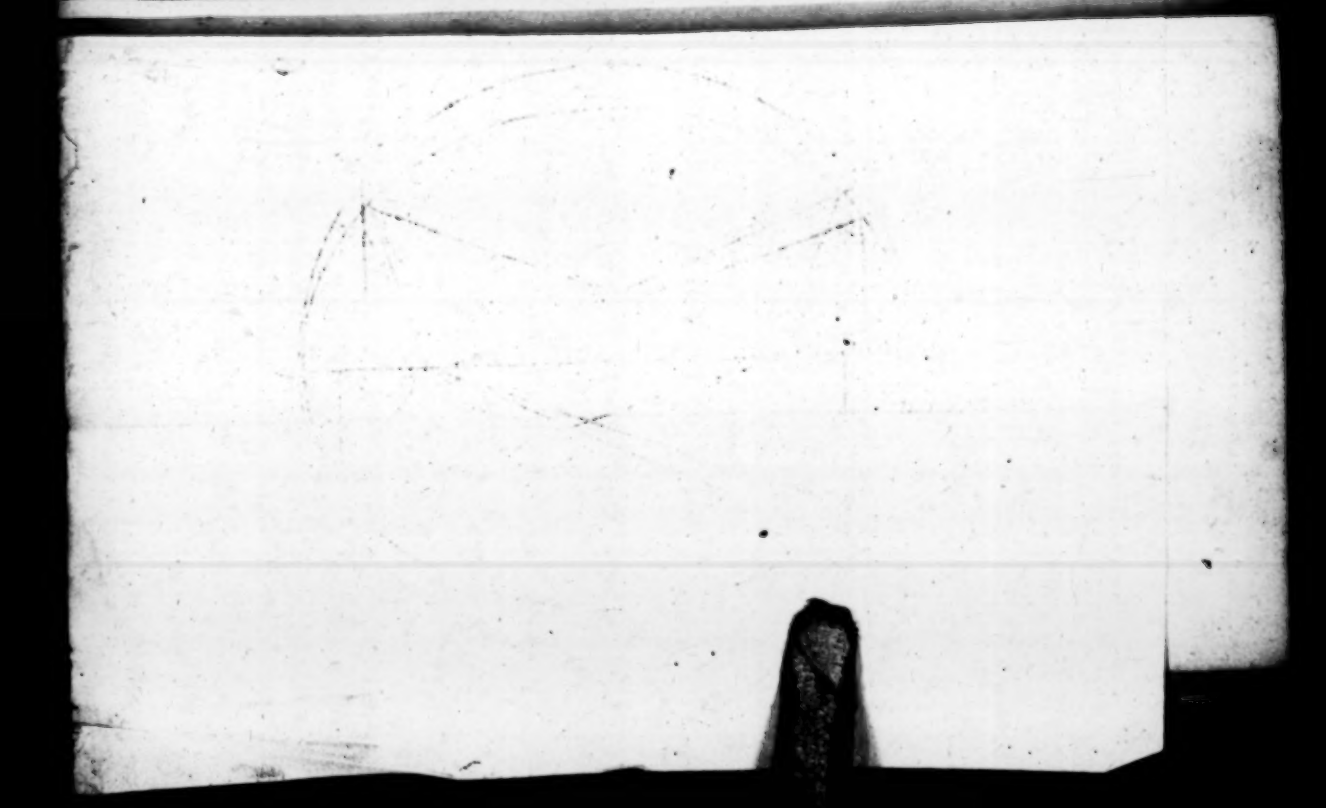
CHAP. XIV.

To find the place of the fixed Stars.

THe annual motion of the fixed Stars is, as hath been shewed, 50 Seconds, hence to find their places at any time assigned, we have exhibited a Table of the Longitude and Latitude of some of the most fixed Stars, from the Catalogue:

9





logue of noble *Tycho* for the year of our Lord 1600 compleat. Now then the motion of the fixed Stars according to our Tables being computed, for the difference of time between 1600 and the time propounded, and subtracted from the place in the Table, when the time given is before 1600, or added to it, when the time given is after; the Summ or difference shall be the place desired. The Latitude and Magnitude are still the same.

Example. Let the given time be 1500, the difference of time is 100 years, and the motion of the fixed Stars for 100 years is 0.38580.

The place of the 1 * in ♈, 1600	7.67129
Motion for 100 years subtract	0.38580
Place required in the year 1500	7.28549

2. *Example.*

Let the time given be	1674.
The place of the first Star in ♈ 1600 was	7.67129
Motion for 60 years is	0.23148
Motion for 14 years is	0.05401
Place required in the year 1674 compl.	7.95678

CHAP. XV.

Of the Theory of the Moon, and the finding the place of her Apogæon, *quantity of excentricity and middle motion.*

THe Moon is a secondary Planet, moving about the Earth, as the Earth and other

Planets do about the Sun, and ſo not only the Earth but the whole Syſtem of the Moon, is alſo carried about the Sun in a year. And hence, according to *Hipparchus*, there ariſes a twofold, but according to *Tycho* a three-fold Inequality in the Moons Motion. The firſt is Periodical and is to be obtained after the ſame manner, as was the excentrick Æquation of the Sun or Earth: in order whereunto, we will firſt ſhew how the place of her *Apogæon* and excentricity may be found.

At *Bononia* in *Italy*, whoſe Longitude is 13 degrees Eaſtward from the Meridian of *London*, *Ricciolus* and others obſerved the apparent times of the middle of three lunar Eclipſes to be as followeth.

The firſt 1642. *April* the 4. at 14 hours and 4 Minutes.
The ſecond 1642, *September* 27 at 16 hours and 46 minutes.
The third 1643. *September* 17 at 7 hours and 31 Minutes.

The equal times reduced to the Meridian of *London*, with the places of the Sun in theſe three obſervations, according to Mr. *Street* in the 25 Page of his *Aſtronomia Carolina*, are thus.

Anno Menſ. D.	h. ′		d.
1642. *April* 4.	13. 37	}	♈. 25. 6. 54
1642. *Septemb.* 27.	15. 57	}	♎. 14. 50. 09
1643. *Septemb.* 17.	6. 46	}	♎. 4. 20. 20

Hence the place of the Moon in the firſt obſervation,

vation is in ♎ 25. 6'. 54. in the second ♈ 14. 50. 9. in the third ♐ 4. 20. 20. Now then in Fig. 10. let the Circle *BHDGFE* denote the Moons Æquant *T* the Center of the Earth, the Semidiameters *TD*, *TE* and *TF* the apparent places of the Moon, in the first, second and third observations, *C* the Center of the Excentrick, *CD*, *CE* and *CF* the Lines of middle motion.

From the first observation to the second there are	176d. 2h. 20'
The true motion of the Moon is deg.	169. 43. 15''
The motion of the *Apogæon* subtract	19. 37. 07
The motion of the true Anomaly is the arch *DE*	150. 06. 08
The motion of the mean Anomaly *DCE*	140. 42. 28
From the first observation to the third, there are	530d. 17h. 9.
The true motion of the Moon is degrees	159. 13. 26
The motion of the *Apogæon* subtract	159. 07. 32

The motion of the true Anomaly is the Arch *DF*	100. 05. 54
The motion of the mean Anomaly *DCF*	93. 46. 45
And deducting the Arch *DGF* from the Arch *DFE*, the remainer is the Arch *FE*	50. 00. 14
And deducting the Angle *DCF* from the Angle *DCE*, the remainer is the Angle *FCE*	46. 55. 43

Suppose 10. 00000000 the Logarithm of *DC*, continue *FC* to *H*, and with the other right Lines compleat the Diagram.

1. In the Triangle *DCH* we have given the Angle *DCH* 86. 13. 15. the complement of *DCF* 93. 46. 45 to a Semicircle. The Angle *DHC* 50. 02. 57. The half of the Arch *DF* and the side *CD* 1000000. To find *CH*.

As the Sine of *DHC* 50. 02. 57	9.88456640
To the Side *DC*, so the Sine of *HDC* 43. 43. 48.	19.83964197
To the Side *CA*	9.95507557

2 In the Triangle *HCE* we have given *CH* as before, the Angle *CHE* 25. 00. 07. The half of the Arch *FE*, the Angle *HCE* 133. 04. 17 the complement of *FCE*, and by consequence the Angle *CEH* 21. 55. 36 To find the Side *CE*.

As

As the Sine of *CEH* 21. 55. 36	9.57219707
To the Side *CH*	19.95507557
So is the Sine of *CHE* 25. 00. 07	9.62597986
	19.58105543
To the Sine *CE*	10.00885836

3. In the Triangle *DCE*, we have given *DC*. *CE* and the Angle *DCE* 140. 42. 28. whose complement 39. 17. 32 is the Summ of the Angles, to find the Angle *CED* and *DE*,

As the greater Side *CE*	10.00885836
Is to the lesser Side *DC*	10.00000000
So is the *Radius*	10.00000000
To the tang. of 44. 24. 54	19.99114164

Which subtracted from 45. 2
the remainer is the half,
Difference of the acute angles 35.16.

As the Radius.

To the tang. of the com. 35. 16	8.01109962
Is to the tang. of the $\frac{1}{2}$ *Z*. 19.38.46	9.55265735
To the tang. of $\frac{1}{2}$ *X*. 00. 12. 35	7.56375697
Their Sum 19. 51. 21. is the angle — *CDE*.	
Their difference 19. 26. 11. is the angle *CED*.	

As the Sine of *CED*. 19. 26. 11.	9.52216126
Is to the Sine of *DCE*. 140. 42. 28.	9.80159290
So is the Side *EC*.	10.00000000
To the Side *DE*.	10.27943164

4. In the Isosceles Triangle *DTE* we have given the Side *DE*, the angle *DTE* 150. 06. 08 whose complement 29. 53. 52 is the Summ of the other two angles, the half whereof is the angle *TDE* 14. 56. 56 which being subtracted from the angle *CDE*. 19. 51. 21 the remainer is the angle *CDT*. 4. 54. 25.

As the Sine of *DTE* 150. 06. 08 *Co. ar.*	0.30237482
Is to the Sine of *DET*. 14. 56. 56	9.41154778
So is the Side *DE*	10.27943164
To the Side *DT*	9.99335424

5. In the Triangle *CDT* we have given *DC*. *DT* and the angle *CDT*, to find *CTD* and *CT*.

As the Side *DT*	9.99335424
Is to the Side *DC*	10.00000000
So is the *Rad.*	10.00000000
To the tang. of 26. 18	10.00664576

Deduct 45.
As the *Radius*.

Is

Is to the Sine of the remainer 0. 26. 18.	7.883686 72
So is the tang. of the ½ *Z* angle 87. 32. 57	11.368 54996
To the tang. ½ *X* angle 10.08.04	9.25223668
Their Summ 97.41.01	is the angle *CTD*

As the Sine of *CTD.* 97.41.01. *Co. ar.*	0.00391693
Is to the Side *DC*	10.00000000
So is the Sine of *CDT* 4. 54. 25	8.93215746
To the Side *CT*	8.93607439

	s. d.
The place of the Moon in the firſt Obſervation	6.25.06.54
The true Anomaly *CTD* ſub.	3.07.41.01
The place of the *Apogæon*	3.17.25.53
☾ place in the firſt Obſervation	6.25.06.54
The Æquation *CDT* Add.	04.54.25
The ☾ mean Longitude	7.00.01.19
From which ſubtract the place of the *Apogeon*	3.27.25.53
There reſts the mean Anomaly *BCD*	3.12.35.26

And for the excentricity in ſuch parts, as the *Radius* of the Æquant is 100000 the Proportion is.

DT	9.99335424
CT	8.93607439
100000	5.00000000
8764	3.94272015

And this is the Method for finding the place of the Moons *Apogæon* and excentricity. And from these and many other Eclipses as well Solar as Lunar, Mr. *Street* limits the place of the ☾ *Apogæon* to be at the time of the first observation 21′. 04″ more, and the mean Anomaly 20. 41″ less, and the excentricity 8765 such parts as the *Radius* of the Æquant is 100000.

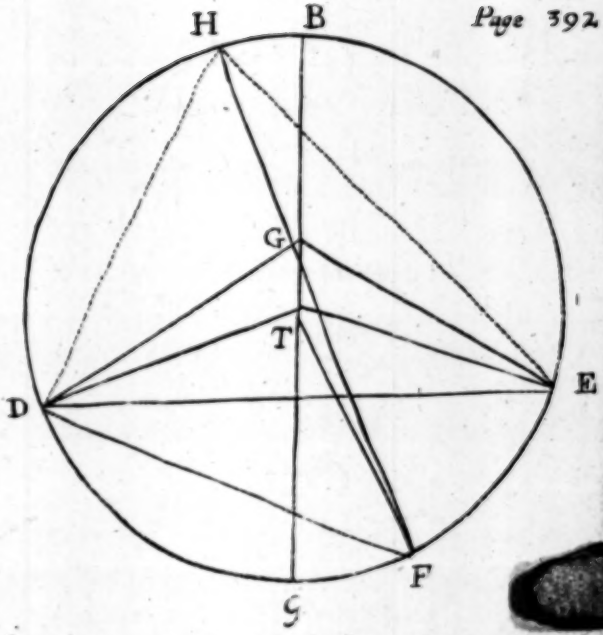
And by comparing sundry observations both antient and modern, he collects the middle motion of the Moon, from her *Apogæon*, to be in the space of four Julian years or 146 days, 53 revolutions, 0 Signes, 7 degrees, 56 minutes, 45 Seconds. And the *Apogæon* from the Æquinox 5 Signes, 12 degrees, 46 minutes. And hence the daily motion of her mean Anomaly will be found to be $13^{d.}$ 03′. 53″. 57‴. $09^{iv.}$ $58^{v.}$ $46^{vi.}$ Of her *Apogæon* 0. 06. 41. 04. 03. 25. 33.

And according to these Measures, if you deduct the motion of the ☾ mean Anomaly for 1641 years *April*

4. hours 13. 37′, *viz.*	8. 22. 02. 00.
from	3. 121. 35. 26
The remainer is	6. 201. 33. 26

from which abating 20′. 41″ the ☾ mean Anom. *Æra Chr.* 6. 20. 12. 45.

In


H
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9

In like manner the motion of her *Apogæon* for the same time is 6.05.311.57
which being deducted from 3.17.25.57

The remainer is 9.11.55.56
To which if you add 21.04
The Sum 91.121.15200
is the place of the ☾ *Apogæum* in the beginning of the Christian *Æra*.

CHAP. XVI.

Of the finding of the place and motion of the Moons Nodes.

Anno Christi 1652, *March* 28, hour. 22. 16′, the Sun and Moon being in conjunction, Mr. *Street* in Page 33, computes the ☾ true place in the Meridian of *London* to be in ♈. 19. 14. 18 with latitude North 46′. 15″.

And *Anno Christi* 1654 *August* 1. hour. 21. 19′. 30″ was the middle of a Solar Eclipse at *London*. at which time the Moons true place was found to be in ♌ 18. 58′. 12″ with North Latitude 32′. 01″.

1654 *August* 1. 21. 19′. 30″ ☾ place ♌ 18. 58. 12
1652 *March* 28. 22. 16. 00 ☾ place ♈ 19. 14. 18

From the first Observation to the second there are 27 years, 4 months, 5 days, 23 hours 03′. 30″.

Mean

Mean motion of the Nodes in that time, deg.	45.19.41
The true motion of the ☾	119.43.54
Their Summ is in Fig. 11. The angle *DPB*	165.03.35

Therefore in the oblique angled Spherical Triangle *DPB* we have given *BP*. 89. 13. 45 the complement of the Moons Latitude in the first observation 2. *PD* 89. 27. 50 the complement of the Moons Latitude in the second observation, and the angle *DPB* 165. 03. 35, whose complement to a Semicircle is *DPF* 14. 56. 25. The angle *PBD* is required.

1. *Proportion.*

As the Cotangent of *PD* 89.27.50	9.97114485
Is to the *Radius*	10.00000000
So is the Cosine of *DPF* 14.56.25	9.98506483
To the tang. of *PF* 89.26.42	12.01191998
BP 89.13.45	

Their *Z* is *FPB* 178.40.27. whose complement Is the Arch *FG* 1.19.33.

2. *Proportion.*

As the Sine of *FP* 89.26.42. *Co. ar.*	0.00002037
Is to the Cotang. of *DPF* 14.56.25	10.57376158
So is the Sine of *FG* 1.19.33	8.36418419
To the Cotang. of *FGD* 85.02.56	8.93796614

FGD=*PBD* inquired.

And

And in the right angled Spherical Triangle $BA\mathfrak{S}$ right angled at A we have given AB 0.46'. 15" the Latitude in the first observation, and the Angle $AB\mathfrak{S} = PBD$ 85. 02. 56. to find $A\mathfrak{S}$ the Longitude of the Moon from the ascending Node.

As the Cot. of $AB\mathfrak{S}$ 85.02.56	8.937966 14
Is to the *Radius*	10.0000000
So is the Sine of AB 0.46'.15"	8.1288229
To the tang. of $A\mathfrak{S}$ 8.49.17	9.1908567

2. *To find the Angle* $A\mathfrak{S}B$.

As the tang. of AB 0.46.15	8.1288621 2
Is to the *Radius*	10.0000000
So is the Sine of $A\mathfrak{S}$ 8.49.17	9.1856971 8
To the Cotang. of $A\mathfrak{S}B$ 5.0.41	11.0568250 6

The angle of the ☾ orbite with the Ecliptick	
The first observed place of the ☾	♈. 19.14.18
$A\mathfrak{S}$ Subtract	8.49.17
There rests the true place of the $\mathfrak{S}$	♈. 10.25.01

The retrograde motion whereof in 4 Julian years or 2461 days, is by other observations found to be Sign 2. deg. 17. 22'. 06". and therefore the daily motion deg. 0. 03'. 10". 38"'. 11$^{iv.}$ 35$^{v.}$

And the motion thereof for 1651 years, *March* 28. h. 22. 16', *viz.* Sign 8. deg. 18. 26'. 58" being added to the place of the Node before found Sig. 0. 10. 25. 01. Their Sum is the place thereof in the beginning of the Christian *Æra* Sign 8. deg. 28. 51'. 59".

But

But the *Rudolphin* Tables as they are corrected by Mr. *Horrox* and reduced to the Meridian of *London*, do differ a little from these measures, for according to these Tables, the Moons mean motions are.

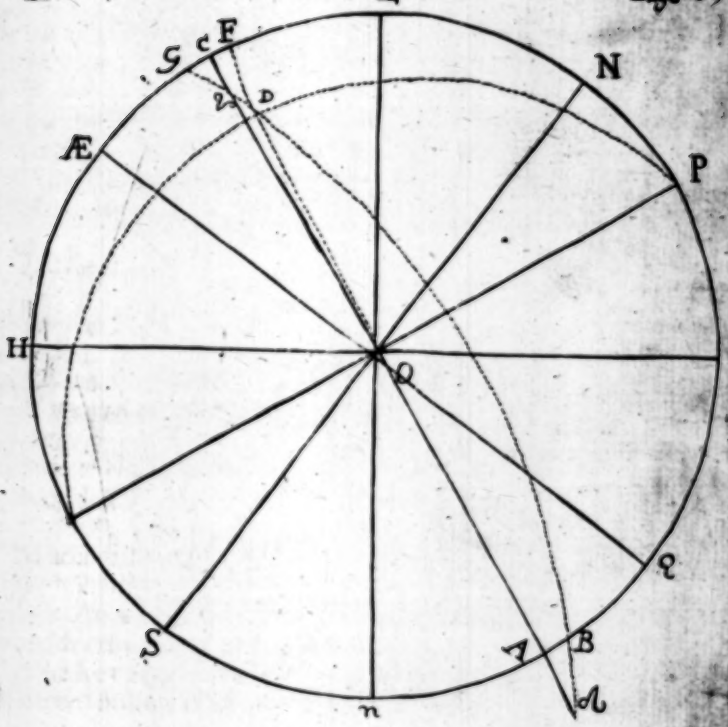
Æra Christi.

The Moons mean Longitude is	Sign.04.deg.02.25.55
The Moons *Apogæon*	Sign.09.deg.13.46.59
The Moons mean Anomaly	Sign.06.deg.18.38.56
The Moons Node Retrograde	Sign.08.deg.28.33.16

And according to these measures, the Moons mean motions in decimal Numbers are.

Æra Christi.

The Moons mean Longitude, deg.	34.00887.345677
The Moons *Apogæon*, deg.	78.82862.654320
The Moons mean Anomaly, deg.	55.18024.691357
The Moons Node Retrograde, deg.	74.69845.679010


Z
F
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n

The ☾ mean motion for one year.

The Moons mean Longitude, deg.	35.94001.44893.1
The Moons *Apogæon*, deg.	11.29551.126365
The Moons mean Anomaly, deg.	24.64450.322566
The Moons Node Retrograde, deg.	05.36900.781604

The ☾ mean motion for one day.

The Moons mean Longitude, deg	03.66010.962873
The Moons *Apogæon*, deg.	00.03094.66c620
The Moons mean Anomaly, deg.	03.62916.302253
The Moons Node Retrograde, deg.	00.01470.961045

And according to these measures are the Tables made shewing the Moons mean Longitude, *Apogæon*, Anomaly, and Node retrograde for Years, Months, Days and Hours.

And hence to compute the Moons true place in her Orbit, I shall make use of the Method, which Mr. *Horrox* in his *Posthumus* works lastly published by Mr. *Flamsted*, in which from the *Rudolphin* Tables he sets down these Dimensions.

The

The Moons mean Semidiameter deg. 00.15'.30"
Her mean distance in Semid. of the Earth Deg. } 11.47.22
The half whereof deg. 5.53.41. he adds 45 the whole is } deg. 50.53.41
Whose Artificial cotangent is 9.91000022
And the double thereof makes this standing Numb. } 9.82000044

The Moons { Greatest 6685.44 / Mean 5523.69 / Least 4361.94 } Excentricity
And her greatest variation 00. 36'. 27".

These things premised his directions for computing the Moons place, are as followeth.

CHAP. XVII.

How to Calculate the Moons true place in her Orbit.

TO the given time find the true place of the Sun, or his Longitude from the Vernal Equinox, as hath been already shewed.

2. From the Tables of the Moons mean motions, write out the *Epocha* next before the given time, and severally under that set the motions, belonging to the years, months and days compleat, and to the hours and scruples current, every one under his like (only remember that in the Bissextile years, after the end of *February*, the days must be increased by one Unite) then adding them all together, the Summ shall be

the

the Moons mean motions for the time given: But in her Node Retragrade you must leave out the *Radix* or first number, and the Summ of the remainer being deducted from the *Radix*, shall be the mean place of her Node required.

3. Deduct the Moons *Apogæon* from the ☉ true place, the rest is the annual Augment, the tangent of whose Complement 180 or 360, being added to the artificial Number given 9.82000044. the Summ shall be the tangent of an Arch, which being deducted from the said Complement, giveth the *Apogæon* Æquation to be added to the mean *Apogæon*, in the first and third quadrants of the annual Augment, and Subtracted in the second and fourth, their Summ or difference is the true *Apogæon*.

4. The true *Apogæon* being Deducted from the ☾ mean Longitude gives the Moons mean Anomaly.

5. Double the annual Augment, and to the Coline thereof add the Logarithm of 1161.75. the difference between the Moons mean and extream Excentricity, *viz.* 3. 06511268, the Summ shall be the Logarithm of a number which being added to the mean Excentricity, if the double annual Augment be in the first or fourth quadrants; or Subtracted from it, if in the second or third quadrants; the Summ or difference shall be the Moons true Excentricity.

6. The Moons true Excentricity being taken for a natural Sine, the Arch answering thereto shall be the ☾ greatest Physical Æquation.

7. To the half of the Moons greatest Physical Æquation add 45 deg. the cotagent of the Summ is the artificial Logarithm of the Excentrick.

To the double whereof if you add the tang. of half the mean Anomaly, the Summ ſhall be the tangent of an Arch, which being added to half the mean Anomaly, ſhall give the Excentrick Anomaly.

8. To the Logarithm of the Excentrick, add the tangent of half the Excentrick Anomaly, the Summ ſhall be the tangent of an Arch, whoſe double ſhall be the Coequated Anomaly, and the difference between this and the mean Anomaly is the terreſtrial Equation, which being added to the Moons mean Longitude, if the mean Anomaly be in the firſt Semicircle, or Subtracted from it, if in the latter, the Summ or difference ſhall be the place of the Moon firſt Equated.

9. From the place of the Moon firſt Equated, Deduct the true place of the Sun, and double the remainer, and to the Sine of the double add the Sine of the greateſt variation 0. 36. 27, *viz.* 8. 02541571, the Summ ſhall be the Sine of the true variation, at that time, which being added to the Moons place firſt Equated, when her ſingle diſtance from the Sun is in the firſt or third quadrants, or Subtracted when in the ſecond or fourth, the Summ or difference ſhall be the Moons true place in her Orbit.

Example.

Let the given time be *Anno Chriſti* 1672. *Feb.* 23. h. 11. 34′.54″ at which time the Suns true place is in ♓ 15. 29133 and the Moons middle motions are as followeth.

☾ Long.

	☾ Longitude	☾ Apogeon	☊ Retrograde
1660.	13.36650.	41.78372.	55.85177
11.	02.66032.	24.31246.	59.08943
January.	13.46339.	00.95934	.45599
D. 23	84.18252.	.71177	.33832
H. 11	1.67755.	.01418	.00674
34′	.08641.	.00072	.00054
54	.00228.	.00012	.00001
☾ Longitude	15.43897.	67.78229	59.89082
			95.96094

These Numbers reduced to the Degrees and Parts of the common Circle are for the ☾ mean Longitude. 55.580292
The ☾ *Apogeon.* 244.015956

The ☉ true place is 345.29133
The ☾ *Apogeon* subtract. 244.01595

The Annual Augment. 101.27538
The Complement whereof is 78.72462

The Tang. of deg. 78.72462 10.70033391
The standing Number. 9.82000044
The Tang. of deg. 73.20288 10.52033435
Their difference. 5.52174 the *Apogeon* Equation.
Mean *Apogeon* 244.01595
Their difference 238.49421 is the true *Apogeon.*

Secondly.

The ☾ mean Longitude.	55. 58029
The true *Apogæon* ſubtract.	238. 49421
Reſts the ☾ mean *Anom.* correct.	177. 08608

Or thus.

The ☾ mean Anomaly in the Tables for the time propounded, will be found to be 67. 78221, which converted into the deg. and parts of the common Circle is	171. 56434
To which the *Apogæon* Equation being added	5. 52174
Their Sum is the mean *Anom.* correct.	177. 08608

And hence it appears that working by the mean Anomaly inſtead of the mean Longitude, the true *Apogæon* Equation muſt be added to the mean Anomaly, in the ſecond and fourth Quadrants of the ☾ Annual Augment, and ſubtracted from it in the firſt and third.

Thirdly.

The Annual Augment. 101. 27538 being doubled is deg. 202. 55076, the Coſine of whoſe exceſs above 180, that is the Coſine of 22. 54076 is	9. 96545577
The Logarithm of 1161. 75	3. 06511268
The Logarithm of 1072. 92	3. 03056845
The ☾ mean Excentr.	5523. 69
Their difference 4450. 77 is the ☾ true Excentricity	

centricity. Which taken as a natural Sine, the Arch answering thereunto Deg. 2. 55094 is the ☾ greatest Physical Equation.

Fourthly.

To the half of the Physical Equation. deg. 01. 27547 add 45 degrees, the Sum is deg. 46. 27547, the Cotangent whereof; *viz.* 9. 98080957 is the Logarithm of the Excentrick, the double of which Logarithm is 9. 96161914

Tangent ½ Anomaly corrected	88. 54304	11. 59455229
Tang. of deg.	88. 40849	11. 55620143

Their Sum deg. 176. 95153 is the excentrick Anomaly.

Fifthly.

The Logarithm of the Excentrick is		9. 98080957
Tang. ½ excent. Anom.	88. 475765	11. 57505878
Tangent of deg.	88. 407268	11. 55586835
The double whereof	176. 814536	is the coequated Anomaly.
M. Anomaly correct.	177. 086080	
Their difference	0. 271544	is the Equation sought to be subst. from ☾ mean Long. 55. 580292
The Remainer	55. 308748	is the ☾ place first Equated.

Sixthly,

From the place of the ☾ first Equated.	55. 308748
Deduct the true place of the Sun	345. 291330
The Remainer is the Distance of the ☾ *a* ☉.	70. 017418
The double whereof is 140. 034836. The Sine of whose Complement to a Semi-circle, 39. 965164 is	9. 80775260
The Sine of the greatest variation	8. 02541571
The Sine of the true var. 0. 390206	7. 83316831
The ☾ place first Equa.	55. 308748
The ☾ place in Orbit 55. 698954 that is in Sexagenary Numbers.	8. 25. 41. 54.

CHAP. XVIII.

To compute the true Latitude of the Moon, and to reduce her place, from her Orbit to the Ecliptick.

THe greatest Obliquity of the Moon's Orb with the Ecliptick or Angle *A* & *B Fig.* 11, is by many Observations confirmed to be 5 Degrees just, at the time of the Conjunction or Opposition of the Sun and Moon, but in her Quarters deg. 5. 18′. Now then to find her Latitude at all times, the said Mr. *Horrox* refers us to *pag.* 87. in the *Rudolphin* Tables, to find from thence the Equation of the Nodes, and Inclination *limitis menstrui*, in this manner.

1. From the mean place of the Node, deduct the

the ☉ true place, the Remainer is the distance of the ☉ from the ☊. with which entring the said Table, he finds the Equation of the Node and Inclination *limitis mensrui*, which being added to or subtracted from the Nodes mean place according to the title, the Sum or difference is the true place of the Node, which being deducted from the place of the Moon in her Orb, the Remainer shall be the Augment of Latitude or Distance of the Moon from the Node, or Leg *A*☊.

2. With the Augment of Latitude, enter the Table of the Moon's Latitude, and take thence her Simple and Latitude and Increase answering to it. Then say, *as the whole excess of Latitude* 18′, *or in Decimals* 30. *is to the Inclination of the Monethly limit: So is the increase of Latitude to the Part Proportional*; which being added to the simple Latitude, will give you the true Latitude of the Moon.

3. With the same Augment of Latitude, enter the Table of Reduction, and take thence the Reduction and Inclination answering thereto: Then say again, *as* 18′.00″. *or* 0.30. *is to the Inclination of the Monethly limit: So is the increase of Reduction, to the Part Proportional*; which being added to the simple Reduction, shall give the true, to be added to, or subtracted from the place of the Moon in the Ecliptick.

Example. By the former Chapter, we found the mean motion of the Node to be 95.96094, which reduced to the Degrees and Parts of the common Circle is 345.459384
And the Suns true place to be 345.291334
Their difference is the distance ☉ *à* ☊ .168050

with which entring the Table, Entituled *Tabula Æquationis Nodorum Lunæ.* I find the Node to need no Equation, and the Inclination *limitis menſtrui* to be deg. 00. 30.

The place of the ☾ in her Orbit	55.698954
The Nodes true place, ſubtract.	345.459384
The Augment of Latitude	70.239570

2. With this Augment of Latitude I enter the Table ſhewing the Moons ſimple Latitude, and thereby find her ſimple Latitude to be Degrees. 04. 70476. North; And the increaſe 00.28234

And therefore the Moons true Latitude is deg. } 4.98610

3. With the ſame Augment of Latitude, I enter the Table of Reduction, and thereby find the Reduction to be 00.06955

And the increaſe of Reduction to be deg. } 00.00855

And therefore the whole Reduction to be ſub. } 00.07810

From the ☾ place in her Orbit	55.69895
The ☾ true place in the Ecliptick	55.62085

That is in Sexagenary Numbers. 8. 25. 37'. 15".

CHAP.

CHAP. XIX.

To find the Mean Conjunction and Oppoſition of the Sun and Moon.

TO this purpoſe we have here exhibited a Table ſhewing the Moons mean motion from the Sun, the conſtruction whereof is this: By the Tables of the Moons mean motions, her mean

Longitude *Æra Chriſti* is	34.0088734567
The ☉ mean Anomaly.	56.6997085185
Præceſſion of the Æquinox.	20.4976851851
Their Sum is the ☉ mean longit. *Æra Chriſti.*	77.4973937036
Which being deducted from the ☾ mean longitude, the remainer is the Moons mean	56.8114797531

diſtance from the Sun, in the beginning of the Chriſtian *Æra.*

In like manner the Moons mean diſtance from the Sun in a year or a day is thus found.

☉ Anomaly for a year.	99.9297857316
Præceſſion of the Æquinox.	0038580246
Their Sum ſubtract.	99.9336437562
From the ☾ mean Longitude.	35.9400144893
Moons diſtance from the ☉.	36.0063707331

Moons

Moons distance from the Sun in a days time.

☉ mean Anomaly.	27378.02348
Præcession of the Æquinox.	1.05699
Their Sum subtract.	27379.08047
From the ☾ mean Longitude.	03.66010.96287
☾ Daily motion from the ☉.	03.38631.88240

And according to these measures are the Tables made, shewing the Moons mean motion from the Sun, by which the mean conjunction of the ☉ and Moon may be thus computed.

To the given year and Month gather the middle motions of the Moon from the Sun, and take the complement thereof to a whole Circle, from which subtracting continually the neareſt leſſer middle motions, the day, hour, and minute enſuing thereto is the mean time of the Conjunction.

Example, *Anno Chriſti* 1676. I would know the time of the mean Conjunction or New Moon in *October*.

Epocha 1660	32.697283
Years Compl. 15.	50.254463
Septemb. Compl.	24.465038
1. day for Leap-year,	03.386318
Their Sum is the Moons motion from the ☉.	10.803102
Complement to a whole Circle.	89.196898
Days 26 Subtract.	88.044289
	1.152609
Hours 8. ſubſtract.	1.128772
	Minutes

	0.023837
Minutes 10 Subtract.	0.023516
The Remainer giveth 8″.	,00321

Therefore the mean Conjunction in *October*, 1676. was the 26 day, 10 min. 8 seconds after 8 at night.

And to find the mean opposition. To the complement of the middle motion, add a semicircle, and then subtract the neareſt leſſer middle motions as before, the day, hour, and minute enſuing thereto, ſhall be the mean oppoſition required.

Example, *Anno Chriſti*, 1676. I deſire to know the mean oppoſition in *November*.

Epocha 1660	32.697283
Years Compl. 15	50.254463
October Compl.	29.440922
1 day for Leap-year.	03.386318
The ☾ mean motion from the ☉.	15.778986
Complement to a whole Circle.	84.221014
To which add a Semicircle.	50.
The Sum is	34.221014
Days 10 ſubtract.	33.863188
	.357826
Hours 2.	.282193
	.075633
Minutes 32.	.075251
The Remainer giveth 9 ſeconds.	.000382

Therefore

Therefore the Full Moon or mean Opposition of the Sun and Moon was *November* the 10th, Hours 2, 32′ 09″. The like may be done for any other.

And here I should proceed to shew the manner of finding the true Conjunction or Opposition of the Sun and Moon, but there being no decimal Canon yet extant, suitable to the Tables of middle motions here exhibited, I chuse rather to refer my Reader to Mr. *Street's Astronomia Carolina*, for instructions in that particular, and what else shall be found wanting in this Subject.

AN

AN INTRODUCTION TO Geography, OR, The Fourth Part of COSMOGRAPHY.

CHAP. I.

Of the Nature and Diviſion of Geography.

GEOGRAPHY is a Science concerning the meaſure and diſtinction of the Earthly Globe, as it is a Spherical Body compoſed of Earth and Water, for that both theſe do together make but one Globe.

2. And

2. And hence the parts of Geography are two, the one concerns the Earthy part, and the other the Water.

3. The Earthy part of this Globe is commonly divided into Continents and Islands.

4. A Continent is a great quantity of Land not separated by any Sea from the rest of the World, as the whole Continent of *Europe*, *Asia*, and *Africa*, or the Continents of *France*, *Spain*, and *Germany*.

5. An Island is a part of Earth environed round about with some Sea or other; as the Isle of *Britain* with the Ocean, the Isle of *Sicily* with the Mediterranean, and therefore in Latine it is called *Insula*, because it is scituate in *Salo*, in the Sea.

6. Both these are subdivided into *Peninsula*, *Isthmus*, *Promontorium*.

7. *Peninsula*, *quasi pene insula*, is a tract of land which being almost encompassed round by water, is joyned to the main land by some little part of Earth.

8. *Isthmus* is that narrow neck of Land which joyneth the *Peninsula* to the Continent.

9. *Promontorium* is a high mountain which shooteth it self into the Sea, the outmost end whereof is called a Cape or Foreland, as *the Cape of Good Hope* in *Africk*.

10. The Watry part of this Globe may be also distinguished by diverse Names, as Seas, Rivers, Ponds, Lakes, and such like.

11. And this Terrestrial Globe may be measured either in whole, or in any particular part.

12. The measure of this Earthly Globe in whole, is either in respect of its circumference, or its bulk and thickness.

13. For

13. For the measuring of the Earths circumference, it is supposed to be compassed with a great Circle, and this Circle in imitation of Astronomers, is divided into 360 degrees or parts, and each degree is supposed to be equal to 15 common German miles, or 60 miles with us in *England*, and hence the circumference of the Earth is found, by multiplying 360 by 15, to be 5400 German miles, or multiplying 360 by 60, the circumference is 21600 English miles.

14. The circumference of the Earth being thus obtained, the Diameter may be found by the common proportion between the Circumference and the Diameter of a Circle, the which according to *Archimedes* is as 22 to 7, or according to *Van Culen* as 1 to 3. 14159. and to bring an Unite in the first place.

As the circumference 3. 14159. is to 1 the Diameter, so is 1 the circumference to 318308 the Diameter, which being multiplied by 5400, the Earths Diameter will be found to be 1718 German miles and 8632 parts, but being multiplied by 21600, the Diameter will be 6875 English miles, and parts 4528.

15. The measure of the Earth being thus found in respect of its whole circumference and Diameter, that which is next to be considered, is the distinction of it into convenient spaces.

16. And this is either Primary or Secondary.

17. The Primary distinction of the Earthly Globe into convenient spaces, is by Circles considered absolutely in themselves, dividing the Globe into several parts without any reference to one another.

18. And

18. And these as in the Celestial Globe, are either great or small.

19. The Great Circles are such, as divide the whole Globe into two Equal Parts or Spaces, and are either more or less Principal.

20. The principal great Circles are such as have a principal or chief use in dividing the Globe into its Parts, and are either fixed or moveable.

21. There is but one fixed or immutable great Circle in the Terrestrial Globe, keeping one and the same place, and without any Multiplication, by reason of the variety of places upon the Earth, and this is called the *Æquator*.

22. The *Æquator* is a great Circle going round about the Terrestrial Globe, from East to West. The use is to shew the Latitude or Distance of any Town or Place from thence towards the North or South Pole, and must be measured by the Degrees in the Meridian.

23. The mutable great Circles are two, the Meridian and the Horizon.

24. The Meridian Circle, is a great Circle drawn from Pole to Pole, or through the North or South Points of every place; so that there are as many Meridians as there are distinct places upon the Earth; but the chief and first Meridian was by the Greeks, and by the Ancients before them placed in the Fortunate Islands, as they were termed of old, from an opinion of some singular blessing imagined by the Ancients upon the *Genius* of those Parts: They are now by the Spaniards called the *Canary* Isles; whereof *Ptolemy*, and *Pliny* out of *Juba* the African King, findeth to be but six; but the late Discoverers meet with seven,

seven, that is, *Lanzerotta*, *Fortaventura*, *Teneriffa*, *Gomera*, *Fierro*, *Palma*, and the *Gran Canary*; which giveth name to the rest: for the Scituation of these Islands, they lie not as *Ptolemy* placed them within one Degree of Longitude, or little less, but more scattering, and lifted up a little above the Tropick of *Cancer*, about the thirtieth Degree of the Northern Latitude, in that part of the Western otherwise called the Atlantick Ocean, which lieth upon the coast of *Africk*, and are therefore by Geographers reckoned among the African Isles. This was the farthest Part of the Earth discovered towards the West to those of about *Ptolemy*'s time: therefore the great Meridian was fixed there, in the Isle *Hera*, or *Junonea*, as then it was called; now *Teneriff*: and from this Meridian all the Longitudes in the Greek Geography are taken.

Our own Geographers, the later especially have affected to transplant this great Meridian out of the Canary Isles into the *Azores*, or otherwise termed the Flemish Isles, because some of them have been famously possessed, and first discovered by them. They are now in number nine: *Tercera*, S. *Michael*, S. *Mary*, S. *George*, *Gratiosa*, *Pico*, *Fayall*, *Corvo*, *Flores*, they are situate in the same Atlantick Ocean, but North West of the *Canaries*, and bending more upon the Spanish Coast, under the 39 Degree of Latitude or thereabout. Through these Isles the late Geographers will have the great Meridian to pass, some of them in the Isles *Corvo*, and *Flores*, the most Western; as *Johnson* in his lesser Globe of the year 1602. others in S. *Michael* and S. *Mary*, the more Eastern of the *Azores*, but *Stevinus* a

Dutch Geographer inclines much to the bringing back the great Meridian to the Fortunate Islands, more particularly to the Peak a Mountain so called from the sharpness in the top, in the Isle *Tenriff*, which is believed to be the highest Mountain in the World; therefore the same *Jobuson* in his greatest Globe of the year 1616, hath drawn the great Meridian in that place, and it were to be wished, that this might be made the common and unchangeable practice.

25. The Horizon is a great Circle, designing so great a Part of the Earth, as a quick sight can discern in an open field; it is twofold Rational and Sensible.

26. The Rational Horizon is that which is supposed to pass through the Center of the Earth, and is represented by the wooden Circle in the Frame, as well of the Celestial, as the Terrestrial Globe, this Rational Horizon belongeth more to Astronomy than Geography.

27. The Sensible Horizon is that before defined, the use of it is to discern the divers risings and settings of the Stars, in divers places of the Earth, and why the days are sometimes longer, and sometimes shorter.

28. The great but less principal Circle upon the Terrestrial Globe is the Zodiack, in which the Sun doth always move. This Circle is described upon Globes and Maps for ornament sake, and to discover under what part of the Zodiack the several Nations lie.

29. The lesser Circles are those which do not divide the Terrestial Globe into two equal, but into two unequal Parts, and these by a general name are called Parallels, or Circles equidistant from

from the Equinoctial; of which as many may be drawn, as there can Meridians, namely 180 if but to each degree, but they are usually drawn to every ten Degrees in each Quadrant from the *Æquator* to the Poles.

30. These Parallels are not of the same Magnitude, but are less and less as they are nearer and nearer to each Pole: and their use is to distinguish the Zones, Climates and Latitudes of all Countries, with the length of the Day and Night in any Part of the World.

31. Again, a Parallel is either named or unnamed.

32. An unnamed Parallel is that which is drawn with small black Circular Lines.

33. A named Parallel is that which is drawn upon the Globe with a more full ruddy and circular Line: such as are the Tropicks of *Cancer* and *Capricorn*, with the Arctick and Antarctick Circles, of which having spoken before in the general description of the Globe, there is no need of adding more concerning them now.

CHAP. II.

Of the Distinction or Dimension of the Earthly Globe by Zones and Climates.

HAving shewed the primary distinction of the Globe into convenient spaces by Circles considered absolutely in themselves, we come now to consider the secondary Dimension or distinction of convenient spaces in the Globe, by the same Circles compared with one another,

and by the ſpaces contained between thoſe Circles.

2. This ſecundary Dimenſion or Diſtinction of the terreſtial Globe into Parts, is either a Zone or a Clime.

3. A *Zone* is a ſpace of the Terreſtial Globe included either between two of the leſſer nominated Circles, or between one and either Pole. They are in Number five, one over hot, two over cold, and two temperate.

4. The over hot or Torrid *Zone*, is between the two Tropicks, continually ſcorched with the preſence of the Sun.

5. The two over cold or Frigid *Zones*, are ſcituated between the two polar Circles and the very Poles, continually wanting the neighbourhood of the Sun.

6. The two temperate *Zones*, are one of them between the Tropick of *Cancer* and the Arctick Circles and the other between the Tropick of *Capricorn* and the Antarctick Circle, enjoyning an indifferency between Heat and Cold; ſo that the parts next the Torrid *Zone* are the hotter, and the parts next the Frigid *Zone* are the Colder.

7. The Inhabitants of theſe *Zones*, in reſpect of the diverſity of their noon Shadows are divided into three kinds, *Amphiſcii*, *Heteroſcii* and *Periſcii*. Thoſe that inhabit between the two Tropicks are called *Amphiſcii*, becauſe that their noon Shadows are diverſly caſt, ſometimes towards the South as when the Sun is more Northward than their vertical point, and ſometimes towards the North, as when the Sun declines Southward from the Zenith.

Thoſe

Thoſe that live between the Tropick of *Cancer* and the Arctick Circle or between the Tropick of *Capricorn* and the Antarctick Circle, are called *Heteroſcii*, becauſe the Shadows at noon are caſt one only way, and that either North or South. They that inhabit Northward of the Tropick of *Cancer* have their Shadows always towards the North, and they that inhabit Southward of the Tropick of *Capricorn*, have their noon Shadows always towards the South.

Thoſe that inhabit between the Poles and the Arctick or Antarctick Circles are called *Periſcii*, becauſe that their Gnomons do caſt their Shadows circulary, and the reaſon hereof is, for that the Sun is carried round about above their *Horizon* in his whole diurnal revolution.

8. The next ſecundary Dimenſion or diſtinction of the earthy Globe into convenient parts or ſpaces, is by Climes.

9. And a Clime or Climate is a ſpace of Earth conteined between three Paralells, the middlemoſt whereof divideth it into two equal parts, ſerving for the ſetting out the length and ſhortneſs of the days in every Country.

10. Theſe Climates and the Parallels by which they are conteined are none of them of equal quantity, for the firſt Clime as alſo the Parallel beginning at the Æquator is larger than the ſecond, and the ſecond is likewiſe greater than the third.

11. The Antients reckoned but ſeven Climates at the firſt, to which Number there were afterward added two more, ſo that in the firſt of theſe Numbers were comprehended fourteen parallels, but in the latter eighteen.

12. *Ptolemy* accounted the Paralells 38 each way from the Equator, that is 38 towards the North, and as many towards the South, 24 of which he reckoned by the difference of one quarter of an hour, 4 by the difference of half an hour, 4 by an whole hours difference, and 6 by a Months difference, but now the parallels being reckoned by the difference of a quarter of an hour, the Climates are 24 in Number till you come to the Latitude of 66 degrees 31 Minutes, to which are afterwards added 6 Climates more unto the Pole it self, where the Artificial day is 6 Months in length.

13. The distances of all both Climates and Parallels, together with their Latitudes from the Æquator, and difference of the quantity of the longest days, are here fully expresst in the Table following.

A Table of the Climates belonging to the three sorts of Inhabitants.

Inhabitants belonging to the several Climes	Climes	Paralells	Length of the Day	Poles Elevation	Best of the Clime
Amphiscii	0	0	12. 0	0. 0	4. 18
		1	12. 15	4. 18	
	1	2	12. 30	8. 34	8. 25
		3	12. 45	12. 43	
	2	4	13. 0	16. 43	7. 50
		5	13. 15	20. 33	
	3	6	13. 30	23. 10	7. 3
		7	13. 45	27. 36	
	4	8	14. 0	30. 47	6. 9
		9	14. 15	33. 45	
	5	10	14. 30	36. 30	5. 17
		11	14. 45	39. 02	
	6	12	15. 0	41. 22	4. 30
		13	15. 15	43. 32	
	7	14	15. 30	45. 29	3. 48
		15	15. 45	47. 20	
	8	16	16. 0	49. 21	3. 13
		17	16. 15	50. 13	
	9	18	16. 30	51. 58	2. 44
		19	15. 45	53. 17	

	Climes	Paralells	Length of the Days	Poles Elevation	Breadth of the Clime
	10	20	17. 00	54. 29	2. 17
Hetzroscii		21	17. 15	55. 34	
	11	22	17. 30	56. 37	2. 0
		23	17. 45	57. 34	
	12	24	18. 00	58. 26	1. 40
		25	18. 15	59. 14	
	13	26	18. 30	59. 59	1. 26
		27	18. 45	60. 40	
	14	28	19. 00	61. 18	1. 13
		29	19. 15	61. 53	
	15	30	19. 30	62. 25	1. 0
		31	19. 45	62. 54	
	16	32	20. 00	63. 22	0. 52
		33	20. 15	63. 46	
	17	34	20. 30	64. 06	0. 44
		35	20. 45	64. 30	
	18	36	21. 00	64. 49	0. 36
		37	21. 15	65. 06	
	19	38	21. 30	65. 21	0. 29
		39	21. 45	65. 35	
	20	40	22. 00	65. 47	0. 22
		41	22. 15	65. 57	
	21	42	22. 30	66. 00	0. 17
		43	22. 45	66. 14	

Clime	Paralells	Length of the Day	Poles Elevation	Breadth of the Clime
22	44	23. 00	66. 20	0. 11
	45	23. 15	66. 25	
23	46	23. 30	66. 28	0. 5
	47	23. 45	66. 30	
24	48	24. 00	66. 31	0. 0

	Here the Climates begin to be accounted by Months, from 66. 31′ where the day is 24 hours long; unto the Pole it self, where it is 6 Months in length.	1	67. 15
		2	69. 30
		3	73. 20
		4	78. 20
		5	84. 0
		6	90. 0

miſcii

14. Hitherto

14. Hitherto we have considered the inhabitants of the Earth in respect of the several Zones and Climes into which the whole Globe is divided; there is yet another distinction behind into which the inhabitants of the Earth are divided in respect of their site and position in reference to one another, and thus the inhabitants of the Earth are divided into the *Perioeci*, *Antoeci* and *Antipodes*.

15. The *Perioeci* are such as dwell in the same Parallel on the same side of the Æquator, how far distant soever they be East and West, the season of the year and the length of the days being to both alike, only the midnight of the one is the noon to the other.

16. The *Antoeci* are such as dwell under the same Meridian and in the same Latitude, or Parallel distance from the Æquator, the one Northward and the other Southward, the days in both places being of the same length, but differ in the Seasons of the year, for when it is Summer in the one it is Winter in the other.

17. The *Antipodes* are such as dwell Feet to Feet, so as a right Line drawn from the one unto the other, passeth from North to South through the Center of the World. These are distant 180 degrees or half the compass of the Earth, they differ in all things, as Seasons of the year, length of days, rising and setting of the Sun and such like. A matter reckoned so ridiculous and impossible in former times, that *Boniface* Arch-Bishop of *Mentz* seeing a Treatise concerning these *Antipodes* written by *Virgilius* Bishop of *Salisburg*, and not knowing what damnable Doctrine might be couched under that strange

ſtrange Name, made complaint firſt to the Duke of *Bohemia*, and after to Pope *Zachary Anno* 745 by whom the poor Biſhop (unfortunate only in being learned in ſuch a time of Ignorance) was condemned of Hereſie, but God hath bleſt this latter age of the World with more underſtanding, whereby we clearly ſee thoſe things, which either were unknown, or but blindly gueſſed at by the Antients.

18. The ſecond part of the Terreſtial Globe is the Water which is commonly divided into theſe parts, or diſtinguiſhed by theſe Names, *Oceanus*, *Mare*, *Fretum*, *Sinus*, *Lacus* and *Flumen*.

19. And firſt *Oceanus* or the Ocean is that general Collection of all Waters, which encompaſſeth the Earth on every ſide.

20. *Mare* the Sea, is a part of the main Ocean, to which we cannot come but through ſome *Fretum* or Strait, as *Mare Mediterraneum*. And it takes it name firſt either from the adjacent Shore, as *Mare Adriaticum*, from the City of *Adria*; or ſecondly from the firſt diſcoverer, as *Mare Magellanicum*, from *Magellanus* who firſt found it, or thirdly from ſome remarkable accident, as *Mare Icarium* from the drowning of *Icarus* the Son of *Dædalus*.

21. *Fretum*, a Strait is a part of the Ocean penned within ſome narrow Bounds, and opening a way into ſome Sea, or out of ſome Sea into the Ocean, as the Strait of *Helleſpont*, *Gibralter*, *&c.*

22. *Lacus*, a Lake is a great body or collection of Water, which hath no viſible Intercourſe with the Sea, or influx into it; as the Lake of

Thraſy-

Thrasymene in *Italy*, and *Lacus Asphaltites*, or the dead Sea in the Land of *Canaan*.

23. *Flumen* or *Fluvius* is a water-course continually running, (whereby it differs from *Stagnum* a standing Pool) issuing from some Spring or Lake, and emptying it self into some part of the Sea, or some other great River, the mouth or outlet of which is called *Ostium*.

And thus we have gon over those particulars both of Earth and Water, which appertain to this Science of Geography in the general; We will now proceed to a more particular Consideration of the several parts into which the Terrestial Globe is commonly divided.

CHAP. III.

Of Europe.

THe Terrestial Globe is divided into two parts, known or unknown.

2. The unknown or the parts of the World not fully discovered, are distinguished into North and South, the unknown parts of the World towards the North, are those which lie between the North part of *Europe* or *America* and the North Pole; and the unknown parts of the World toward the South, are those which ly between the South part of *America* and the South Pole.

3. The known parts of the World were antiently these three, *Europe*, *Asia* and *Africk*, to which in latter ages a fourth hath been added which is called *America*.

4. *Europe*

4. *Europe* is bounded on the North with the Northern Ocean, and on the South with the Mediterranean Sea, on the East with the River *Tanais*, and on the West with the Western Ocean, and is contained between the Tropick of *Cancer*, and the Pole Arctick, or 44 degrees as most do say, taking its beginning Southward from *Sicily* where the Pole is elevated 36 degrees, and is thence continued to 80 degrees of North Latitude, and so the whole Latitude of *Europe* is in English miles 2640, but some allow to *Europe* 45 degrees of Latitude, that is in English miles 2700.

5. The Longitude of *Europe* is reckoned from the furthest part of *Spain* and the *Atlantick* Ocean, to the River *Tanais*, which some reckon to be 60 Degrees, to one of which Degrees passing through the middle of *Europe*, they allow fifteen German miles almost, or sixty English, and so the Longitude in German miles is 900, in English 3600.

6. *Europe* though the least of all the four Quarters of the World, is yet of most renown amongst us: First, because of the temperature of the Air, and fertility of the Soil: Secondly, from the study of Arts, both ingenuous and mechanical: Thirdly, of the Roman and Greek Monarchies: Fourthly, from the purity and sincerity of the Christian Faith: Fifthly, because we dwell in it, and so give it the first place.

7. *Europe* may be considered as it stands divided into the Continent and the Islands: the Continent lying all together, containeth these Countries. 1. *Spain*. 2. *France*. 3. *Germany*. 4. *Italy*, and the *Alpes*. 5. *Belgium*. 6. *Denmark*,

mark, 7. *Swethland*. 8. *Russia*. 9. *Poland*. 10. *Hungary*. 11. *Sclavonia*. 12. *Dacia*, and 13. *Greece*. Of each of which I will give some short account; as also of the chief Islands as they are dispersed, in the Greek, Ægæan, Cretan and Ionian Seas, with those in the Adriatick, Mediterranean, and in the British and Northern Ocean.

8. Amongst these I give *Spain* the first place, as being the most Western Part of all the Continent of *Europe* environed on all sides with the Sea, except towards *France*; from which it is separated by the Pyrenæan Hills: but more particularly, it is bounded upon the North with the Cantabrian, on the West with the Atlantick Ocean, on the South with the Straits of *Gibralter*, on the East with the Mediterranean, and on the North East with the said Pyrenæan Hills. The Figure of it is compared by *Strabo* to an Oxes hide spread upon the Ground; the Neck whereof being that *Isthmus* which unites it to *France*.

9. The greatest length hereof, it reckoned at 800 miles, the breadth where it is broadest at 500, the whole Circumference 2480 Italian miles: but *Mariana* measuring the compass of it by the bendings of the Pyrenæan Hills, and the creeks and windings of the Sea, makes the full circuit of it to be 2816 miles of Italian measure.

10. It is situate in the more Southerly Part of the Northern temperate Zone, and almost in the midst of the fourth and sixth Climates; the longest day being 15 hours and a quarter in length in the most Northern Parts hereof: but in the extream South near to *Gibralter* not above fourteen,

fourteen, which Situation of this Country, rendreth the Air here very clear and calm, seldom obscured with mists and vapours, and not so much subject to Diseases as the more Northern Regions are.

11. This Continent is subdivided into the Kingdoms of *Navarr.* 2. *Biscay.* 3. *Guipuscoa.* 4. *Leon* and *Oviedo.* 5. *Gallicia.* 6. *Corduba.* 7. *Granada.* 8. *Murcia.* 9. *Toledo.* 10. *Castile.* 11. *Portugal.* 12. *Valentia.* 13. *Catalonia.* 14. *Majorca.* And 15. *Aragon*; but all of them are now united in the Monarchy of *Spain.*

12. *France* according to the present dimensions of it, is bounded on the East with a Branch of the *Alpes* which divide *Dauphine* and *Piemont*, as also with the Countries of *Savoy*, *Switzerland*, and some Parts of *Germany* and the *Netherlands.* On the West with the Aquitanick Ocean, and a Branch of the Pyrenæan Mountains which divide it from *Spain.* On the North with the English Ocean, and some Parts of *Belgium*, and on the South with the rest of the Pyrenean Mountains, and the Mideterranean.

13. The Figure of it is almost square, each side of the Quadrature being reckoned 600 miles in length, but they that go more exactly to work upon it, make the length thereof to be 660 Italian miles, the breadth 570, the whole Circumference 2040. It is seated in the Northern temperate Zone, between the middle Parallel of the first Clime, where the longest day is 15 hours, and the middle Parallel of the eighth Clime, where the longest day is 16 hours and a half.

14. The Principal Provinces in this flourish-

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ing Country, are. 1. *France* ſpecially ſo called. 2. *Champagne*. 3. *Picardy*. 4. *Normandy*. 5. *Bretagne*. 6. The Eſtates of *Angiou*. 7. *La Beauſie*. 8. *Nivernois*. 9. The Dukedom of *Bourbon*. 10. *Berry*. 11. *Poicton*. 12. *Limoſin*. 13. *Piregort*. 14. *Quercu*. 15. *Aquitain*. 16. *Languedoc*. 17. *Provence*. 18. *Daulphine*. 19. *La Breſſe*. 20. *Lionnois*. 21. The Dutchy. 22. The County of *Burgundy*. 23. The Iſlands in the Aquitanick and Gallick Ocean: Thoſe of moſt note are theſe ſix. 1. *Oleron*. 2. *Ree*. 3. *Jarſey*. 4. *Gernſey*. 5. *Sarke*. 6. *Aldernay* on the ſhores of *Normandy*, of which the four laſt are under the Kings of *England*.

15. *Italy* once the Empreſs of the greateſt part of the then known World, is compaſſed with the Adriatick, Ionian and Tyrrhenian Seas, except it be towards *France* and *Germany*, from which it is parted by the *Alpes*; ſo that it is in a manner, a *Peninſula*, or a Demi-Iſland. But more particularly it hath on the Eaſt the lower part of the *Adriatick*, and the *Ionian* Sea, by which it is divided from *Greece*; on the Weſt the River *Varus*, and ſome part of the *Alpes*, by which it is parted from *France*, on the North in ſome part the *Alpes* which divide it from *Germany*; and on the other, part of the Adriatick, which divides it from *Dalmatia*; and on the South the Tyrrhenian and Tuſcan Seas, by which it is ſeparated from the main Land of *Africa*.

16. It containeth in length from *Auguſta Prætoria*, now called *Aoſt*, at the foot of the *Alpes*, unto *Otranto* in the moſt Eaſtern Point of the Kingdom of *Naples* 1020 miles; in breadth from the

the River *Varo*, which parts it from *Provence*, to the mouth of the River *Arſia* in *Friuli*, where it is broadeſt, 410 miles; about *Otranto*, where it is narroweſt not above 25 miles; and in the middle parts from the mouth of *Peſcara* in the *Adriatick* or upper Sea to the mouth of *Tiber* in the *Tuſcan* or lower Sea, 126 miles. The whole compaſs by Sea reckoning in the windings and turnings of the ſhore, comes to 3038 miles; which added to the 410 which it hath by Land, make up in all 3448 miles: but if the Coaſts on each ſide be reckoned by a ſtraight Line, then as *Caſtaldo* computes it, it comes to no more than 2550 miles.

17. The whole Country lieth under the firſt and ſixth Climates of the Northern temperate Zone, which it wholly taketh up: ſo that the longeſt day in the moſt Northern Parts is 15 hours and three firſt parts of an hour; the longeſt in the Southern Parts, falling ſhort a full hour of that length.

18. *Italy* as it ſtands now is divided into the Kingdoms of *Naples*, *Sicily* and *Sardinia*. 2. The Land or Patrimony of the Church. 3. The great Dukedom of *Tuſcany*. 4. The Commonwealths of *Venice*, *Genoa* and *Luca*. 5. The Eſtates of *Lombardy*, that is the Dukedoms of 1. *Millain*. 2. *Mantua*. 3. *Modena*. 4. *Parma*. 5. *Montferrat*, and the Principality of *Piedmont*.

19. To the *Peninſula* of *Italy* belong the *Alpes*, a ridge of Hills, wherewith as with a ſtrong and defenſible Rampart *Italy* is aſſured againſt *France* and *Germany*. They are ſaid to be five days Journey high, covered continually with Snow,

from the whiteneſs whereof they took this name, it doth contain the Dukedom of *Savoy*; the Seigniory of *Geneva*; the Country of *Walliſland*, *Switzerland* and the *Griſons*.

20. *Belgium*, or the *Netherlands*, is bounded on the East with *Weſtphalin*, *Gulick*, *Cleve*, and the Land of *Triers*, Provinces of the higher *Germany*; on the West with the main Ocean, which divides it from *Britain*; on the North with the River *Ems*, which parts it from *Eaſt-Friezeland*; on the South with *Picardie* and *Campagne* two French Provinces; upon the South-East with the Dukedom of *Lorrain*.

21. It is in compaſs 1000 Italian or 280 German miles, and is ſituated in the Northern temperate Zone, under the ſeventh, eighth and ninth Climates: the longeſt day in the midſt of the ſeventh Climate where it doth begin, being 16 hours, in the beginning of the ninth Climate increaſed to 16 hours 3 quarters, or near 17 hours.

22. It containeth thoſe Provinces which in theſe later Ages were poſſeſſed by the Houſe of *Burgundy*, that is the Lordſhip of *Weſt-Friezeland*, given to the Earls of *Holland* by *Charles* the Bald; the Earldom of *Zutphen* united unto that of *Gelder* by Earl *Otho* of *Naſſau*, and finally the Eſtate of *Groening*, *Over-Yſſel*, and ſome part of *Utrecht*, by *Charles* the Fifth. As it ſtands now divided between the *Spaniards* and the States it containeth the Provinces of 1. *Flanders*. 2. *Artois*. 3. *Hainault*. 4. The Biſhoprick of *Cambray*. 5. *Namur*. 6. *Luxemburg*. 7. *Limbourg*. 8. *Luyckland*, or the Biſhoprick of *Leige*. 9. *Brabant*. 10. *Marquiſate*. 11. *Mechlin*. The reſt of the *Netherlands*

therlands which have now for ſometime withdrawn their obedience from the Kings of *Spain*, are 1. *Holland*. 2. *Zeland*. 3. *Weſt-Friezeland*. 4. *Utrecht*. 5. *Over-Yſſel*. 6. *Gelderland*. 7. *Zutphen*. 8. *Groening*.

23. *Germany* is bounded on the Eaſt with *Pruſſia*, *Poland*, and *Hungary*; on the Weſt with *France*, *Switzerland* and *Belgium*; on the North with the *Baltick* Seas, the Ocean, and ſome part of *Denmark*; on the South with the *Alps* which part it from *Italy*.

24. The length from Eaſt to Weſt, that is from the *Viſtula* or *Weiſſel* to the *Rhine*, is eſtimated at 840 Italian miles, the breadth from North to South, that is from the Ocean to the Town of *Brixen* in *Tyrol*, 740 of the ſame miles. So that the Figure of it being near a Square, it may take up 3160 miles in compaſs, or thereabouts. Situate in the Northern temperate Zone, between the middle Parallels of the ſixth and tenth Climates; the longeſt day in the moſt Southern Parts being 15 hours and an half, and in the moſt Northern 17 hours and a quarter.

25. The Principal Parts of this great Continent, are 1. *Cleveland*. 2. The Eſtates of the three ſpiritual Electors, *Colen*, *Mentz*, and *Triers*. 3. The *Palatinate* of the *Rhine*. 4. *Alſatia*. 5. *Lorrain*. 6. *Suevia* or *Schwaben*. 7. *Bavaria*. 8. *Auſtria* and its Appendices. 9. The Confederation of *Waderaw*. 10. *Franconia*. 11. *Wirtenberg*. 12. *Baden*. 13. The *Palatinate* of *Northgoia*, or the Upper *Palatinate*. 14. *Bohemia* and the Incorporate Provinces. 15. *Pomerania*. 16. *Mecklenburg*. 17. The *Marquiſate* of *Brandenburg*.

burg. 18. *Saxony*, and the Members of it. 19. The Dukedom of *Brunſwick* and *Luneburg.* 20. The *Langravedom* of *Haſſia.* 21. *Weſtphalen.* 22. *Eaſt-Friezeland.*

26. *Denmark* or *Danemark*, reckoning in the Additions of the Dukedom of *Holſtein*, and the great Continent of *Norway*, with the Iſles thereof, now all united and incorporated into one Eſtate is bounded on the Eaſt with the Baltick Sea and ſome part of *Sweden*; on the Weſt with the main Weſtern Ocean; on the North-Eaſt with a part of *Sweden*; full North with the main frozen Seas; and on the South with *Germany*, from which it is divided on the South-Weſt by the River *Albis*, and on the South-Eaſt by the *Trave*; a little *Iſthmus* or neck of Land uniting it to the Continent.

27. It lieth partly in the Northern temperate Zone, and partly within the Arctick Circle; extending from the middle Parallel of the tenth Clime, or 55 degree of Latitude where it joyneth with *Germany*, as far as the 71 degree where it hath no other bound but the frozen Ocean; by which account the longeſt day in the moſt Southern Parts is 17 hours and a quarter, but in the Parts extreamly North, they have no Night for two whole Moneths, three Weeks, one Day, and about ſeven hours; as on the other ſide no day for the like quantity of time, when the Sun is moſt remote from them, in the other Tropick.

28. The whole Body of the Eſtate conſiſteth chiefly of three Members. *viz.* 1. The Dukedom

dom of *Holſtein*; containing *Waggerland*, *Dilmarſh*, *Starmaria*, and *Holſtein*, eſpecially ſo called. 2. The Kingdom of *Denmark*; comprehending both *Jnitlands*, part of *Scandia*, and the *Hemodes*, or Baltick Iſlands. 3. The Kingdom of *Norway* conſiſting of *Norway* it ſelf, and the Iſlands of the Northern Ocean.

29. *Swethland* is bounded on the Eaſt with *Muſcovy*, on the Weſt with the *Deſerine* Hills, which divide it from *Norway*; on the North with the great frozen Ocean ſpoken of before; on the South with *Denmark*, *Lieſland*, and the Baltick Sea.

30. It is ſituate under the ſame Parallels and Degrees with *Norway*, that is, from the firſt Parallel of the 12 Clime, where the Pole is elevated 58 degrees 26 minutes, as far as to the 71 degree of Latitude, by which account the longeſt day in the Southern Point is but 18 hours, whereas on the fartheſt North of all the Countrey, they have no Night for almoſt three whole Moneths together.

31. The whole Kingdom is divided into two Parts, the one lying on the Eaſt, the other on the Weſt of the Bay or Gulf of *Bodener*, being a large and ſpacious branch of the *Baltick* Sea, extending from the moſt Southerly Point of *Gothland*, as far as to *Lapland* on the North. According to which Diviſion we have the Provinces of 1. *Gothland*. 2. *Sweden* lying on the Weſt ſide of the Gulph. 3. *Lapland* ſhutting it up upon the North. 4. *Bodia* or *Bodden*. 5. *Finland* on the Eaſt ſide thereof. 6. The *Sweediſh* Iſlands, where it mingleth with the reſt of the *Baltick* Seas.

32. *Ruſſia* is bounded on the Eaſt by *Tartary*,

on the West with *Livonia* and *Finland*, from which it is divided by great mountains and the River *Poln*, on the North by the frozen Ocean, and some part of *Lapland*, and on the South by *Lituania* a Province of the Kingdom of *Poland*, and the Crim Tartar inhabiting on the Banks of *Palus Maeotis*, and the Euxine Sea. It standeth partly in *Europe* and partly in *Asia*, the River *Tanais* or *Don* running through it, the common boundary of those great and noted parts of the world.

33. It is scituate North within the Artick Circle so far, that the longest day in Summer will be full six months, whereas the longest day in the southern parts is but 16 hours and an half.

34. It is divided into the Provinces of 1. *Moscovy* specially so called. 2. *Smoleusko*, 3. *Mosaisky*, 4. *Plesco*, 5. *Novagrod* the great, 6. *Corelia*, 7. *Biarmia*, 8. *Petzora*, 9. *Condora*, 10. *Obdora*, 11. *Jugria*, 12. *Severia*, 13. *Permia*, 14. *Rozan*, 15. *Wiathka*, 16. *Casan*, 17. *Astracan*, 18. *Novogordia* inferiour, 19. *The Morduits* or *Mordua*, 20. *Worotime*, 21. *Tuba*, 22. *Wolodomir*, 23. *Duina*, 24. the Russian Islands.

35. *Poland* is bounded on the East with *Russia*, and the Crim-Tartar, from whom it is parted by the River *Borysthenes*; on the West with *Germany*, on the North with the Baltick Sea and some part of *Russia*, on the South with the *Carpathian* Mountains, which divide it from *Hungary*, *Transilvania*, and *Moldavia*. It is of figure round in compass 2600 miles, scituate under the 8 and 12 Climates, so that the longest day in the southern parts is but 16 hours, and about 18 hours

hours in the parts most North.

36. The several Provinces of which this Kingdom doth consist, are 1. *Livonia*, 2. *Samogitia*, 3. *Lituania*, 4. *Volkinia*, 5. *Podolia*, 6. *Russia nigra*, 7. *Massovia*, 8. *Podlassia*, 9. *Prussia*, 10. *Pomerellia*, 11. *Poland* specially so called.

37. *Hungary* is bounded on the East with *Transilvania* and *Walachia*, on the West with *Sterria*, *Austria* and *Moravia*, on the North with the *Carpathian* mountains which divide it from *Poland*, and on the South with *Sclavonia*, and some part of *Dacia*: it extendeth in length from *Presburg* along the *Danow* to the borders of *Transilvania*, for the space of 300 English miles, and 190 of the same miles in breadth.

38. It lieth in the Northern temperate Zone, betwixt the middle parallels of the 7 and 9 Climates, so that the longest Summers day in the Southern parts is but 15 hours and an half, and not above 16 hours in the parts most North.

40. This Country is commonly divided into the upper *Hungary* and the lower, the upper lying on the North of the River *Danow*, the lower lying on the South of that River, comprehending all *Pannonia* inferior and part of Superior, and is now possessed by the King of *Hungary* and the Great Turk, who is Lord of the most part by Arms and Conqueſt.

04. *Sclavonia* is bounded on the East with *Servia*, *Macedonia* and *Epirus*, from which it is parted by the River *Drinus*, and a line drawn from thence unto the Adriatick, on the West with *Carniola* in *Germany*, and *Istria* in the Seig-

niory of *Venice*, from which laſt it is divided by the River *Arſia*; on the North with *Hungary*, on the South with the Adriatick Sea.

41. It is ſcituate in the Northern temperate Zone, between the middle Parallels of the ſixth and ſeventh Climates, ſo that the longeſt day in Summer is about 15 hours and an half.

42. This Country as it came at laſt to be divided, between the Kings of *Hungary* and the State of *Venice*; is diſtinguiſhed into 1. *Windiſchland*, 2. *Croatia*, 3. *Boſnia*, 4. *Dalmatia*, 5. *Liburnia* or *Cantado di Zara*, and 6. The Sclavonian Iſlands.

43. *Dacia* is bounded on the Eaſt with the Euxine Sea and ſome part of *Thrace*; on the Weſt with *Hungary* and *Sclavonia*; on the North with *Podolia*, and ſome other members of the Realm of *Poland*, on the South with the reſt of *Thrace* and *Macedonia*.

44. It lieth on both ſides of the *Danow* fronting all along the upper and the lower *Hungary*, and ſome part of *Sclavonia*; extended from the 7 Climate to the 10; ſo that the longeſt Summers day in the moſt northern parts thereof, is near 17 hours, and in the moſt ſouthern 15 hours 3 quarters.

45. The ſeveral Provinces comprehended under the name of *Dacia*, are 1. *Tranſilvania*, 2. *Moldavia*, 3. *Walachia*, 7. *Raſcia*, 5. *Servia*, 6. *Bulgaria*, the firſt four in old *Dacia*, on the North ſide of the *Danow*; the two laſt in new *Dacia*, on the South thereof.

45. *Greece* in the preſent Latitude and extent thereof, is bounded on the Eaſt with the Propontick,

Propontick, Helleſpont, and Ægean Seas, on the Weſt with the Adriatick; on the North with Mount *Hemus* which parteth it from *Bulgaria*, *Servia* and ſome part of *Illyricum*; and on the South with the Sea *Ionian*; ſo that it is in a manner a *Peninſula* or Demi-Iſland, environed on three ſides by the Sea, on the fourth only united to the reſt of *Europe*.

46. It is ſcituate in the northern temperate Zone, under the fifth and ſixth Climates, the longeſt day being 15 hours.

47. In this Country formerly ſo famous for learning and government, the ſeveral Provinces are 1. *Peloponneſus*, 2. *Achaia*, 3. *Epirus*, 4. *Albania*, 5. *Macedon*, 6. *Thrace*, 7. The Iſlands of the Propontick; 8. Ægean, and 9. The Ionian Seas, and 10. finally the Iſle of *Crete*.

And thus I have given you a brief deſcription of thoſe Countries which are comprehended in the Continent of *Europe*; the Iſlands in this part of the world are many; I will mention only ſome few. Theſe two in the Britiſh and Northern Ocean, known by the names of *Great Britain* and *Ireland* are the moſt famous, to which may be added *Greenland*. In the Mediterranaen Sea you have the Iſlands of *Sicilia*, *Sardinia*, *Corſica* and *Crete*, which is now called *Candia* the greater and the leſs: As for the other Iſlands belonging to this part of the world, the Reader may expect a more particular deſcription from them who have or ſhall write more largely of this ſubject: This we deem ſufficient for our preſent purpoſe. Let this then ſuffice for the deſcription of the firſt part of the World called *Europe*.

CHAP.

CHAP. IV.

Of Asia.

Asia is bound on the West with the Mediteranean and Ægæan Seas, the *Hellespont*, *Propontis*, Thracian *Bosphorus* and the Euxine Sea, the *Palus Mæotis*, the Rivers *Tanais* and *Duina*, a Line being drawn from the first of the two said Rivers unto the other, by all which it is parted from *Europe*; on the North it hath the main Scythick Ocean; but on the East the Indian Ocean, and *Mare del Eur* by which it is separated from *America*; on the South the Mediterranean, or that part of it, which is called the Carpathian Sea, washing the shoars of *Anatolia*, and the main Southern Ocean, passing along the Indian, Persian and Arabian Coasts: and finally on the south-west, the red Sea or Bay of *Arabia*, by which it is parted from *Affrick*. Environed on all sides with the Sea, or some Sea like Rivers, except a narrow *Isthmus* in the south-west, which joyns it to *Africk*, and the space of ground (whatsoever it be) between *Duina* and *Tanais*, on the North-west which unites it to *Europe*.

2. It is situated East and West, from the 52 to the 163 degree of Longitude; and North and South from the 82 degree of Latitude to the very Æquator; some of the Islands only lying on the South of that Circle: so that the longest summers day in the southern parts, is but twelve hours, but in the most northern parts hereof almost four whole Months together.

3. This

3. This Country hath heretofore been had in ſpecial honour; 1. For the creation of Man, who had his firſt making in this part of the World. 2. Becauſe in this part of it ſtood the Garden of *Eden*, which he had for the firſt place of his habitation. 3. Becauſe here flouriſhed the four firſt great Monarchies of the Aſſyrians, Babylonians, Medes and Perſians. 4. Becauſe it was the Scene of almoſt all the memorable Actions which are recorded by the penmen of the Scriptures. 5. Becauſe our Saviour Chriſt was borne here, and here wrought his moſt divine Miracles, and accompliſhed the great work of our Redemption. 6. And finally, becauſe from hence all Nations of the World had their firſt beginning, on the diſperſion which was made by the Sons of *Noah* after their vain attempt at *Babel*.

4. This part of the World for the better underſtanding of the Greek and the Roman Stories and the eſtate of the Aſſyrian, Babylonian and the Perſian Monarchies, to which the holy Scriptures do ſo much relate, we ſhall conſider as divided into the Regions of 1. *Anatolia or Aſia minor*. 2. *Cyprus*. 3. *Syria*. 4. *Arabia*. 5. *Chaldea*. 6. *Aſſyria*. 7. *Meſopotamia*. 8. *Turcomania*. 9. *Media*. 10. *Perſia*. 11. *Tartaria*. 12. *China*. 13. *India*. and 14. the Oriental Iſlands.

Anatolia *or* Aſia minor.

Anatolia or *Aſia minor*, is bounded on the Eaſt with the River *Euphrates*, by which it is parted from the greater *Aſia*; on the Weſt with the Thracian *Boſphorus*, *Propontis*, *Helleſpont*, and the

the Ægean Sea, by which it is parted from *Europe*; on the North with *Pontus Euxinus*, called alſo the black Sea, and *Mare Maggiore*, and on the South by the Rhodian, Lydian and Pamphilian Seas, ſeveral parts of the Mediterranean. So that it is a Demi-Iſland or *Peninſula* environed on all ſides with water, excepting a ſmall *Iſthmus* or Neck of Land extending from the head of *Euphrates* to the Euxine Sea, by which it is joyned to the reſt of *Aſia*.

It reacheth from the 51 to the 72 degree of Longitude, and from the 36 to the 45 degree of Latitude, and lyeth almoſt in the ſame poſition with *Italy*, extending from the middle Parallel of the fourth Clime, to the middle Parallel of the ſixth, ſo that the longeſt ſummers day in the Southern Parts, is about 14 hours and a half; and one hour longer in thoſe parts which lie moſt towards the North.

The Provinces into which it was divided before the Roman Conqueſt were 1. *Bithynia*. 2. *Pontus*. 3. *Paphlagonia*. 4. *Galatia*. 5. *Cappadocia*. 6. *Armenia Major & Minor*. 7. *Phrygia minor*. 8. *Phrygia major*. 9. *Myſia* the greater and the leſs. 10. *Aſia* ſpecially ſo called, comprehending *Æolis* and *Ionia*. 11. *Lydia*. 12. *Caria*. 13. *Lycia*. 14. *Lycaonia*. 15. *Piſidia*. 16. *Pamphylia*. 17. *Iſauria*. 18. *Cilicia*. 19. The Province of the Aſian Iſles, whereof the moſt principal are 1. *Tenedos*. 2. *Chios*. 3. *Samos*. 4. *Choos*. 5. *Icaria*. 6. *Leſbos*. 7. *Patmos*. 8. *Claros*. 9 *Carpathos*. 10. *Rhodes*.

Cyprus

Cyprus.

Cyprus is ſituated in the Syrian and Cilician Seas, extended in length from Eaſt to Weſt 200 miles, in breadth 60 the whole compaſs reckoned 550, diſtant about 60 miles from the rocky Shores of *Cilicia* in *Aſia minor*, and about one hundred from the main Land of *Syria*.

It is ſituated under the fourth Climate, ſo that the longeſt day in Summer is no more than 14 hours and a half.

Divided by *Ptolemy* into the 4 provinces of 1. *Paphia*. 2. *Amathaſia*. 3. *Lepathia*. 4. *Salamine*.

Syria.

Syria is bounded on the Eaſt with the River *Euphrates* by which it is parted from *Meſopotamia*; on the Weſt with the Mediterranean Sea; on the North with *Cilicia* and *Armenia minor*, parted from the laſt by mount *Taurus*; and on the South with *Paleſtine*, and ſome parts of *Arabia*. The length hereof from Mount *Taurus* to the Edge of *Arabia*, is ſaid to be 525 Miles; the breadth from the Mediterranean to the River *Euphrates* 470 Miles, drawing ſomewhat near unto a Square.

The whole Country was antiently divided into theſe ſix parts. 1. *Phœnicia*. 2. *Paleſtine*. 3. *Syria* ſpecially ſo called. 4. *Comagena*. 5. *Palmyrene*. and *Caloſyria*, or *Syria Cava*.

Arabia

Arabia.

Arabia hath on the East *Chaldæa* and the Bay or Gulf of *Persia*; on the West *Palestine*, some part of *Egypt*, and the whole course of the red Sea, on the North the River *Euphrates* with some parts of *Syria* and *Palestine*, and on the South the main southern Ocean. It is in circuit about 4000 Miles, but of so unequal and heteregeneous Composition, that no general Character can be given of it, and therefore we must look upon it as it stands divided into *Arabia Deserta*, 2. *Arabia Petræa*. 3. *Arabia Felix* and 4. The *Arabick* Islands.

Chaldea.

Chaldea is bounded on the East with *Susiana* a Province of *Persia*; on the West with *Arabia deserta*; on the North with *Mesopotamia*; and on the South with the Persian Bay and the rest of *Deserta*.

Assyria.

Assyria is bounded on the East with *Media*, from which it is parted by the Mountain called *Coathras*; on the West with *Mesopotamia*, from which it is divided by the River *Tygris*; on the South with *Susiana*; and on the North with some part of *Turcomania*; it was antiently divided into six parts. 1. *Arraphachitis*. 2. *Adiabene*. 3. *Calacine*. 4. *Aobelites*. 5. *Apolloniates*.

Mesopotamia

Meſopotamia.

Meſopotamia is bounded on the Eaſt with the River *Tygris* by which it is parted from *Aſſyria*; on the Weſt with *Euphrates* which divides it from *Comagena* a Province of *Syria*; on the North with Mount *Taurus*; by which it is ſeparated from *Armenia major*; and on the South with *Chaldea* and *Arabia deſerta* from which laſt it is parted by the bendings of *Euphrates* alſo. It was antiently divided into, 1. *Anthemaſia*. 2. *Chalcitis*. 3. *Caulanitis*. 4. *Acchabene*. 5. *Ancorabitis* and 6. *Ingine*.

Turcomania.

Turcomania is bounded on the Eaſt with *Media* and the Caſpian *Sea*; on the Weſt with the *Euxine* Sea, *Cappadocia* and *Armenia minor*; on the North with *Tartary*, and on the South with *Meſopotamia* and *Aſſyria*. A Countrey which conſiſteth of four Provinces. 1. *Armenia major* or *Turcomania* properly and ſpecially ſo called. 2. *Colchis*. 3. *Iberia*. 4. *Albania*.

Media.

Media is bounded on the Eaſt with *Parthia*, and ſome part of *Oxyrcania*, Provinces of the Perſian Empire; on the Weſt with *Armenia major*, and ſome part of *Aſſyria*; on the North with the Caſpian Sea and thoſe parts of *Armenia major*, which now paſs in the account of *Iberia*, *Georgia*; and on the South with *Perſia*. It is now divided

divided into two Provinces. 1. *Atropatia.* 2. *Media major.*

Perſia.

Perſia is bounded on the Eaſt with *India*; on the Weſt with *Media*, *Aſſyria*, and *Chaldea*; on the North with *Tartary*, on the South with the main Ocean.

It is divided into the particular Provinces of 1. *Suſiana.* 2. *Perſis.* 3. *Ormus.* 4. *Carmania.* 5. *Gedroſia.* 6. *Drangiana.* 7. *Arachoſia.* 8. *Paropamiſus.* 9. *Aria.* 10. *Parthia.* 11. *Hyrcania.* 12. *Margiana* and 13. *Baltria.*

Tartaria.

Tartaria is bounded on the Eaſt with *China*, the Oriental Ocean, and the Straits of *Anian*, by which it is parted from *America*, on the Weſt with *Ruſſia* and *Podolia*, a Province of the Realm of *Poland*; on the North with the main Scythick or frozen Ocean; and on the South with part of *China*, from which it is ſeparated by a mighty Wall, ſome part of *India*, the River *Oxus* parting it from *Baltria* and *Margiana*, two Perſian Provinces; the *Caſpian* Sea which ſeparates it from *Media* and *Hyrcania*; the *Caucaſian* Mountains interpoſing between it and *Turcomania*; and the Euxine Sea which divideth it from *Anatolia* and *Thrace.*

It reacheth from the 50 degree of Longitude to the 195 which is 145 degrees from Weſt to Eaſt; and from the 40 degree of Northern Latitude, unto the 80, which is within 10 degrees

grees of the Pole it ſelf, By which accompt it lieth from the beginning of the ſixth Clime, where the longeſt day in Summer is 15 hours, till they ceaſe meaſuring the Climates, the longeſt day in the moſt Northern parts hereof being full ſix Months, and in the winter half of the Year, the night as long.

It is now divided into theſe five parts. 1. *Tartaria Precopenſis.* 2. *Aſiatica.* 3. *Antiqua.* 4. *Zagathay.* 5. *Cathay.*

China.

China is bounded on the North with *Altay* and the Eaſtern *Tartars*, from which it is ſeparated by a continued Chain of Hills, part of thoſe of *Ararat*, and where that chain is broken off or interrupted, with a great wall extended 400 Leagues in length; on the South partly with *Cauchin China* a Province of *India*, partly with the Ocean; on the Eaſt with the oriental Ocean, and on the Weſt with part of *India* and *Cathay*.

It reacheth from the 130 to the 160 degree of Longitude, and from the Tropick of *Cancer* to the 53 degree of Latitude; ſo that it lieth under all the Climes from the third to the ninth incluſively. The longeſt ſummers day in the ſouthern parts being 13 hours and 40 Minutes increaſed in the moſt northern parts to 16 hours and 3 quarters.

It containeth no fewer than 15 Provinces. 1. *Canton.* 2. *Foquien.* 3. *Olam.* 4. *Siſnam* 5. *Tolenchia.* 6. *Cauſay.* 7. *Minchian.* 8. *Ochian.* 9. *Honan.* 10. *Pagnia.* 11. *Taitan.* 12. *Quinchen.* 13. *Chagnian* 14. *Suſnan.* 15. *Cuniſay.*

niſay. Beſides the provinces of *Suchuen*, the Iſland of *Chorea* and the Iſland of *Cheaxan*.

India.

India is bounded on the Eaſt with the Oriental Ocean and ſome part of *China*; on the Weſt with the Perſian Empire; on the North with ſome Branches of Mount *Taurus*, which divide it from *Tartary*; on the South with the Indian Ocean.

Extended from 106 to 159 degrees of Longitude, and from the *Æquator* to the 44th degree of Northern Latitude, by which account it lieth from the beginning of the firſt to the end of the ſixth Clime, the longeſt Summers day in the ſouthern Parts being 12 hours onely, and in the parts moſt North 15 hours and a half.

The whole Country is divided into two main parts, *India intra Gangem*, and *India extra Gangem*.

The Oriental Iſlands.

The Oriental Iſlands are 1. *Japan*. 2. The *Philippine* and Iſles adjoyning. 3. The Iſlands of *Bantam*. 4. The *Moluccoes*. 5. Thoſe called *Sinda* or the *Celebes*. 6. *Java*. 7. *Borneo*. 8. *Sumatra*. 9. *Ceilan*. and 10. others of leſs note.

Chap. V.

CHAP. V.

Of Africk.

Africk is bounded on the East by the Red Sea, and Bay of *Arabia*, by which it is parted from *Asia*; on the West by the main Atlantick Oceans interposing between it and *America*; on the North by the Mediterranean Sea, which divides it from *Europe* and *Anatolia*; and on the South with the Æthiopick Ocean, separating it from *Terra Australis incognita* or the southern continent, parted from all the rest of the World except *Asia* only, to which it is joyned by a narrow *Isthmus* not above 60 miles in length.

It is situate for the most part under the Torrid Zones, the *Æquator* crossing it almost in the midst. It is now commonly divided into these seven parts. 1. *Ægypt*. 2. *Barbary* or the Roman Africk. 3. *Numidia*. 4. *Lybia*. 5. *Terra Nigritarum*. 6. *Æthiopia superior*. and 7. *Æthiopia inferior*.

Ægypt.

Ægypt is bounded on the East with *Idumea*, and the Bay of *Arabia*, on the West with *Barbary*, *Numidia*, and part of *Lybia*, on the North with the Mediterranean Sea; on the South with *Æthiopia superior*, or the *Abyssyn* Emperor; it is situate under the second and fifth Climates, so that the longest day in Summer is but thirteen hours and a half.

Barbary.

Barbary is bounded on the East with *Cyrenaica*; on the West with the Atlantick Ocean; on the North with the Mediterranean Sea, the Straits of *Gibralter* and some part of the Atlantick also; on the South with Mount *Atlas*, by which it is separated from *Lybia inferior* or the Desarts of *Lybia*.

It is situated under the third and fourth Climates: so that the longest Summers day in the parts most South, amounteth to 13 hours and 3 quarters, and in the most northern parts it is 14 hours and a quarter. This country is now reduced to the Kingdoms of 1. *Tunis*. 2. *Tremesch* or *Algiers*. 3. *Fesse* and 4. *Morocco*.

Numidia.

Numidia is bounded on the East with *Egypt*, on the West with the Atlantick Ocean; on the North with Mount *Atlas*, which parteth it from *Barbary* and *Cyrene*; on the South with *Lybia Deserta*.

Lybia.

Lybia is either *Interior* or *Deserta*, *Libia interior* is bounded on the North with Mount *Atlas* by which it is parted from *Barbary* and *Cyrenaica*; on the East with *Lybia Marmarica* interposed between it and *Egypt*, and part of *Æthiopia superior*, or the Habassine Empire; on the South with *Æthiopia inferior*, and the Land of the Negroes

groes; and on the West with the main Atlantick Ocean.

Lybia deserta is bounded on the North with *Numidia* or *Biledulgerid*; on the South with the Land of the Negroes; and on the West with *Gulata* another Province of the Negroes interposed between it and the Atlantick.

Terra Nigritarum.

Terra Nigritarum or the Land of the Negroes is bounded on the East with *Æthiopia Superior*; on the West with the Atlantick Ocean; on the North with *Lybia deserta* and on the South with the Ethiopick Ocean, and part of *Æthiopia Inferior*.

Æthiopia Superior.

Æthiopia Superior is bounded on the East with the Red Sea and the *Sinus Barbaricus*; on the West with *Lybia Interior*, the Realm of *Nubia* in the Land of the Negroes and part of the Kingdoms of *Congo* in the other *Æthiopia*; on the North with *Egypt* and *Lybia Marmarica*, and on the South with the Mountains of the Moon, by which it is parted from the main Body of *Æthiopia Inferior*.

It is situate on both sides of the Æquinoctial, extending from the South Parallel of seven degrees, where it meeteth with some part of the other *Æthiopia* to the Northern end of the Isle of *Meroz* situated under the fifth Parallel on the North of that Circle.

Æthiopia Inferior.

Æthiopia inferior is bounded on the East with the Red Sea; on the West with the Ethiopick Ocean; on the North with *Terra Nigritarum*, and the higher *Æthiopia*; and on the South where it endeth, is a point of a *Conus*, with the main Ocean parting it from the Southern undiscovered Continent. This in *Ptolemyes* time went under the name of *Terra incognita*.

CHAP. IV.

Of America.

America the fourth and last part of the World is bounded on the East with the Atlantick Ocean and the *Vergivian* Seas, by which it is parted from *Europe* and *Africa*; on the West with the *Pacifick* Ocean, which divides it from *Asia*; on the South with some part of *Terra Australis incognita*, from which it is separated by a long, but narrow Strait, called the Straits of *Magellan*; the North bounds of it hitherto not so well discovered, as that we can certainly affirm it to be Island or Continent.

It is called by some and that most aptly, The new World; New for the late discovery, and World for the vast greatness of it. The whole is naturally divided into two great Peninsules, whereof that towards the North is called *Mexicana*. That towards the South hath the name of *Peruana*: the *Isthmus* which joyneth these two together

together is very long, but narrow in some places not above 120 miles from Sea to Sea, in many not above seventeen.

The Northern *Peninsula* called *Mexicana*, may be most properly divided into the Continent and Islands: The Continent again into the several Provinces of 1. *Estotiland*, 2. *Nova Francia*, 3. *Virginia*, 4. *Florida*, 5. *California*, 6. *Nova Gallicia*, 7. *Nova Hispania*, 8. *Guntimala*. The Southern *Peninsula* called *Peruana*, taking in some part of the *Isthmus*, hath on the Continent the Provinces of 1. *Castella Aurea*, 2. *Nova Granada*, 3. *Peru*, 4. *Chile*, 5. *Paraguay*, 6. *Brasil*, 7. *Guiana*, and 8. *Paria*. The Islands which belong to both, are dispersed either in the Southern Ocean called *Mare del Zur*, where there is not any one of Note but those called *Los Ladrones* and the Islands of *Solomon*. Or in the Northern Ocean called *Mare del Noords*, reduced unto the *Caribes*, *Porto-Rico*, *Hispaniola*, *Cuba* and *Jamaica*. And thus much concerning the real and known parts of the Terrestrial Globe.

CHAP. XV.

Of the Description of the Terrestrial Globe by Maps Universal and Particular.

HItherto we have spoken of the true and real Terrestrial Globe, and of the measure thereof by Circles, Zones, and Climates, as it is usually represented by a Sphere or Globe; which must be confessed to be the neareſt and the most commensurable to nature: Yet it may also be de-

deſcribed upon a plain, in whole or in part many ſeveral ways: But thoſe which are moſt uſeful and artificial are theſe two, by Parallelogram and by Planiſphere.

2. The deſcription thereof by Parallelogram is thus, the Parallelogram is divided in the midſt by a line drawn from North to South, paſſing by the *Azores* or *Canaries* for the great Meridian. Croſs to this and at eight Angles, another line is drawn from Eaſt to Weſt for the *Æquator*; then two parallels to each to comprehend the figure, in the ſquares whereof there are ſet down four parts of the world rather than the whole: And this way of deſcription though not exact or near to the natural, hath yet been followed by ſuch as ought ſtill to be accounted excellent, and is the form of our plain Charts, and in places near the Æquinoctial may be uſed without committing any great error; becauſe the Meridians about the Æquinoctial are equi-diſtant, but as they draw up towards the Pole, they do upon the Globe come nearer and nearer together, to ſhew that their diſtance is proportionably diminiſhed till it come to a concurrence, and anſwerably the Parallels as they are deeper in latitude, ſo they grow leſs and leſs with the Sphere; ſo that at 60 degrees, the Equinoctial is double to the parallel of Latitude, and ſo proportionably of the reſt.

3. Hence it followeth, that if the picture of the earth be drawn upon a Parallelogram, ſo that the Meridians be equally diſtant throughout, and the Parallels equally extended, the Parellel of 60 degrees ſhall be as great as the line of the *Æquator* it ſelf is, and he that coaſteth about the world

in

in the latitude of 60 degrees, shall have as far to go by this Map, as he that doth it in the *Æquator*, though the way be but half as long. For the longitude of the Earth in the *Æquator* it self, is 21600; but in the Parallel of 60 but 10800 miles. So two Cities under the same parallel of 60, shall be of equal Longitude to other two under the Line, and yet the first two shall be but 50, the other two an hundred miles distant. So two Ships departing from the *Æquator* at 60 miles distance, and coming up to the Parallel of 60, shall be thirty miles nearer, and yet each of them keep the same Meridians and sail by this Card upon the very points of the Compass at which they set forth. This was complained of by *Martin Cortez* and others, and the learned *Mercator* considering well of it, caused the degrees of the Parallel to increase by a proportion towards the Pole. The Mathematical Generation whereof, Mr. *Wright* in the second Chapter of his *Correction of Errors in Navigation*, hath sought by the inscription of a Planisphere into a Concave Cylinder. And this description of the Earth upon a Parallelogram, may indeed be so ordered by Art, as to give a true account of the scituation and distance of the parts, but cannot be fitted to represent the figure of the whole.

4. The description therefore of the whole by Planisphere is much better, because it represents the face of the Earth upon a plain, in its own proper Spherical Figure as upon the Globe it self. This description cannot well be contrived upon so few as one Circle or more than two.

Suppose then the Globe to be divided into two

two equal parts or Hemiſpheres, which cannot be done but by a great Circle: And therefore it muſt be done by the *Æquator* or Meridian (for the Colure is all one with the Meridian) the Horizon cannot fix, and the Zodiack hath nothing to do here.

5. Suppoſe then the Globe to be flatted upon the plain of the *Æquator*, and you have the firſt way of projection dividing the Globe into the North and South Hemiſpheres.

In this projection the Pole is the Centre, the *Æquator* is the Circumference divided into 360 degrees of Longitude, the Parallels are whole Circles, the Meridians are ſtreight lines, the Parallels are Parallels indeed, and the Meridians equi-diſtantly concur, and therefore all the degrees are equal. After this way of projection, *Ptolemy* deſcribes that part of the habitable world which was diſcovered to his time.

6. Suppoſe the Globe to be flatted upon the plain of the Meridian, and you have the other way of projection; the *Æquator* here is a ſtreight line, the great Meridian a whole Circle, in this Section the Meridians do not equi-diſtantly concur, the Parallels are not Parallels indeed, and therefore the degrees are all unequal.

However, this latter way is that which is now moſt and indeed altogether in uſe.

7. Particular Maps are but limbs of the Globe, and therefore though they are drawn aſunder, yet are they ſtill to be done with that proportion, as a remembring eye may ſuddenly acknowledge, and joyn them to the whole Body.

The

The Projection is moſt commonly upon a Parallelogram, in which the Latitude is to be expreſſed by Parallels from North to South, and the Longitude by Meridians from Weſt to Eaſt at 10 or 15 degrees diſtance, as you pleaſe, and may be drawn either by circle or right Lines; but if they be right Lines, the Meridians are not to be drawn parallel, but inclining and concurring, to ſhew the nature of the whole, whereof they are ſuch parts. For the Graduation; the degrees of Longitude are moſt commonly divided upon the North and South ſides of the Parallelogram; the degrees of Latitude upon the Eaſt and Weſt ſides, or otherwiſe upon the moſt Eaſtern or Weſtern Meridian of the Map, within the ſquare. But it hath ſeemed good to ſome in theſe particular deſcriptions to make no graduation or projection at all; but to put the matter off to a ſcale of Miles, and leave the reſt to be believed.

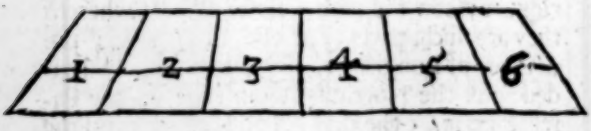
The difference of Miles in ſeveral Countries is great, but it will be enough to know that the Italian and Engliſh, are reckoned for all one, and four of theſe do make a German Mile; two a French League. The Swediſh or Daniſh Mile conſiſteth of 5 Miles Engliſh and ſomewhat more. Sixty common Engliſh and Italian Miles anſwer to a degree of a great Circle.

Now as the Miles of ſeveral Countries do very much differ, ſo thoſe of the ſame do not very much agree: and therefore the ſcales are commonly written upon with *Magna*, *Mediocria* and *Parva*, to ſhew the difference. In ſome Maps you ſhall find the Miles thus hiddenly ſet down, and the meaning is, that you ſhould meaſure the

Milliaria

Milliaria magna upon the lowermoſt Line, the *Parva* upon the uppermoſt, and the *Mediocria* upon the middlemoſt.

Scala Milliarium.



The uſe of the Scale is for the meaſuring the diſtances of places in the Map, by ſetting one foot of your Compaſſes in the little circle repreſenting one place, and the other foot in the like little circle repreſenting another, the Compaſſes kept at that diſtance being applied to the Scale, will ſhew the number of great or middle Miles according as the inhabitants of thoſe places are known to reckon.

Soli Deo Gloria.

A View

A View of the more Notable *Epochæ*

Epochæ.	Years of the Julian Period.	Months
The Julian Period	1	*Jan.* 1
Creation of the World	765	*Jan.* 1
Æra of the *Olympiades*	3938	*July* 8
The building of *Rome*	4961	*Ap.* 21
Epochæ of *Nabonaſſer*	3667	*Feb.* 26
The beginning of *Metons* Cyrcle.	4281	*June* 26
The beginning of the periods of *Calippus*	4384	*June* 28
The Death of *Alexander* the great	4390	*No.* 12
Æra of the *Caldees*	4403	*Oct.* 15
The *Æra* of *Dionyſius*	4429	*Mar.* 25

The beginning of the Chriſtian *Æra* falls in the 4713 year of the Julian Period.	Years of Chriſt	Month
The Dioclefian *Æra*	284	*Aug.* 29
The Turkiſh *Æra* or *Hijra*	622	*July* 16
The Perſian *Æra* from *Iſdagird*	632	*June* 16
The *Æra* from the Perſian *Sultan*	1079	*Mar.* 14

Days in the Year of

Julian Accompt				*Ægypt* and Perſian Accompt			
1	0	0	0	365	2	5	0
2	0	0	0	730	5	0	0
3	0	0	0	1095	7	5	0
4	0	0	0	1461	0	0	0
5	0	0	0	1826	2	5	0
6	0	0	0	2191	5	0	0
7	0	0	0	2556	7	5	0
8	0	0	0	2922	0	0	0
9	0	0	0	3287	2	5	0
10	0	0	0	3652	5	0	0

1	0	0	0	365	0	0	0
2	0	0	0	730	0	0	0
3	0	0	0	1095	0	0	0
4	0	0	0	1460	0	0	0
5	0	0	0	1825	0	0	0
6	0	0	0	2190	0	0	0
7	0	0	0	2555	0	0	0
8	0	0	0	2920	0	0	0
9	0	0	0	3285	0	0	0
10	0	0	0	3650	0	0	0

Days in Julian Months			Days in Ægyptian Months		Days in Persian Months	
Comon		Bissex				
January	31	30	*Thoth*	30	*Pharvadin*	30
February	59	60	*Paophi*	60	*Aripehast*	60
March	90	91	*Athyr*	90	*Chortat*	90
April	120	121	*Chœat*	120	*Tirma*	120
May	151	152	*Tybi*	150	*Mertat*	150
June	181	182	*Michir*	180	*Sachriur*	180
July	212	213	*Phamenoth*	210	*Macherma*	210
August	243	244	*Pharmuthi*	240	*Aptnina Wahak*	245
September	273	274	*Pachon*	270		
October	304	305	*Payny*	300	*Aderma*	275
November	334	335	*Epephi*	330	*Dima*	305
December	365	366	*Mesori*	339	*Pechmam*	335
			Epagomena	365	*Aphander*	365

Days in Turkish or Arabical Years						Days in Turkish Months	
1	354		21		7442	*Muharran*	30
2	709		22		7796	*Sapher*	59
3	·1063		23		8150	*Rabit* 1.	89
4	·1417		24		8505	*Rabit* 2.	118
5	·1772		25		8859	*Giumadi* 1.	148
6	·2126		26		9213	*Giumadi* 2.	177
7	·2480		27		9568	*Regeb*	207
8	·2835		28		9922	*Sahaben*	236
9	·3189		29		10276	*Ramaddan*	266
10	·3543		30	0	10631 0	*Scheval*	295
11	·3898		60	0	21262 0	*Dulkadati*	325
12	·4252		90	0	31893 0	*Dulhajati*	354
13	·4607		120	0	42524 0	*Dsilhittscbe true*	
14	·4961		150	0	53155 0		
15	·5315		180	0	63786 0	*In anno Abundanti*	355
16	·5670		210	0	74417 0		
17	·6024		240	0	05048 0		
18	·6378		270	0	95679 0		
19	·6733		300	0	106310 0		
20	·7087						

	January			February			March		
1	3	A	Circumcis.		D	Purificat	3	D	
2		B		11	E			E	
3	11	C		19	F		11	F	
4		D		8	G			G	
5	19	E			A		19	A	
6	8	F	Epiphany	16	B		8	B	
7		G		5	C			C	
8	16	A			D		16	D	
9	5	B		13	E		5	E	
10		C		2	F			F	
11	13	D			G		13	G	
12	2	E		10	A		2	A	
13		F			B			B	
14	10	G		18	C		10	C	
15		A		7	D			D	
16	18	B			E		18	E	
17	7	C		15	F		7	F	
18		D		4	G			G	
19	15	E			A		15	A	
20	4	F		12	B		4	B	
21		G		1	C			C	
22	12	A			D		12	D	
23	1	B		9	E		1	E	
24		C			F			F	
25	9	D	Conv.S.Paul	17	G	S.Matthias	9	G	Anunc.
26		E		6	A			A	
27	17	F			B		17	B	
28	6	G		14	C		6	C	
29		A						D	
30	14	B					14	E	
31	3	C					3	F	

April				May				Junt			
1		G		11	B	Phil.& Jac.			E		
2	11	A			C			19	F		
3		B		19	D			8	G		
4	19	C		8	E			16	A		
5	8	D			F			5	B		
6	16	E		16	G				C		
7	5	F		5	A			13	D		
8		G			B			2	E		
9	13	A		14	C				F		
10	2	B		2	D			10	G		
11		C			E				A		S.Barnaby
12	10	D		10	F			18	B		
13		E			G			7	C		
14	18	F		18	A				D		
15	7	G		7	B			15	E		
16		A			C			4	F		
17	15	B		15	D				G		
18	4	C		4	E			12	A		
19		D			F			1	B		
20	12	E		12	G				C		
21	1	F		1	A			9	D		
22		G			B				E		
23	9	A		9	C			17	F		
24		B			D			6	G		S.John B.
25	17	C	Mark Evang.	17	E				A		
26	6	D		6	F			14	B		
27		E			G			3	C		
28	14	F		14	A				D		
29	5	G		3	B			11	E		Pet. Ap.
30		A			C				F		
31				11	D						

	July			August			September		
1	15	G		8	C		16	F	
2	8	A		16	D		5	G	
3		B		5	E			A	
4	16	C			F		13	B	
5	5	D		13	G		2	C	
6		E		2	A			D	
7	13	F			B		10	E	
8	2	G		10	C			F	
9		A			D		18	G	
10	10	B		18	E		7	A	
11		C		7	F			B	
12	18	D			G		15	C	
13	7	E		15	A		4	D	
14		F			B			E	
15	15	G		4	C		12	F	
16	4	A		12	D		1	G	
17		B		1	E			A	
18	12	C			F		9	B	
19	1	D		9	G			C	
20		E	Margaret		A		17	D	
21	9	F		17	B		6	E	S. Matth.
22		G		6	C			F	
23	17	A			D		14	G	
24	6	B		14	E	Barthol.	3	A	
25		C		3	F			B	
26	14	D		11	G		11	C	
27	3	E		19	A		19	D	
28		F			B		8	E	
29	11	G		8	C			F	S. Mich.
30	9	A			D			G	
31		B			E				

October				November			December			
1	16	A			D	All Saints	13	F		
2	5	B		13	E	All Souls	2	G		
3	13	C		2	F			A		
4	2	D			G		10	B		
5		E		10	A	P. Conspir.		C		
6	10	F			B		18	D		
7		G		18	C		7	E		
8	18	A		7	D			F		
9	7	B			E		15	G		
10		C		15	F		4	A		
11	15	D		4	G			B		
12	4	E			A		12	C		
13		F		12	B		1	D		
14	12	G		1	C			E		
15	13	A			D		9	F		
16		B		9	E			G		
17	9	C			F		17	A		
18		D			G		6	B		
19	17	E	Luke Evang.	17	A			C		
20	6	F		6	B		14	D		
21		G		14	C		3	E	S. Thomas	
22	14	A		3	D			F		
23	3	B			E		11	G		
24		C		11	F		19	A		
25	11	D		19	G			B	Chri. Nat.	
26	19	E			A		8	C	S. Steph.	
27		F		8	B			D	S. John	
28	8	G	Sim. & Jude		C		16	E	Innocents	
29		A		16	D		5	F		
30	16	B		5	E	S. Andrew		G		
31	5	C					13	A	Sylvester	

	January		February		March	
1	*	A	XXIX	D	*	D
2	XXIX	B	XXVIII	E	XXIX	E
3	XXVIII	C	XXVII	F	XXVIII	F
4	XXVII	D	25.XXVI	G	XXVII	G
5	XXVI	E	XXV.XXIV	A	XXVI	A
6	25.XXV	F	XXIII	B	25.XXV	B
7	XXIV	G	XXII	C	XXIV	C
8	XXIII	A	XXI	D	XXIII	D
9	XXII	B	XX	E	XXII	E
10	XXI	C	XIX	F	XXI	F
11	XX	D	XVIII	G	XX	G
12	XIX	E	XVII	A	XIX	A
13	XVIII	F	XVI	B	XVIII	B
14	XVII	G	XV	C	XVII	C
15	XVI	A	XIV	D	XVI	D
16	XV	B	XIII	E	XV	E
17	XIV	C	XII	F	XIV	F
18	XIII	D	XI	G	XIII	G
19	XII	E	X	A	XII	A
20	XI	F	IX	B	XI	B
21	X	G	VIII	C	X	C
22	IX	A	VII	D	IX	D
23	VIII	B	VI	E	VIII	E
24	VII	C	V	F	VII	F
25	VI	D	IV	G	VI	G
26	V	E	III	A	V	A
27	IV	F	II	B	IV	B
28	III	G	I	C	III	C
29	II	A			II	D
30	I	B			I	E
31	*	C			*	F

	April		May		June	
1	XXIX	G	XXVIII	B	XXVII	E
2	XXVIII	A	XXVII	C	25.XXVI	F
3	XXVII	B	XXVI	D	XXV.XXIV	G
4	25.XXVI	C	25.XXV	E	XXIII	A
5	XXV.XXIV	D	XXIV	F	XXII	B
6	XXIII	E	XXIII	G	XXI	C
7	XXII	F	XXII	A	XX	D
8	XXI	G	XXI	B	XIX	E
9	XX	A	XX	C	XVIII	F
10	XIX	B	XIX	D	XVII	G
11	XVIII	C	XVIII	E	XVI	A
12	XVII	D	XVII	F	XV	B
13	XVI	E	XVI	G	XIV	C
14	XV	F	XV	A	XIII	D
15	XIV	G	XIV	B	XII	E
16	XIII	A	XIII	C	XI	F
17	XII	B	XII	D	X	G
18	XI	C	XI	E	IX	A
19	X	D	X	F	VIII	B
20	IX	E	IX	G	VII	C
21	VIII	F	VIII	A	VI	D
22	VII	G	VII	B	V	E
23	VI	A	VI	C	IV	F
24	V	B	V	D	III	G
25	IV	C	IV	E	II	A
26	III	D	III	F	I	B
27	II	E	II	G	*	C
28	I	F	I	A	XXIX	D
29	*	G	*	B	XXVIII	E
30	XXIX	A	XXIX	C	XXVII	F
31			XXVIII	D		

	July		August		September	
1	XXVI	G	XXV. XXIV	C	XXIII	F
2	25.XXV	A	XXIII	D	XXII	G
3	XXIV	B	XXII	E	XXI	A
4	XXIII	C	XXI	F	XX	B
5	XXII	D	XX	G	XIX	C
6	XXI	E	XIX	A	XVIII	D
7	XX	F	XVIII	B	XVII	E
8	XIX	G	XVII	C	XVI	F
9	XVIII	A	XVI	D	XV	G
10	XVII	B	XV	E	XIV	A
11	XVI	C	XIV	F	XIII	B
12	XV	D	XIII	G	XII	C
13	XIV	E	XII	A	XI	D
14	XIII	F	XI	B	X	E
15	XII	G	X	C	IX	F
16	XI	A	IX	D	VIII	G
17	X	B	VIII	E	VII	A
18	IX	C	VII	F	VI	B
19	VIII	D	VI	G	V	C
20	VII	E	V	A	IV	D
21	VI	F	IV	B	III	E
22	V	G	III	C	II	F
23	IV	A	II	D	I	G
24	III	B	I	E	*	A
25	II	C	*	F	XXIX	B
26	I	D	XXIX	G	XXVIII	C
27	*	E	XXVIII	A	XXVII	D
28	XXIX	F	XXVII	B	25.XXVI	E
29	XXVIII	G	XXVI	C	XXV. XXIV	F
30	XXVII	A	25.XXV	D	XXIII	G
31	25.XXVI	B	XXIV	E		

	October		November		December	
1	XXII	A	XXI	D	XX	F
2	XXI	B	XX	E	XIX	G
3	XX	C	XIX	F	XVIII	A
4	XIX	D	XVIII	G	XVII	B
5	XVIII	E	XVII	A	XVI	C
6	XVII	F	XVI	B	XV	D
7	XVI	G	XV	C	XIV	E
8	XV	A	XIV	D	XIII	F
9	XIV	B	XIII	E	XII	G
10	XIII	C	XII	F	XI	A
11	XII	D	XI	G	X	B
12	XI	E	X	A	IX	C
13	X	F	IX	B	VIII	D
14	IX	G	VIII	C	VII	E
15	VIII	A	VII	D	VI	F
16	VII	B	VI	E	V	G
17	VI	C	V	F	IV	A
18	V	D	IV	G	III	B
19	IV	E	III	A	II	C
20	III	F	II	B	I	D
21	II	G	I	C	*	E
22	I	A	*	D	XXIX	F
23	*	B	XXIX	E	XXVIII	G
24	XXIX	C	XXVIII	F	XXVII	A
25	XXVIII	D	XXVII	G	XXVI	B
26	XXVII	E	25.XXVI	A	25.XXV	C
27	XXVI	F	XXV. XXIV	B	XXIV	D
28	25.XXV	G	XXIII	C	XXIII	E
29	XXIV	A	XXII	D	XXII	F
30	XXIII	B	XXI	E	XXI	G
31	XXII	C			XX	A

A Table shewing the Dominical Letter, Golden Number and Epact, according to the Julian account for ever, and in the Gregorian, till the Year 1700.

1672	1	GF	CB
1673	2	E	A
1674	3	D	G
1675	4	C	F
1676	5	BA	ED
1677	6	G	C
1678	7	F	B
1679	8	E	A
1680	9	DC	GF
1681	10	B	E
1682	11	A	D
1683	12	G	C
1684	13	FE	BA
1685	14	D	G
1686	15	C	F
1687	16	B	E
1688	17	AG	DC
1689	18	F	B
1690	19	E	A
1691	20	D	G
1692	21	CB	FE
1693	22	A	D
1694	23	G	C
1695	24	F	B
1696	25	ED	AG
1697	26	C	F
1698	27	B	E
1699	28	A	D

Year	G N	Julian Epact	Gregor. Epact
1672	1	11	1
1673	2	22	12
1674	3	3	23
1675	4	14	4
1676	5	25	15
1677	6	6	26
1678	7	17	7
1679	8	28	18
1680	9	9	29
1681	10	20	10
1682	11	1	21
1683	12	12	2
1684	13	23	13
1685	14	4	24
1686	15	15	5
1687	16	26	16
1688	17	7	17
1689	18	18	8
1690	19	29	19

The anticipation of the Gregorian Calendar.

From 5 *October* 1582	D. 10
From 24 *Feb.* 1700	D. 11
From 24 *Feb.* 1800	D. 12
From 24 *Feb.* 1900	D. 13
From 24 *Feb.* 2100	D. 14
From 24 *Feb.* 2200	D. 15
From 24 *Feb.* 2320	D. 16

	October		November		December	
1	XXII	A	XXI	D	XX	F
2	XXI	B	XX	E	XIX	G
3	XX	C	XIX	F	XVIII	A
4	XIX	D	XVIII	G	XVII	B
5	XVIII	E	XVII	A	XVI	C
6	XVII	F	XVI	B	XV	D
7	XVI	G	XV	C	XIV	E
8	XV	A	XIV	D	XIII	F
9	XIV	B	XIII	E	XII	G
10	XIII	C	XII	F	XI	A
11	XII	D	XI	G	X	B
12	XI	E	X	A	IX	C
13	X	F	IX	B	VIII	D
14	IX	G	VIII	C	VII	E
15	VIII	A	VII	D	VI	F
16	VII	B	VI	E	V	G
17	VI	C	V	F	IV	A
18	V	D	IV	G	III	B
19	IV	E	III	A	II	C
20	III	F	II	B	I	D
21	II	G	I	C	*	E
22	I	A	*	D	XXIX	F
23	*	B	XXIX	E	XXVIII	G
24	XXIX	C	XXVIII	F	XXVII	A
25	XXVIII	D	XXVII	G	XXVI	B
26	XXVII	E	25.XXVI	A	25.XXV	C
27	XXVI	F	XXV. XXIV	B	XXIV	D
28	25.XXV	G	XXIII	C	XXIII	E
29	XXIV	A	XXII	D	XXII	F
30	XXIII	B	XXI	E	XXI	G
31	XXII	C			XX	A

A Table shewing the Dominical Letter, Golden Number and Epact, according to the Julian account for ever, and in the Gregorian, till the Year 1700.

1672	1	GF	CB
1673	2	E	A
1674	3	D	G
1675	4	C	F
1676	5	BA	ED
1677	6	G	C
1678	7	F	B
1679	8	E	A
1680	9	DC	GF
1681	10	B	E
1682	11	A	D
1683	12	G	C
1684	13	FE	BA
1685	14	D	G
1686	15	C	F
1687	16	B	E
1688	17	AG	DC
1689	18	F	B
1690	19	E	A
1691	20	D	G
1692	21	CB	FE
1693	22	A	D
1694	23	G	C
1695	24	F	B
1696	25	ED	AG
1697	26	C	F
1698	27	B	E
1699	28	A	D

Year	G N	Julian Epact	Gregor. Epact
1672	1	11	1
1673	2	22	12
1674	3	3	23
1675	4	14	4
1676	5	25	15
1677	6	6	26
1678	7	17	7
1679	8	28	18
1680	9	9	29
1681	10	20	10
1682	11	1	21
1683	12	12	2
1684	13	23	13
1685	14	4	24
1686	15	15	5
1687	16	26	16
1688	17	7	17
1689	18	18	8
1690	19	29	19

The anticipation of the Gregorian Calendar.

From 5 *October* 1582	D.10
From 24 *Feb.* 1700	D.11
From 24 *Feb.* 1800	D.12
From 24 *Feb.* 1900	D.13
From 24 *Feb.* 2100	D.14
From 24 *Feb.* 2200	D.15
From 24 *Feb.* 2300	D.16

		III	IV	V	VI	VII	VIII
1	P	*	XI	XXII	III	XIV	XXV
2	N	XXIX	X	XXI	II	XIII	XXIV
3	M	XXVIII	IX	XX	I	XII	XXIII
4	H	XXVII	VIII	XIX	*	XI	XXII
5	G	XXVI	VII	XVIII	XXIX	X	XXI
6	F	XXV	VI	XVII	XXVIII	IX	XX
7	E	XXIV	V	XVI	XXVII	VIII	XIX
8	D	XXIII	IV	XV	XXVI	VII	XVIII
9	C	XXII	III	XIV	XXV	VI	XVII
10	B	XXI	II	XIII	XXIV	V	XVI
11	A	XX	I	XII	XXIII	IV	XV
12	u	XIX	*	XI	XXII	III	XIV
13	t	XVIII	XXIX	X	XXI	II	XIII
14	s	XVII	XXVIII	IX	XX	I	XII
15	r	XVI	XXVII	VIII	XIX	*	XI
16	q	XV	XXVI	VII	XVIII	XXIX	X
17	p	XIV	XXV	VI	XVII	XXVIII	IX
18	n	XIII	XXIV	V	XVI	XXVII	VIII
19	m	XII	XXIII	IV	XV	XXVI	VII
20	l	XI	XXII	III	XIV	XXV	VI
21	k	X	XXI	II	XIII	XXIV	V
22	i	IX	XX	I	XII	XXIII	IV
23	h	VIII	XIX	*	XI	XXII	III
24	g	VII	XVIII	XXIX	X	XXI	II
25	f	VI	XVII	XXVIII	IX	XX	I
26	e	V	XVI	XXVII	VIII	XIX	*
27	d	IV	XV	XXVI	VII	XVIII	XXIX
28	c	III	XIV	XXV	VI	XVII	XXVIII
29	b	II	XIII	XXIV	V	XVI	XXVII
30	a	I	XII	XXIII	IV	XV	XXVI

IX	X	XI	XII	XIII	XIV	XV
VI	XVII	XXVIII	IX	XX	I	XII
V	XVI	XXVII	VIII	XIX	*	XI
IV	XV	XXVI	VII	XVIII	XXIX	X
III	XIV	XXV	VI	XVII	XXVIII	IX
II	XIII	XXIV	V	XVI	XXVII	VIII
I	XII	XXIII	IV	XV	XXVI	VII
*	XI	XXII	III	XIV	25	VI
XXIX	X	XXI	II	XIII	XXIV	V
XXVIII	IX	XX	I	XII	XXIII	IV
XXVII	VIII	XIX	*	XI	XXII	III
XXVI	VII	XVIII	XXIX	X	XXI	II
XXV	VI	XVII	XXVIII	IX	XX	I
XXIV	V	XVI	XXVII	VIII	XIX	*
XXIII	IV	XV	XXVI	VII	XVIII	XXIX
XXII	III	XIV	XXV	VI	XVII	XXVIII
XXI	II	XIII	XXIV	V	XVI	XXVI
XX	I	XII	XXIII	IV	XV	XXVII
XIX	*	XI	XXII	III	XIV	25
XVIII	XXIX	X	XXI	II	XIII	XXIV
XVII	XXVIII	IX	XX	I	XII	XXIII
XVI	XXVII	VIII	XIX	*	XI	XXII
XV	XXVI	VII	XVIII	XXIX	X	XXI
XIV	XV	VI	XVII	XXVIII	IX	XX
XIII	XXIV	V	XVI	XXVII	VIII	XIX
XII	XXIII	IV	XV	XXVI	VII	XVIII
XI	XXII	III	XIV	25	VI	XVII
X	XXI	II	XIII	XXIV	V	XVI
IX	XX	I	XII	XXIII	IV	XV
VIII	XIX	*	XI	XXII	III	XIV
VII	XVIII	XIX	X	XXI	II	XIII

	XVI	XVII	XVIII	XIX	I	II
P	XXIII	IV	XV	XXVI	VIII	XIX
N	XXII	III	XIV	25	VII	XVIII
M	XXI	II	XIII	XXIV	VI	XVII
H	XX	I	XII	XXIII	V	XVI
G	XIX	*	XI	XXII	IV	XV
F	XVIII	XXIX	X	XXI	III	XIV
E	XVII	XXVIII	IX	XX	II	XIII
D	XVI	XXVII	VIII	XIX	I	XII
C	XV	XXVI	VII	XVIII	*	XI
B	XIV	25	VI	XVII	XXIX	X
A	XIII	XXIV	V	XVI	XXVIII	IX
u	XII	XXIII	IV	XV	XXVII	VIII
t	XI	XXII	III	XIV	XXVI	VII
r	X	XXI	II	XIII	25	VI
r	IX	XX	I	XII	XXIV	V
q	VIII	XIX	*	XI	XXIII	IV
p	VII	XVIII	XXIX	X	XXII	III
n	VI	XVII	XXVIII	IX	XXI	II
m	V	XVI	XXVII	VIII	XX	I
l	IV	XV	XXVI	VII	XIX	*
k	III	XIV	25	VI	XVIII	XXIX
i	II	XIII	XXIV	V	XVII	XXVIII
h	I	XII	XXIII	IV	XVI	XXVII
g	*	XI	XXII	III	XV	XXVI
f	XXIX	X	XXI	II	XIV	25
e	XXVIII	IX	XX	I	XIII	XXIV
d	XXVII	VIII	XIX	*	XII	XXIII
c	XXVI	VII	XVIII	XXIX	XI	XXII
b	25	VI	XVII	XXVIII	X	XXI
a	XXIV	V	XVI	XXVII	IX	XX

Anni Christi.			
N	1		
P	320		
P	580	Biſſ.	
a	800	Biſſ.	C
b	1100	Biſſ.	C
c	1400	Biſſ.	C
	Detractis decem diebus.		
D	1484		
D	1600	Biſſ.	
C	1700		
C	1800		CC
B	1900		
B	2000	Biſſ.	
B	2100		C
A	2200		
u	2300		
A	2400	Biſſ.	C
u	2500		
t	2600		
t	2700		C
t	2800	Biſſ.	
ſ	2900		
ſ	3000		C
r	3100		
r	3200	Biſſ.	
r	3300		C
q	3400		
p	3500		

Anni Christi.			
q	3600	Biſſ.	C
p	3700		
n	3800		
n	3900		
n	4000	Biſſ.	C
m	4100		
l	4200		
l	4300		CC
l	4400	Biſſ.	
k	4500		
k	4600		C
i	4700		
i	4800	Biſſ.	
i	4900		C
h	5000		
g	5100		
h	5200	Biſſ.	C
g	5300		
f	5400		
f	5500		C
f	5600	Biſſ.	
e	5700		
e	5800		C
d	5900		
d	6000	Biſſ.	
d	6100		C
c	6200		
b	6300		
c	6400	Biſſ.	C
b	6500		

A Table shewing the Dominical Letter both in the Julian and the Gregorian account for ever.

Cy. ⊙	1	2	3	4	5	6	7
1	C B	D C	E D	F E	G F	A G	B A
2	A	B	C	D	E	F	G
3	G	A	B	C	D	E	F
4	F	G	A	B	C	D	E
5	E D	F E	G F	A G	B A	C B	D C
6	C	D	E	F	G	A	B
7	B	C	D	E	F	G	A
8	A	B	C	D	E	F	G
9	G F	A G	B A	C B	D C	E D	F E
10	E	F	G	A	B	C	D
11	D	E	F	G	A	B	C
12	C	D	E	F	G	A	B
13	B A	C B	D C	E D	F E	G F	A G
14	G	A	B	C	D	E	F
15	F	G	A	B	C	D	E
16	E	F	G	A	B	C	D
17	D C	E D	F E	G F	A G	B A	C B
18	B	C	D	E	F	G	A
19	A	B	C	D	E	F	G
20	G	A	B	C	D	E	F
21	F E	G F	A G	B A	C B	D C	E D
22	D	E	F	G	A	B	C
23	C	D	E	F	G	A	B
24	B	C	D	E	F	G	A
25	A G	B A	C B	D C	E D	F E	G F
26	F	G	A	B	C	D	E
27	E	F	G	A	B	C	D
28	D	E	F	G	A	B	C
Anni	1582 1600	1790	1800	1900 2000	2100	2200	2300 2400
Chr.	2500	2600	2700 2800	2900	3000	3100 3200	3300

			LXX	Ash.	East.	Asci.	Pent.	Corp. Christi.	Adv.
16	XXIII		Jan.	Feb.	Mar.	Apr.	May.	May	Nov.
5	XXII	d	18	4	22	30	10	21	29
	XXI	e	19	5	23	Ma. 1	11	22	30
13	XX	f	20	6	24	2	12	23	De. 1
2	XIX	g	21	7	25	3	13	24	2
	XVIII	a	22	8	26	4	14	25	3
10	XVII	b	23	9	27	5	15	26	No. 27
	XVI	c	24	10	28	6	16	27	28
18	XV	d	25	11	29	7	17	28	29
7	XIV	e	26	12	30	8	18	29	30
	XIII	f	27	13	31	9	19	30	Dec. 1
15	XII	g	28	14	Ap. 1	10	20	31	2
4	XI	a	29	15	2	11	21	Jun. 1	3
	X	b	30	16	3	12	22	2	No. 27
12	IX	c	31	17	4	13	23	3	28
1	VIII	d	Feb. 1	18	5	14	24	4	29
	VII	e	2	19	6	15	25	5	30
9	VI	f	3	20	7	16	26	6	Dec. 1
	V	g	4	21	8	17	27	7	2
17	IV	a	5	22	9	18	28	8	3
6	III	b	6	23	10	19	29	9	No. 27
	II	c	7	24	11	20	30	10	28
14	I	d	8	25	12	21	31	11	29
3	*	e	9	26	13	22	Jun. 1	12	30
	XXIX	f	10	27	14	23	2	13	Dec. 1
11	XXVIII	g	11	28	15	24	3	14	2
	XXVII	a	12	Ma. 1	16	25	4	15	3
19	25. XXVI	b	13	2	17	26	5	16	No. 27
8	XXV. XXIV	c	14	3	18	27	6	17	28
		d	15	4	19	28	7	18	29
		e	16	5	20	29	8	19	30
		f	17	6	21	30	9	20	Dec. 1
		g	18	7	22	31	10	21	2
		a	19	8	23	Jun. 1	11	22	3
		b	20	9	24	2	12	23	No. 27
		c	21	10	25	3	13	24	28

A Table to convert Sexagenary Degrees and Minutes into Decimals and the contrary.

1	00	37	10	73	20	109	30	145	40	181	50
2		38		74		110		146		182	
3		39		75		111		147		183	
4	01	40	11	76	21	112	31	148	41	184	51
5		41		77		113		149		185	
6		42		78		114		150		186	
7		43		79		115		151		187	
8	02	44	12	80	22	116	32	152	42	188	52
9		45		81		117		153		189	
10		46		82		118		154		190	
11	03	47	13	83	23	119	33	155	43	191	53
12		48		84		120		156		192	
13		49		85		121		157		193	
14		50		86		122		158		194	
15	04	51	14	87	24	123	35	159	44	195	54
16		52		88		124		160		196	
17		53		89		125		161		197	
18	05	54	15	90	25	126	35	162	45	198	55
19		55		91		127		163		199	
20		56		92		128		164		200	
21		57		93		129		165		201	
22	06	58	16	94	26	130	36	166	46	202	56
23		59		95		131		167		203	
24		60		96		132		168		204	
25		61		97		133		169		205	
26	07	62	17	98	27	134	37	170	47	206	57
27		63		99		135		171		207	
28		64		100		136		172		208	
29	08	65	18	101	28	137	38	173	48	209	58
30		66		102		138		174		210	
31		67		103		139		175		211	
32		68		104		140		176		212	
33	09	69	19	105	29	141	39	177	49	213	59
34		70		106		142		178		214	
35		71		107		143		179		215	
36	10	72	20	108	30	144	40	180	50	216	60

A Table to Convert Sexagenary Degrees and Minutes into Decimals and the contrary.

217	60	253	70	289	80	325	90	277777778
218		254		290		326		555555555
219		255		291		327		833333333
220	61	256	71	292	81	328	91	111111111
221		257		293		329		388888889
222		258		294		330		666666667
223	—	259	—	295	—	331	—	944444444
224	62	260	72	296	82	332	92	222222222
225		261		297		333		500000000
226		262		298		334		777777778
227	63	263	73	299	83	335	93	055555555
228		264		300		336		333333333
229	—	265	—	301	—	337	—	611111111
230		266		302		338		888888889
231	64	267	74	303	84	339	94	166666667
232		268		304		340		444444444
233		269		305		341		722222222
234	65	270	75	306	85	342	95	000000000
235		271		307		343		277777778
236		272		308		344		555555555
237		273		309		345		833333333
238	66	274	76	310	86	346	96	111111111
239		275		311		347		388888889
240		276		312		348		666666667
241	—	277	—	313	—	349	—	944444444
242	67	278	77	314	87	350	97	222222222
243		279		315		351		500000000
244		280		316		352		777777778
245	68	281	78	317	88	353	98	055555555
246	—	282	—	318	—	354	—	333333333
247		283		319		355		611111111
248		284		320		356		888888889
249	69	285	79	321	89	357	99	166666667
250		286		322		358		444444444
251		287		323		359		722222222
252	70	288	80	324	90	360	100	000000000

A Table to Convert Sexagenary Minutes into Decimals and the contrary.

	Minutes	Seconds	Thirds
1	00462962	00007716	00000128
2	00925925	15432	257
3	01388889	23148	385
4	01851851	30864	515
5	02314814	00038580	00000643
6	02777778	46296	771
7	03240740	54012	900
8	03703703	61728	1028
9	04166667	69444	1157
10	04629629	00077160	00001286
11	05092592	084876	1414
12	05555555	092592	1543
13	06018518	100308	1671
14	06481480	108024	1800
15	06944444	00115740	1929
16	07409407	123456	2057
17	07870370	131172	2186
18	08333333	138889	2314
19	08796296	146604	2443
20	09259259	00154320	2572
21	00722222	162036	2700
22	10185185	169752	2829
23	10648148	177468	2957
24	11111111	185184	3086
25	11574074	00192900	3215
26	12037037	200616	3343
27	12500000	208332	3472
28	12962962	216048	3600
29	13425926	223764	3729
30	13888889	00231481	00003858

	Minutes	Seconds	Thirds
31	14351852	00239670	00003986
32	14814814	246913	4115
33	15277777	254629	4243
34	15747040	262345	4372
35	16203703	270061	4501
36	16666666	00277777	00004629
37	17129629	285493	4758
38	17592592	293209	4886
39	18055555	300925	5015
40	18518518	308640	5144
41	18981481	00316356	00005272
42	19444414	324072	5401
43	19907407	331788	5529
44	20370370	339504	5658
45	20833333	347220	5787
46	21296296	00354936	00005915
47	21759259	362652	6044
48	22222222	370370	6172
49	22685185	378084	6301
50	23148148	385802	6430
51	23611111	00393518	00006558
52	24074074	401234	6687
53	24537037	408950	6815
54	25000000	416666	6944
55	25462963	424382	7073
56	25925926	00432098	00007201
57	26388888	439814	7330
58	26851852	447530	7458
59	27314814	455256	7587
60	27777777	00462962	00007716

A Table Converting Hours and Minutes into Degrees and Minutes of the *Æquator*, and into

	Hours.		Minutes
1	04.16666667	1	0.06944444
2	08.33333333	2	0.13888888
3	12.5	3	0.20833333
4	16.16666667	4	0.27777777
5	20.83333333	5	0.34722222
6	25.0	6	0.41666666
7	29.16666667	7	0.48611111
8	33.33333333	8	0.55555555
9	37.5	9	0.625
10	41.66666667	10	0.69444444
11	45.83333333	11	0.76388888
12	50.	12	0.83333333
13	54.16666667	13	0.90277777
14	58.33333333	14	0.97222222
15	62.5	15	1.04166666
16	66.66666667	16	1.11111111
17	70.83333333	17	1.18055555
18	75.00	18	1.25
19	79.16666667	19	1.31944444
20	83.33333333	20	1.38888888
21	87.5	21	1.45833333
22	91.66666667	22	1.52777777
23	95.83333333	23	1.59722222
24	100.00000000	24	1.66666666
		25	1.73611111
		26	1.80555555
		27	1.875
		28	1.94444444
		29	2.01388888
		30	2.08333333

The Decimal parts of a Day and the contrary.

Seconds		Minutes	Seconds
.00115740	31	2.15277777	.03587963
.00231481	32	2.22222222	.03703704
.00347222	33	2.29166666	.03819444
.00462962	34	2.36111111	.03935185
.00578703	35	2.43055555	.04050926
.00694444	36	2.5	.04166666
.00810184	37	2.56944444	.04282407
.00925925	38	2.63888888	.04398148
.01041666	39	2.70833333	.04513888
.01157405	40	2.77777777	.04629629
.01273148	41	2.84722222	.04745370
.01388888	42	2.91666666	.04861111
.01504630	43	2.98611111	.04976852
.01620371	44	3.05555555	.05092592
.01736111	45	3.125	.05208333
.01851853	46	3.19444444	.05324074
.01967593	47	3.26388888	.05439814
.02083333	48	3.33333333	.05555555
.02199074	49	3.40277777	.05671296
.02314810	50	3.47222222	.05787037
.02430555	51	3.54166666	.05902777
.02546295	52	3.61111111	.06018518
.02662037	53	3.68055555	.06134259
.02777777	54	3.75	.0625
.02893518	55	3.81944444	.06365741
.03009259	56	3.88888888	.06481481
.03125000	57	3.95833333	.06597222
.03240741	58	4.02777777	.06712963
.03356482	59	4.09722222	.06828704
.03472222	60	4.16666666	.06944444

A Catalogue of some of the most eminent Cittes and Towns in *England* and *Ireland* wherein is shewed the difference of their Meridian from *London*, with the hight of the Pole.

Names of Cities	Differ. Merid.	Hight Pole
	o 1 s	55.55
St. *Albons*	o 6 s	55.49
Barwick	o 2 s	52.18
Bedford	o 11 s	51.32
Briſtol	o o	53. 2
Boſton	o 1 e	52.17
Cambridge	o 5 e	51.27
Canterbury	o 10 s	54.57
Carlile	o 11 s	53.20
Cheſter	o 4 s	52.30
Coventry	o 17 s	52.2
Carmarthen	o 3 s	50.56
Chicheſter	o 5 e	52.4
Colcheſter	o 5 s	53.6
Darby	o 26 s	53.11
Dublin in *Ireland*	o 5 s	54.45
Dureſme	o 15 s	50.32
Dartmouth	o 1 e	52.20
Ely	o 2 s	52.58
Grantham	o 9 s	52.00
Gloceſter	o 6 s	52.49
Haleſax	o 1 s	52.50
Hartford	o 11 s	52.14
Hereford	o 1 s	52.19
Huntington	o 1 s	53.58
Hull	o 11 s	54.08
Lancaſter	o 4 s	52.40
Leiceſter		

Names of the Cities	Differ. Merid.	Hight Pole
Lincoln	0 1 3	53. 12
Middle of the Isle of Man	0 17 3	54. 22
Nottingham	0 4 3	53. 03
Newark	0 3 3	53. 02
Newcastle	0 6 3	54. 58
N. Luffingham	0 3 3	52. 41
Norwich	0 4 2	52. 44
Northampton	0 4 3	52. 18
Oxford	0 5 3	51. 54
Okenham	0 3 3	52. 44
Peterborough	0 2 3	52. 35
Richmond	0 6 3	54. 26
Rochester	0 3 2	51. 28
Ross	0 10 3	52. 07
St. Michaels Mount in Cornwal	0 23 3	50. 38
Stafford	0 8 3	52. 55
Stamford	0 2 3	52. 41
Shrewsbury	0 11 3	52. 48
Tredah in Ireland	0 27 3	53. 28
uppingham in Rutland	0 3 3	52. 40
Warwick	0 6 3	52. 25
Winchester	0 5 3	50. 10
waterford in Ireland	0 27 3	52. 22
Worcester	0 9 3	52. 20
Yarmouth in Suffolk	0 6 2	52. 45
York	0 4 3	54. 00
London	0 00	51. 32

The Suns mean Longitude and mean Anomaly in Ægyptian Years.

	☉ Mean Longitude	☉ Mean Anomaly
1	99.93364 37563	99.92889 33116
2	99.86728 75126	99.85778 66232
3	99.80093 12690	99.78667 99348
4	99.73457 50253	99.71557 32465
5	99.66821 87816	99.64446 65581
6	99.60186 25380	99.57335 98697
7	99.53550 62943	99.50225 31814
8	99.46915 00506	99.42114 64930
9	99.40279 38070	99.36003 98046
10	99.33643 75633	99.28893 31162
100	93.36437 56334	92.88933 11628
1000	33.64375 63341	28.89331 16289

The Suns Mean Anomaly and Præcession of the Æquinox in Ægyptian Years.

Year.	☉ Mean Anomaly	Præcession Æquinox.
1	99.92978 57316	00.00385 80246
2	99.85957 14632	00.00771 60493
3	99.78935 71949	00.01157 40740
4	99.71914 29265	00.01543 20987
5	99.64892 86582	00.01929 01234
6	99.57871 43898	00.02314 81481
7	99.50850 01114	00.02700 61728
8	99.29785 73164	00.03086 41975
9	99.36807 15847	00.03472 22221
10	99.29785 73164	00.03858 02469
100	92.97857 31642	00.38580 24691
1000	9.78573 16427	03.85802 46913

The Suns mean Longitude and mean Anomaly in Julian Years.

	☉ Mean Longitude	☉ Mean Anomaly
1	99.93364 37563	99.92889 33116
2	99.86728 75126	99.85778 66232
3	99.80093 12689	99.78667 99348
B 4	00.00836 58301	99.98929 01234
5	99.94200 95864	99.91818 34350
6	99.87565 33427	99.84707 67466
7	99.80929 70990	99.77597 00583
B 8	00.01673 16602	99.97858 02468
9	99.95037 54165	99.90747 35584
10	99.88401 91728	99.83636 68700
11	99.81766 29291	99.76525 91816
B 12	00.02509 74903	99.96787 03702
13	99.95874 12466	99.89676 36818
14	99.89238 50029	99.82565 69934
15	99.82602 87592	99.75455 03050
B 16	00.03346 33205	99.95716 04036
17	99.96710 70768	99.88605 48052
18	99.90075 08331	99.81494 81168
19	99.83439 45894	99.74384 14284
B 20	00.04182 91506	99.91645 06171
40	00.08365 83012	99.89290 12342
60	00.12548 74518	99.83935 18513
80	00.16731 66024	99.78580 24684
100	00.20914 57530	99.73225 30855
200	00.41820 15060	99.46450 61710
300	00.62743 72590	99.19675 92565
400	00.83658 30120	98.92901 23420
500	01.04572 87650	98.66126 54275
600	01.25487 45180	98.39351 85130
700	01.46402 02710	98.12577 15985

The ☉ mean Longitude and Anomaly

Æra	☉ mean Longitude	☉ mean Anomaly
Chr.	77.22400.86419	58.24289.56790
1600	80.54391.97529	53.95880.62961
1620	80.59074.89035	53.90525.69132
1640	80.63257.80541	53.85170.75303
1660	80.67440.72047	53.79815.81474
1680	80.71623.63553	53.74460.87645
1700	80.75806.55059	53.69105.93816
1720	80.79989.46665	53.63750.99987
1740	80.84172.38171	53.58396.06158
1760	80.88265.29677	53.53041.12329
	☉ mean Lon. in Mon.	☉ mean Ano. in Mo.
Janu.	08.48751.45488	08.48711.14867
Febr.	16.15365.74832	16.15288.96037
Mar.	24.64117.24320	24.64060.10904
April	32.85489.65760	32.85333.47872
May	41.34241.15248	41.34044.62739
June	49.55613.56688	49.55377.99708
July	58.04365.06176	58.04089.14575
Aug.	66.53116.55664	66.52800.29442
Sept.	74.74488.97104	74.74133.66410
Octo.	83.23240.46592	85.22844.81277
Nov.	91.44612.88032	91.44178.18245
Dec.	99.93364.37563	99.92889.33116

In Anno Bissentili, post Februarium adde unum diem & unius dies motum.

The Suns mean Longitude and mean Anomaly in Days		
	☉ mean Longitude	☉ mean Anomaly
1	0.2737908048	0.2737777898
2	0.5475816096	0.5475555796
3	0.8213724144	0.8213333694
4	1.0951632192	1.0951111592
5	1.3689540240	1.3688889490
6	1.6427448288	1.6426667388
7	1.9165356336	1.9164445286
8	2.1903264384	2.1902223184
9	2.4641172432	2.4640001082
10	2.7379080480	2.7377778980
11	3.0116988528	3.0115556878
12	3.2854896576	3.2853334776
13	3.5592804624	3.5591112674
14	3.8330712672	3.8328890572
15	4.1068620720	4.1066668470
16	4.3806428768	4.3804446368
17	4.6544436816	4.6542224266
18	4.9282344864	4.9280002164
19	5.2020252912	5.2017780062
20	5.4758160960	5.4755557960
21	5.7496069008	5.7493335858
22	6.0233977056	6.0231113756
23	6.2971885104	6.2968891654
24	6.5709793152	6.5706669552
25	6.8447701200	6.8444447450
26	7.1185609248	7.1182225348
27	7.3923517296	7.3920003246
28	7.6661425344	7.6957781144
29	7.9399333392	7.9395559042
30	8.2137241440	8.2133336940
31	8.4875149488	8.4871114838

The Suns mean Longitude and mean Anomaly in Days

	☉ Mean Longitude	☉ Mean Anomaly.
1	0.0114079502	0.0114074079
2	0.0228159004	0.0228148158
3	0.0342238506	0.0342222237
4	0.0456318008	0.0456296316
5	0.0570397510	0.0570370395
6	0.0684477012	0.0684444474
7	0.0798556514	0.0798518553
8	0.0912636016	0.0912592632
9	0.1026715518	0.1026666711
10	0.1140795020	0.1140740790
11	0.1254874522	0.1254814869
12	0.1368954024	0.1368888948
13	0.1483033526	0.1482963027
14	0.1597113028	0.1597037106
15	0.1711192530	0.1711111185
16	0.1825272032	0.1825185264
17	0.1939351534	0.1939259343
18	0.2053431036	0.2053333422
19	0.2167510538	0.2167407501
20	0.2281590040	0.2281481580
21	0.2395669542	0.2395555659
22	0.2509749044	0.2509629738
23	c.2623828546	0.2623703817
24	0.2737777048	0.2737777896

The Suns mean Anomaly and Præceſſion of the Æquinox.

Æra	☉ Anomaly.	Præceſſ. Æquinox
Chr.	56.69976.85185	20.49768.51851
1600	53.87323.10751	26.67052.46907
1620	53.83789.15687	26.74768.51845
1640	53.80255.20623	26.82484.56783
1660	53.76721.25559	26.90200.61721
1680	53.73187.30495	26.97916.66659
1700	53.69653 35431	27.05632.71597
1720	53.66119.40367	27.13348.76535
1740	53.65585.45303	27.21064.81473
1760	53.59051.50230	27.28780.86411
	☉ Anomaly in Months	Præceſſ. Æquinox in Months
Janu	08.48718.72813	0.00032.76678
Febr.	16.15303.38579	0.00062.36258
Mar.	24.64022.11392	0.00095.12937
April	32.85362.81857	0.00126.83916
May	41.34081.54670	0.00159.60594
June	49.55422.25134	0.00191.31573
July	58.04140.97947	0.00224.08251
Aug.	66.52859.70760	0.00256.84929
Sept.	74.74200.41225	0.00288.55908
Octo.	83.22919.14038	0.00321.32587
Nov.	91.44259.84502	0.00353.03566
Dec.	99.92978.57315	0.00385.80244

The ⊙ mean Anomaly, and Præceſſion of the Æquinox in Julian Years.

	⊙ mean Anomaly	Præceſſ. Æquinox
1	99.92978573 16	00.00385802 46
2	99.85957146 32	00.00771604 93
3	99.78935719 49	00.01157407 40
B 4	99.99292316 86	00.01543209 87
5	99.92270890 02	00.01929012 33
6	99.85249463 18	00.02314814 79
7	99.78228036 34	00.02700617 25
B 8	99.98584633 72	00.03086419 74
9	99.91563206 88	00.03472222 20
10	99.84541780 04	00.03858024 66
11	99.77520353 21	00.04243827 14
B 12	99.97876950 58	00.04629629 61
13	99.90855523 74	00.05015432 07
14	99.83834096 90	00.05401234 53
15	99.76812660 66	00.05787036 99
B 16	99.97169267 44	00.06172839 48
17	99.90147840 60	00.06558641 94
18	99.83126473 76	00.06944444 40
19	99.76104986 92	00.07330246 86
B 20	99.96461584 34	00.07716049 38
40	99.92923068 68	00.15432098 76
60	99.89384653 02	00.23148148 14
80	99.85846237 36	00.30864197 52
100	99.82707821 70	00.38580246 90
200	99.64615643 40	00.77160493 80
300	99.46923465 10	01.15740740 70
400	99.29231286 80	01.54320987 60
500	99.11539108 50	01.92901234 50
600	98.93846930 20	02.31481481 40
700	98.76154751 90	02.70061728 30

The Suns mean Anomaly and Præc. of the Æqui. in Days.

D	☉ Anomaly	Præcef. Æquinox
1	0.2737802348	0.0000105699
2	0.5475604697	0.0000211398
3	0.8213407046	0.0000317097
4	1.0951209395	0.0000422797
5	1.3689011744	0.0000528496
6	1.6426814092	0.0000634195
7	1.9164616441	0.0000739894
8	2.1902418790	0.0000845593
9	2.4640221139	0.0000951292
10	2.7378023488	0.0001056993
11	3.0115825836	0.0001162692
12	3.2853628184	0.0001268391
13	3.5591430532	0.0001374090
14	3.8329232880	0.0001479789
15	4.1067035228	0.0001585488
16	4.3804837576	0.0001691187
17	4.6542639924	0.0001796886
18	4.9280442272	0.0001902585
19	5.2018244620	0.0002008284
20	5.4756046976	0.0002113986
21	5.7493849324	0.0002219685
22	6.0231651672	0.0002325384
23	6.2969454020	0.0002431083
24	6.5707256368	0.0002536782
25	6.8445058716	0.0002642481
26	7.1182861064	0.0002748180
27	7.3920663412	0.0002853879
28	7.6658455766	0.0002959580
29	7.9396258115	0.0003065279
30	8.2134070464	0.0003170979
31	8.4871872813	0.0003276678

The Suns mean Anomaly and Præc. of the Æqui. in Hours

D	☉ mean Anomaly	Præcess. Æquinox
1	0.0114075097	0.0000004404
2	0.0228150195	08808
3	0.0342225293	13212
4	0.0456300391	17616
5	0.0570375489	22020
6	0.0684450587	0.0000026424
7	0.0798525684	30828
8	0.0912600782	35232
9	0.1026675881	39636
10	0.1140750978	44041
11	0.1254826075	0.0000048445
12	0.1368901174	0.0000052849
13	0.1482976271	57253
14	0.1597051368	61657
15	0.1711126465	66061
16	0.1825201562	70465
17	0.1939276659	74869
18	0.2053351761	0.0000079272
19	0.2167426858	83677
20	0.2281501955	88081
21	0.2395577052	92485
22	0.2509652149	96889
23	0.2623727246	101293
24	0.2737802348	0.0000105698

THE TABLES OF THE MOONS MEAN MOTIONS.

The Moons mean Longitude and Apogeon

Æra	☾ Mean Longitude	☾ Apogæon
Chr.	34.0088734567	78.8286265432
1600	02.0644290122	63.5892746911
1620	39.1651134566	89.6540895059
1640	76.2658079010	15.7189033207
1660	13.3665023454	41.7837191355
1680	50.4671967898	67.6485339503
1700	87.5677912342	93.9133487651
1620	29.6685801230	19.9781635799
1740	61.7692801230	46.0429783947
1760	98.8699745674	72.1077932095
	☾ Mean Long. in Mon.	☾ Apogæon in Mont.
Janu.	13.4633984897	00.9593447922
Febr.	15.9484670933	01.8258497658
Mar.	29.4098665830	02.7851945580
April	39.2131554440	03.7135927440
May.	52.6765539337	04.6729375362
June	62.4798427947	05.6013357222
July	75.9432412844	06.5606805144
Aug.	89.4066397741	07.5200253066
Sept.	99.2099286451	08.4484234926
Octo.	12.6733271348	09.4077682848
Nov.	22.4766159958	10.3361664708
Dec.	35.9400144893	11.2955112636

The Moons mean Anomaly and Node Retrograde

Æra	☾ Mean Anomaly	☾ Node Retrograde
Chr.	55.1802469135	74.6984567901
1600	38.4751543211	78.2198302468
1620	49.5110239507	70.7638117283
1640	60.5469035803	63.3077932098
1660	71.5827832099	55.8517746913
1680	82.6186628395	48.3957561728
1700	93.6545424691	40.9397376543
1720	04.6904220987	33.4837191358
1740	15.7263017283	26.0277006173
1760	26.7621813579	18.5716820988
	☾ Mean Ano. in Mon.	Node Ret. in Mont.
Jann.	12.5040536975	00.4559979224
Febr.	14.7206183275	00.8678670136
Mar.	27.2246720250	01.3238649360
April	35.4995627000	01.7651532480
May.	48.0036163975	02.2211511704
June	56.8785070725	02.6624394824
July	69.3825607700	03.1184374048
Aug.	81.8866144675	03.5744353272
Sept.	90.7615051425	04.0157236392
Octo.	03.2655588400	04.4717215616
Nov.	12.1404495150	04.9130098736
Dec.	24.6445032256	05.3690078960

The Moons mean Motions in Julian Years.

	☾ Mean Longitude	☾ Apogæon
1	35.9400144893	11.2955112636
2	71.8800289786	22.5910225272
3	07.8200434679	33.8865337908
B 4	47.4201388888	45.2129629629
5	83.3601533781	56.5084742265
6	19.3001678674	67.8039854901
7	55.2401823567	79.0994967537
B 8	91.8402777777	90.4259259258
9	30.7802922670	01.7214371894
10	66.7203067563	13.0169484530
11	02.6603212456	24.3124597166
B 12	42.2604166666	35.6388888888
13	78.2004311559	46.9344001524
14	14.1404456652	58.2299114160
15	50.0804601545	69.5254226706
B 16	89.6805555555	80.8518518518
17	25.6205700448	91.1473631154
18	61.5605845341	02.4428743790
19	97.5005990234	13.7383856426
B 20	37.1006944124	26.0648148148
40	74.2013888888	52.1296296296
60	11.3020833333	78.1944444434
80	48.4027777779	04.2592592592
100	85.5034722222	30.3240740740
200	71.0069444444	60.6481481481
300	56.5104166666	90.9722222202
400	42.0138888888	21.2962962962
500	27.5173611111	51.6203703702
600	13.0208333333	81.9444444442
700	98.5243055555	12.2685185182

The Moons mean Motions in Julian Years

	☾ Mean Anomaly	☾ Nodes Retrograde
1	24.6445032256	05.3690078260
2	49.2890064512	10.7380156520
3	73.9335096768	16.1070234780
B 4	02.2071759259	21.4912037037
5	26.8516791515	26.7602115297
6	51.4951823771	32.1292193557
7	76.1396856027	37.4982271817
B 8	04.4143518518	42.9824074074
9	29.0588550774	48.3514152334
10	53.7033583030	53.7204230594
11	78.3478615286	59.0894308854
B 12	06.6215277777	64.4736111111
13	31.2660310033	69.8426189371
14	55.9105342289	74.2116267631
15	80.5550374545	79.5806345891
B 16	08.8287037037	85.9648148148
17	33.4732069293	91.3338226408
18	58.1177101549	96.7028304668
19	82.7622133805	02.0718382928
B 20	11.0958796297	07.4560185185
40	22.0717592594	14.9120370370
60	33.1076388891	22.3680555555
80	44.1435185188	29.8240740740
100	55.1793981487	37.2800925925
200	110.3587962974	74.5601851850
300	165.5381944462	111.8402777775
400	220.7175925948	149.1203703700
500	275.8969907435	186.4004629629
600	331.0763888922	223.6805555555
700	386.2557870409	260.9606481480

The Moons mean Motions in Days.

Days	☾ Mean Longitude	☾ Apogæon
1	03.6601096287	00.0309466062
2	07.3202192574	00.0618932124
3	10.9803288861	00.0928398186
4	14.6404385148	00.1237864248
5	18.3005481435	00.1547330310
6	21.9606577722	00.1856796372
7	25.6207674009	00.2166262434
8	29.2808770296	00.2475728496
9	32.9409866583	00.2785194558
10	36.6010962870	00.3094660620
11	40.2612059157	00.3404126682
12	43.9213155444	00.3713592744
13	47.5814251731	00.4023058806
14	51.2415348018	00.4332524868
15	54.9016444305	00.4641990930
16	58.5617540592	00.4951456992
17	62.2218636879	00.5260923054
18	65.8819733166	00.5570389116
19	69.5420829453	00.5879855178
20	73.2021925740	00.6189321240
21	76.8623022037	00.6498787302
22	80.5224118314	00.6808233364
23	84.1825214601	00.7117719426
24	87.8426310898	00.7427185488
25	91.5027407175	00.7736651550
26	95.1628503462	00.8046117612
27	98.8229599749	00.8355583674
28	02.4830696036	00.8665049736
29	06.1431792323	00.8974515798
30	09.8032888610	00.9283981860
31	13.4633984897	00.9593447922

The Moons mean Motions in Days.

Days	☾ Mean Anomaly	☾ Node Retrograde
1	03.6291630225	00.0147096104
2	07.2583260450	00.0294192208
3	10.8874890675	00.0441288312
4	14.5166520900	00.0588384416
5	18.1458151125	00.0735480520
6	21.7749781350	00.0882576624
7	25.4041411575	00.1029672728
8	29.0333041800	00.1176768832
9	32.6624672025	00.1323864936
10	36.2916302250	00.1470961040
11	39.9207932475	00.1618057144
12	43.5499562700	00.1765153248
13	47.1791192925	00.1912249352
14	50.8082823150	00.2059345456
15	54.4374453375	00.2206441560
16	58.0666083600	00.2353537664
17	61.6957713825	00.2500633768
18	65.3249344050	00.2647729872
19	68.9540974275	00.2794825976
20	72.5832604500	00.2941922080
21	76.2124234725	00.3089018184
22	79.8415864950	00.3236114288
23	83.4707495175	00.3383210392
24	87.0999125400	00.3530306496
25	90.7290755625	00.3677402600
26	94.3582385850	00.3824498704
27	97.9874016075	00.3971594808
28	01.6165646300	00.4118690912
29	05.2457276525	00.4265787016
30	08.8748906750	00.4412883120
31	12.5040536975	00.4559979224

The Moons mean Motions in Hours.

Hours	☾ Mean Longitude.	☾ Apogæon
1	00.15250456 78	00.00128944 19
2	00.30500913 57	00.00257888 38
3	00.45751370 35	00.00386832 57
4	00.61001827 13	00.00415776 76
5	00.76252283 91	00.00644720 95
6	00.91502740 71	00.00773665 15
7	01.06753197 49	00.00902609 34
8	01.22003654 27	00.01031553 53
9	01.37254111 05	00.01160497 72
10	01.52504567 86	00.01289441 92
11	01.67755024 64	00.01418386 11
12	01.83005481 43	00.01547330 31
13	01.98255938 21	00.01676274 50
14	02.13506394 99	00.01805218 69
15	02.28756851 77	00.01934162 88
16	02.44007308 55	00.02063107 07
17	02.59257765 33	00.02192051 26
18	02.74508222 11	00.02320995 45
19	02.89758678 91	00.02449939 64
20	03.05009135 69	00.02578883 84
21	03.20259592 50	00.02707828 03
22	03.35510049 28	00.02836772 22
23	03.50760506 07	00.02965716 42
24	03.66010962 85	00.03094660 61

The Moons mean Motions in Hours.

Hours	☾ Mean Anomaly	☾ Node Retrograde
1	00.1512151259	00.0006129004
2	00.3024302518	00.0012258008
3	00.4536453778	00.0018387013
4	00.6048605037	00.0024516017
5	00.7560756296	00.0030645021
6	00.9072907556	00.0036774026
7	01.0585058815	00.0042903030
8	01.2097210074	00.0049032034
9	01.3609361333	00.0055161038
10	01.5121512593	00.0061290043
11	01.6633663852	00.0067419047
12	01.8145815112	00.0073548052
13	01.9657966371	00.0079677056
14	02.1170117630	00.0085806060
15	02.2682268889	00.0091935064
16	02.4194420148	00.0098064068
17	02.5706571407	00.0104193072
18	02.7218722666	00.0110722076
19	02.8730873926	00.0116451081
20	03.9243025185	00.0122580085
21	03.1755176445	00.0128709090
22	03.3267327704	00.0134838094
23	03.4779478964	00.0140967099
24	03.6291630223	00.0147096103

The Moons mean Motions in Minutes of an Hour

M.	☾ M. Long.	☾ Apog.	☾ M. An.	☊ Retrog.
1	.0025414	.0000214	.0025202	.0000102
2	.0050828	.0000429	.0050405	.0000204
3	.0076242	.0000643	.0075607	.0000306
4	.0101656	.0000859	.0100810	.0000408
5	.0127070	.0001074	.0126012	.0000510
6	.0152484	.0001288	.0151214	.0000612
7	.0177898	.0001502	.0176416	.0000714
8	.0203312	.0001716	.0201618	.0000816
9	.0228726	.0001930	.0226820	.0000918
10	.0254141	.0002149	.0252025	.0001021
11	.0279555	.0002363	.0277227	.0001123
12	.0304969	.0002577	.0302429	.0001225
13	.0330383	.0002791	.0327631	.0001327
14	.0355797	.0003004	.0352833	.0001429
15	.0381211	.0003218	.0378035	.0001531
16	.0406624	.0003432	.0403237	.0001633
17	.0432038	.0003646	.0428439	.0001735
18	.0457452	.0003860	.0453641	.0001837
19	.0482867	.0004079	.0478843	.0001939
20	.0508284	.0004298	.0504045	.0002041
21	.0533696	.0004512	.0529247	.0002143
22	.0559110	.0004726	.0554449	.0002245
23	.0584524	.0004940	.0579651	.0002347
24	.0609938	.0005154	.0604853	.0002442
25	.0635352	.0005368	.0630055	.0002544
26	.0660766	.0005582	.0655257	.0002642
27	.0686180	.0005795	.0680459	.0002744
28	.0711594	.0006008	.0705661	.0002846
29	.0737008	.0006222	.0730863	.0002948
30	.0762422	.0006437	.0756075	.0003064

The Moons mean Motions in Seconds.

	☾ M.Long.	☾ Apog.	☾ M. An.	☊ Retrog.
1	0000423	0000003	0000420	0000002
2	0000847	0000007	0000840	0000003
3	0001270	0000010	0001260	0000005
4	0001693	0000013	0001680	0000006
5	0002116	0000016	0002100	0000009
6	0002539	0000019	0002520	0000010
7	0002969	0000022	0002940	0000012
8	0003392	0000025	0003360	0000013
9	0003815	0000028	0003780	0000015
10	0004275	0000035	0001200	0000017
11	0004658	0000038	0004620	0000019
12	0005078	0000041	0005040	0000020
13	0005504	0000044	0005460	0000022
14	0005930	0000047	0005880	0000023
15	0006357	0000050	0006300	0000025
16	0006784	0000053	0006720	0000027
17	0007207	0000056	0007140	0000028
18	0007630	0000059	0007560	0000029
19	0008050	0000062	0007980	0000031
20	0008470	0000065	0008400	0000033
21	0008893	0000068	0008820	0000035
22	0009316	0000071	0009240	0000036
23	0009736	0000074	0009660	0000038
24	0010156	0000077	0010080	0000039
25	0010582	0000080	0010500	0000041
26	0011008	0000083	0010920	0000043
27	0011434	0000086	0011340	0000044
28	0011860	0000089	0011760	0000047
29	0012287	0000092	0012180	0000049
30	0012714	0000095	0012600	0000051

☉	Sig. 0. &. 6		1 & 7		2 & 8		☉
♎	Æqu. ☊ Addi	Inclin. limitis	Æqu. ☊ Addi	Inclin. limitis	Æqu. ☊ Addi	Inclin. limitis	♎
0	0.00000	30000	1.06500	20000	1.08500	15000	30
1	0.00000	30000	1.12888	25722	1.01888	14527	29
2	0.00055	30000	1.19277	25472	0.95305	14055	28
3	0.00194	29972	1.25222	25166	0.88666	13583	27
4	0.00416	29944	1.30833	24888	0.82055	13138	26
5	0.00888	29888	1.36166	24583	0.75333	12666	25
6	0.01472	29833	1.41055	24277	0.68694	12194	24
7	0.02305	29777	1.45666	23972	0.62416	11722	23
8	0.02416	29722	1.49916	23638	0.56444	11250	22
9	0.04805	29638	1.53666	23305	0.50555	10750	21
10	0.06500	29555	1.58000	22972	0.44666	10250	20
11	0.08555	29444	1.60027	22638	0.38888	09750	19
12	0.10944	29361	1.62527	22277	0.34000	09250	18
13	0.13666	29250	1.64472	21916	0.28972	08750	17
14	0.16833	29111	1.65771	21555	0.25083	08250	16
15	0.20250	28972	1.66277	21222	0.20972	07750	15
16	0.24111	28833	1.65805	20833	0.17388	07250	14
17	0.27472	28667	1.64527	20444	0.14138	06750	13
18	0.32944	28567	1.62638	20055	0.11305	06250	12
19	0.37916	28361	1.60194	19666	0.08805	05722	11
20	0.43277	28194	1.58222	19277	0.06722	05222	10
21	0.48888	18027	1.53972	18861	0.04916	04694	9
22	0.54833	27833	1.50333	18444	0.03416	04166	8
23	0.60694	27611	1.46222	18027	0.02333	03638	7
24	0.66833	27416	1.41777	17611	0.01500	03138	6
25	0.73555	27194	1.37027	17194	0.00888	02611	5
26	0.79805	26972	1.31861	16750	0.00416	02083	4
27	0.86416	26722	1.26527	16333	0.00194	01555	3
28	0.93083	26500	1.20833	15888	0.00055	01027	2
29	0.99611	26250	1.14750	15444	0.00000	00527	1
30	1.06500	26000	1.08500	15000	0.00000	00000	0
	Subtract.		Subtract.		Subtract.		
	Sig. 5. & 11		4 & 10		3 & 9		

Aug.	Sig. 0. N.	Incr.	1. North	Incr.	2. North	Incr.	
Lat.	Sig. 6. S. Latit.	or Exc.	7. South Latit.	or Exc.	8. South Latit.	or Exc.	
0	0.00000	00000	2.49750	15000	4.32888	26000	30
1	0.08722	00527	2.57277	15444	4.37166	26250	29
2	0.17444	01027	2.64722	15888	4.41361	26500	28
3	0.26166	01555	2.72083	16333	4.45388	26722	27
4	0.34861	02083	2.79361	16416	4.49277	26972	26
5	0.43555	02611	2.86555	17194	4.53055	27194	25
6	0.52222	03138	2.93638	17611	4.56666	27416	24
7	0.60888	03638	3.00666	18027	4.60166	27611	23
8	0.69527	04166	3.07583	18444	4.63500	27833	22
9	0.78138	04694	3.14416	18861	4.66722	28027	21
10	0.86722	05222	3.21166	19277	4.69777	28194	20
11	0.95277	05722	3.27805	19666	4.72694	28361	19
12	1.03833	06250	3.34333	20055	4.75472	28527	18
13	1.12333	06750	3.41055	20444	4.78111	28666	17
14	1.20805	07250	3.47111	20833	4.80583	28833	16
15	1.29250	07750	3.53333	21222	4.82916	28972	15
16	1.37666	08250	3.59444	21555	4.85111	29111	14
17	1.46027	08750	3.65472	21916	4.87166	29250	13
18	1.54333	09250	3.71361	22277	4.89055	29361	12
19	1.62611	09750	3.77138	22638	4.90777	29444	11
20	1.70805	10250	3.82833	22972	4.92388	29555	10
21	1.78972	10750	3.88388	23305	4.93833	29638	9
22	1.87111	11250	3.93805	23638	4.95111	29722	8
23	1.95166	11722	3.99138	23972	4.96250	29777	7
24	2.03166	12194	4.04333	24277	4.97250	29833	6
25	2.11083	12666	4.09416	24583	4.98083	29888	5
26	2.18972	13138	4.14361	24888	4.98777	29944	4
27	2.26777	13583	4.19166	25166	4.99301	29972	3
28	2.34500	14055	4.23861	25472	4.99694	29972	2
29	2.42166	14527	4.28444	25722	4.99916	30000	1
30	2.49750	15000	4.32888	26000	5.00000	30000	0
	Sig. 11. S.		Sig. 10. S.		9. South		
	Sig. 3. N.		4 N.		3. North		

As Lat.	Sig. 0. 6 Red.	Incr.	S. 1. 7 Red.	Incr.	S. 2. 8 Red.	Incr.	As Lat.
0	.00000	00000	09444	01166	09472	01166	30
1	.00388	00055	09638	01194	09277	01138	29
2	.00750	00111	09805	01222	09055	01111	28
3	.01138	00166	09972	01250	08833	01083	27
4	.01527	00222	10111	01277	08611	01055	26
5	.01888	00250	10250	01277	08388	01027	25
6	.02277	00305	10388	01277	08138	01000	24
7	.02638	00333	10500	01305	07861	00972	23
8	.03000	00361	10583	01305	07611	00944	22
9	.03361	00416	10665	01305	07305	00916	21
10	.03722	00472	10750	01333	07027	00861	20
11	.04083	00527	10805	01333	06722	00833	19
12	.04444	00555	10861	01333	06416	00805	18
13	.04777	00611	10888	01333	06111	00777	17
14	.05111	00638	10916	01361	05805	00722	16
15	.05444	00666	10916	01361	05472	00666	15
16	.05777	00722	10916	01361	05138	00638	14
17	.06111	00777	10862	01361	04805	00611	13
18	.06416	00805	10861	01361	04444	00555	12
19	.06722	00833	10805	01333	04111	00527	11
20	.07000	00861	10750	01333	03750	00472	10
21	.07305	00916	10694	01305	03388	00416	9
22	.07583	00944	10611	01305	03027	00361	8
23	.07888	00972	10500	01305	02692	00333	7
24	.08111	01000	10388	01277	02611	00305	6
25	.08361	01027	10277	01277	02222	00250	5
26	.08583	01055	10138	01277	01527	00222	4
27	.08823	01083	10000	01250	01138	00166	3
28	.09055	01111	09833	01222	00750	00111	2
29	.09250	01138	09638	01194	00388	00055	1
30	.09444	01166	09472	01166	00000	00000	0
	11.5		10.4		9. 3		

A Table shewing the mean Motion of the Moon from the Sun in Years and Months.

Æra	☾ à ☉ in Years		☾ à ☉ in Years
Chr.	56.8114797531	1	36.0063707331
1600	21.5296732464	2	72.0127414663
1620	58.5795367034	3	08.0191121993
1640	95.6384101604	4	47.4117836215
1660	32.6972836174	5	83.4181543546
1680	69.7561560744	6	19.4245250877
1700	06.8150365314	7	55.4308958208
1720	43.8739039884	8	94.8235672430
1740	80.9027774454	9	40.8298373761
1760	17.9916509024	10	76.8361087092
	Motion of the Moon from the Sun in Months.	11	02.8426794423
		12	42.2353508645
		13	78.2417215976
		14	14.2480923307
		15	50.2544630638
Jan.	04.9758835440	16	89.6471344860
Feb.	99.7928106160	17	25.6535052191
Mar.	04.7686941680	18	61.6598759522
April	06.3582588800	19	97.6662466853
		20	37.0589181075
May	11.3541424240	40	74.1178362150
June	12.9237071440	60	11.1767543225
July	17.8995906880	80	48.2356724300
Aug.	22.8754742320	100	85.2945905375
		200	70.5891810750
Sept.	24.4650389520	300	55.8837716125
Octo.	29.4409224960	400	41.1783621500
Nov.	31.0304872160	500	26.4729526875
Dec.	36.0063707331	600	11.7675432250
		700	97.0621337625

A Table shewing the mean Motion of the Moon from the Sun in Days and Hours.

	☾ ☉ in Days.		☾ ☉ in Hours.
1	03.3863188240	1	00.1410966176
2	06.7726376480	2	00.2821932352
3	10.1589564720	3	00.4232898520
4	13.5452752960	4	00.5643864706
5	16.9315941200	5	00.7054830882
6	20.3179129440	6	00.8465797060
7	23.7042317680	7	00.9876763236
8	27.0905505920	8	01.1287729412
9	30.4768694160	9	01.2698695588
10	33.8631882400	10	01.4109661766
11	37.2495070640	11	01.5520627942
12	47.6358258880	12	01.6931594120
13	44.0221447720	13	01.8342560296
14	47.4084635360	14	01.9753526472
15	50.7947823600	15	02.1164492648
16	54.1811011840	16	02.2575458824
17	57.5674200080	17	02.3986425000
18	60.9537388320	18	02.5397391176
19	64.3400576560	19	02.6808357354
20	67.7263764800	20	02.8219323520
21	71.1126953040	21	02.9630289708
22	74.4990141280	22	03.1041255884
23	77.8853329520	23	03.2452222062
24	81.2716517760	24	03.3863188240
25	84.6579706000		
26	88.0442894240		
27	91.4306082480		
28	94.8169270720		
29	98.2032458960		
30	01.5895647200		
31	04.9758835440		

A Table shewing the mean Motion of the Moon from the Sun in Minutes.

	☾ ☌ ☉ in Minutes.		☾ ☌ ☉ in Minutes.
1	00.0023516102	31	00.0728999183
2	00.0047032205	32	00.0752515088
3	00.0070548308	33	00.0776031390
4	00.0094064411	34	00.0799547492
5	00.0117580513	35	00.0823063591
6	00.0141096617	36	00.0846579696
7	00.0164612719	37	00.0870095798
8	00.0188128822	38	00.0893611900
9	00.0211644924	39	00.0917128002
10	00.0235161029	40	00.0940644104
11	00.0258677131	41	00.0964160206
12	00.0282193233	42	00.0987676308
13	00.0305709335	43	00.1011192410
14	00.0329225437	44	00.1034708512
15	00.0352741539	45	00.1058224614
16	00.0376257641	46	00.1081740716
17	00.0399773746	47	00.1105256818
18	00.0423289848	48	00.1128772920
19	00.0446805950	49	00.1152289022
20	00.0470322052	50	00.1175805124
21	00.0493838154	51	00.1199321226
22	00.0517354256	52	00.1222837328
23	00.0540870358	53	00.1246353430
24	00.0564386460	54	00.1269869532
25	00.0587902562	55	00.1293385634
26	00.0611418664	56	00.1316901736
27	00.0634934766	57	00.1340417838
28	00.0658450868	58	00.1363933940
29	00.0681966970	59	00.1387450050
30	00.0705483080	60	00.1410966152

A Table shewing the mean Motion of the Moon from the Sun in Seconds.

	☾ − ☉ in Seconds		☾ − ☉ in Seconds
1	00.0000391955	31	00.0012149985
2	00.0000783870	32	00.0012541920
3	00.0001175805	33	00.0012933855
4	00.0001567740	34	00.0013325790
5	00.0001959675	35	00.0013717725
6	00.0002351610	36	00.0014109660
7	00.0002743545	37	00.0014501595
8	00.0003135480	38	00.0014893530
9	00.0003527415	39	00.0015285465
10	00.0003919350	40	00.0015677400
11	00.0004311285	41	00.0016069335
12	00.0004703220	42	00.0016461270
13	00.0005095155	43	00.0016853205
14	00.0005487090	44	00.0017245140
15	00.0005879025	45	00.0017637075
16	00.0006270960	46	00.0018029010
17	00.0006662895	47	00.0018420945
18	00.0007054830	48	00.0018812880
19	00.0007446765	49	00.0019204815
20	00.0007838700	50	00.0019596750
21	00.0008230635	51	00.0019988685
22	00.0008622570	52	00.0020380620
23	00.0009014505	53	00.0020772555
24	00.0009406440	54	00.0021164490
25	00.0009798375	55	00.0021556425
26	00.0010190310	56	00.0021948360
27	00.0010582245	57	00.0022340295
28	00.0010974180	58	00.0022732230
29	00.0011366115	59	00.0023124165
30	00.0011758050	60	00.0023516100

A Catalogue of some of the most notable fixed Stars according to the observations of *Tycho Brahe*, and by him rectified to the beginning of the Year of Mans Redemption, 1601.

The Names of the Stars	Longit.	Latit.
The first Star of Aries.	07.671 ♈	7.8. N 4
The bright Star in the top of the head of Aries.	20.583 ♉	9.57.N 3
The South Eye of Taurus.	01.169 ♉	5.31. S 1
The North Eye of Taurus.	00.801 ♊	5.31. S 1
The bright Star of the Pleiades.	06.620 ♉	2. 6. S 3
The higher head of Gemini.	04.078 ♋	4.11.N 5
The lower head of Gemini.	04.921 ♋	10.2.N 2
The bright foot of Gemini.	01.059 ♋	6.38.N 2
In the South Arm of Cancer.	02.238 ♌	6.48. S 2
The bright Star in the neck of Leo.	06.662 ♌	5.8. S 3
The heart of Leo.	06.745 ♌	8.47.N 2
In the extream of the tail of Leo.	04.458 ♍	0.26.N 1
In Virgo's *Wing*; Vindemiatrix.	01.217 ♎	12.18.N 1
Virgins Spike.	05.074 ♎	16.15.N 3
South Ballance.	02.643 ♏	1.59. S 1
North Ballance.	03.833 ♎	0.26. N 2
The highest in the Forehead of Scorpio.	07.388 ♏	8.35. N 2
The Scorpions heart.	01.171 ♐	1.05. N 3
Former of the 3 in the head of Sagittarius.	02.203 ♑	4.27. S 1
Northern in the former horn of Capricorn.	07.861 ♑	1.24. N 4
The left Shoulder of Aquarius.	04.949 ♒	7.22. N 3
In the mouth of the South Fish.	03.620 ♓	8.42.N 3
The Polar Star or last Star in the tail of the lesser Bear.	05.400 ♑	9.4. N 5 66.02.N 2

The Names of the Stars.	Longit.		Latit.		
The last Star in the tail of the great Bear,	05.888	♍	54.25.N	2
The Tongue of the Dragon.	05.259	♍	76.17.N	4
Arcturus *in the skirt of his Garment.*	05.181	♎	31.2. N	1
The bright Star of the North Crown.	01.845	♍	44.23.N	2
The Head of Hercules.	02.921	♐	37.23.N	3
The bright Star of the Harp.	03.699	♑	61.47.N	1
The Head of Medusa.	05.727	♉	22.22.N	3
The bright Star in the Goats left Shoulder.	04.518	♊	22.50.N	1
The middle of the Serpents Neck.	04.583	♏	25.35.N	2
The bright Star in the Eagles Shoulder.	07.264	♈	29.21.N	2
The bright Star in the Dolphins Tail.	02.370	♒	29. 8.N	3
The mouth of Pegasus.	07.984	♓	22. 7.N	3
The head of Andromeda.	01.410	♈	25.42.N	2
In the top of the Triangle.	00.366	♉	26.49.N	4
In the Snout of the Whale.	02.643	♉	7.50. S	
The bright Star in the Whales Tail.	07.481	♓	20.47. S	2
Bright Shoulder of Orion.	06.444	♊	16.06 S	2
Middlemost in the belt of Orion.	04.972	♊	24.33. S	2
The last in the tail of the Hen.	07.324	♊	38.26. S	4
The great Dogs mouth Sirius.	02.386	♋	38.30. S	1
The lesser Dog Procyon.	03.641	♋	16.57. S	2
In the top of the Ships Stern.	01.636	♋	43.18. S	3
Brightest in Hydra's *Heart.*	06.044	♌	22.24. S	1

FINIS.

THE

CONTENTS

OF THE

First Part,

CONTAINING

The Practical Geometry or the Art of Surveying.

CHapter 1. *Of the Definition and Division of Geometry.*
Chap. 2. *Of Figures in the General, more particularly of a Circle and the Affections thereof.*
Chap. 3. *Of Triangles.*
Chap. 4. *Of Quadrangular and Multangular Figures.*
Chap. 5. *Solid Bodies.*

The Contents.

Chap. 6. *Of the measuring of Lines both Right and Circular.*

Chap. 7. *Of the measuring of a Circle.*

Chap. 8. *Of the measuring of plain Triangles.*

Chap. 9. *Of the measuring of Heights and Distances.*

Chap. 10. *Of the taking of Distances.*

Chap. 11. *How to take the Plot of a Field at one Station, &c.*

Chap. 12. *How to take the Plot of a Wood, Park, or other Champian Plane, &c.*

Chap. 13. *The Plot of a Field being taken by an Instrument, how to compute the Content thereof in Acres, Roods, and Perches.*

Chap. 14. *How to take the Plot of mountainous and uneven Ground, &c.*

Chap. 15. *To reduce Statute measure into*

into Customary, and the contrary.
Chap. 16. *Of the measuring of solid Bodies.*

Tables.

Page.
A Table of Squares. 99
A Table for the Gauging of Wine Vessels. 114
A Table for the Gauging of Beer and Ale Vessels. 120
A Table shewing the third part of the Areas *of Circles, in Foot measure and Decimal parts of a Foot.* 132
A Table shewing the third part of the Area *of any Circle in Foot measure, not exceeding 10 f. circumf.* 136
A Table for the speedy finding of the length or Circumference answering to any Arch in Degrees and Decimal parts. 151
A Common Divisor for the speedy con-

converting of the Table, shewing the Areas of the Segments of a Circle whose Diameter is 2 &c. 154

A Table shewing the Ordinates, Arches, and Areas of the Segments of a Circle, whose Diameter is 1 &c. 156

The Contents of the Second Part of this Treatise, of the Doctrine of the PRIMUM MOBILE.

CHap. 1. *Of the General Subject of Astronomy.*

Chap. 2. *Of the Distinctions and Affections of Spherical Lines and Arches.*

Chap. 3. *Of the kind and parts of Spherical Triangles, and how to project the same upon the Plane of the Meridian.*

Chap. 4. *Of the solution of Spherical Triangles.*

Chap. 5. *Of such Spherical Problems as are of most general Use in the Doctrine of the* Primum Mobile, &c.

The Contents of the Third Part of this Treatise, being an Account of the Civil Year with the reason of the difference between the Julian and Gregorian Calendars, and the manner of Computing the Places of the Sun and Moon.

Chap. 1. *Of the Year Civil and Astronomical.*

Chap. 2. *Of the Cycle of the Moon, what it is, how placed in the Calendar, and to what purpose.*

Chap. 3. *Of the use of the Golden Number in finding the Feast of Easter.*

Chap. 4. *Of the Reformation of the Calen-*

Calendar by Pope Gregory *the Thirteenth*, &c.

Chap. 5. *Of the Moons mean Motion and how the Anticipation of the New Moons may be discovered by the Epacts.*

Chap. 6. *To find the Dominical Letter and Feast of Easter, according to the Gregorian Account.*

Chap. 7. *How to reduce Sexagenary Numbers into Decimals, and the contrary.*

Chap. 8. *Of the difference of Meridians.*

Chap. 9. *Of the Theory of the Suns or Earths motion.*

Chap. 10. *Of the finding of the Suns Apogæon, quantity of Excentricity and middle Motion.*

Chap. 11. *Of the quantity of the tropical and sydereal Year.*

Chap. 12. *Of the Suns mean Motion otherwise stated.*

Chap.

Chap. 13. *How to calculate the Suns true place by either of the Tables of middle Motion.*

Chap. 14. *To find the place of the fixed Stars.*

Chap. 15. *Of the Theory of the Moon and the finding the place of her Apogæon, quantity of Excentricity, and middle motion.*

Chap. 16. *Of the finding of the place and motion of the Moons Nodes.*

Chap. 17. *How to calculate the Moons true place in her Orbs.*

Chap. 18. *To compute the true Latitude of the Moon, and to reduce her place from her Orbit to the Ecliptick.*

Chap. 19. *To find the mean Conjunctions and Oppositions of the Sun and Moon;*

The

The Fourth Part, or an Introduction to Geography.

Chap. 1. Of *the Nature and Division of Geography.*
Chap. 2. Of *the Distinction or Dimension of the Earthly Globe by Zones and Climates.*
Chap. 3. Of Europe.
Chap. 4. Of Asia.
Chap. 5. Of Africk.
Chap. 6. Of America.
Chap. 7. Of *the description of the Terrestrial Globe, by Maps Universal and Particular.*

A Table of the view of the most notable *Epochas.*

Page.
The Julian Calendar. 461
The Gregorian Calendar. 466

Page.

A Table to convert Sexagenary Degrees and Minutes into Decimals and the contrary. 476

A Table converting hours and minutes into degrees and minutes of the Æquator. 480

A Table of the Longitudes and Latitudes of some of the most eminent Cities and Towns in England *and* Ireland. 482

A Table of the Suns mean Longitude and Anomaly in both Ægyptian and Julian Years, Months, Days, Hours and Minutes. 484

Tables of the Moons mean motion. 493

A Catalogue of some of the most notable fixed Stars, according to the observation of Tycho Brahe, *rectified to the year* 1601. 511

Books

Books Printed for and sold by Thomas Passinger *at the* Three Bibles *on the middle of* London-Bridge.

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